Working papers series

WP ECON 06.22

The Optimal Degree of Exchange Rate Flexibility: A Target Zone Approach

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JEL Classification Numbers: E52, F31, F33.
Keywords: Target Zones, Exchange Rate Agreements, Monetary Policy, Time Consistency.

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August 30th 2006

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Abstract: This paper presents a benchmark model that rationalizes the choice of the degree of exchange rate flexibility. We show that the monetary authority may gain efficiency by reducing volatility of both the exchange rate and the interest rate at the same time. Furthermore, the model is consistent with some known stylized facts in the empirical literature on target zones that previous models were not able to generate jointly, namely, the positive relation between the exchange rate and the interest rate differential, the degree of non-linearity of the function linking the exchage rate to fundamentals and the shape of the exchange rate stochastic distribution.

Keywords: Target zones, exchange rate agreements, monetary policy, time consistency.
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Acknowledgements: We are particularly indebted to the editor, Menzie Chinn, and two anonymous referees for very useful comments and suggestions. We also thank all participants at the Workshop on Exchange Rates of centrA (Málaga, November 28th-29th 2001), and the Workshop on Macroeconomics of Universitat Autonoma de Barcelona (April 18th 2002) for helpful comments and suggestions. Specially, the comments from Consuelo Gámez, José García Solanes, Javier Pérez, Simón Sosvilla, Mark Taylor and José Luis Torres have been very valuable. All remaining errors are ours. Rodríguez-Mendizábal aknowledges financial support from the Spanish Ministry of Science and Technology through grant SEC2003-0036 and from the Barcelona Economics Program of XREA. Rodríguez-López aknowledges financial support from the Spanish Ministry of Science and Technology through grant SEC2003-04028/ECO. Both authors thank centrA for valuable financial aid.

1 Introduction

This paper analyzes the choice of the degree of flexibility allowed to the exchange rate. Recent work by Calvo and Reinhart [9], Reinhart [31], Reinhart and Rogoff [32], Fischer [14] and Levy-Yeyati and Sturzenegger [25] have concluded that many of the *de jure* free floaters strongly intervene to soften the fluctuations of the nominal exchange rate. This has been labeled as *fear of floating*. They also observed that the degree of intervention in the exchange market varies across countries so that different degrees of flexibility are allowed for different currencies.

We construct a target zone model to analyze the choice of exchange rate flexibility. We believe the target zone approach is useful for several reasons. First, many currencies in fact fluctuate or have fluctuated in an explicit target zone. This is the case of historical episodes like the EMS. It is also the case of the current ERM2. This system is working as a "hub-and-spokes" zone between the euro and those currencies of the European Union countries not participating in the Monetary Union (United Kingdom, Denmark and Sweden). Also, accession countries to the European Union have to commit to a target zone arrangement during the transition period. As we explain below, our paper contributes to this literature on explicit target zones.

Second, the choice of exchange rate flexibility can be viewed under a target zone perspective. In our view, such a flexibility can be approximated by the bandwidth where the exchange rate is allowed to fluctuate. Broad classes of exchange rate regimes can then be seen as special cases of this bandwidth. For instance, a pegged rate can be interpreted as a band of fluctuation with a zero width and a pure floating rate is equivalent to an infinite fluctuation band. This is, of course, a very crude simplification of the different exchange rate regimes. For example, it does not distinguish between different forms of hard pegs. However, it should be clear that we will not try to map every single exchange rate regime into a corresponding target zone. Instead, what we do is to construct a general model that summarizes the degree of exchange rate flexibility in a single parameter, i.e. the bandwidth. To the extent that broad classes of regimes differ in the degree of variability allowed to the exchange rate, we believe this simplification to be useful for the analysis and comparison of these broad classes of exchange systems. In this sense, it is important to notice that we use the term target zone in a broad sense to refer to any system that limits the movements of the exchange rate within a band of fluctuation either explicitly or implicitly.

As mentioned before, the *fear of floating* literature has pointed out that countries reveal a preference towards smoothing the dynamics of the exchange rate. Intermediate regimes seem to be defining the current world so that completely fixed or fully flexible rates (see Fischer [14]) are seldom observed. Pure fixed exchange rate regimes are rarely used since they compromise a large amount of monetary independence. A monetary commitment that pegs the exchange rate to a low inflation foreign currency can establish a disciplining effect that motivates a higher degree of credibility and price stability (see Giavazzi and

Pagano [19]). However, given the evidence on the choice of currency regimes by central banks, it seems that the gains from stability of fixed exchange rates have not been big enough to dampen the losses from less monetary independence (see Svensson [39] for a detailed discussion). On the other hand, pure free floating regimes are not observed either. Monetary authorities use to play *leaning against the wind* policies in an attempt to control the exchange rate around some target level, official or unofficially. Thus, it seems that hybrid regimes that limit exchange rate fluctuations have been widely used on the basis that it apparently reaps the benefits of both flexible and fixed exchange rates.

Krugman [23] presented the model that has become the standard tool to study target zone regimes. The dynamics of the exchange rate were derived from a linear asset pricing relation and an arbitrage condition given by the uncovered interest rate parity. His model assumed that the central bank intervenes so as to maintain the exchange rate within the band. In this paper, we extend Krugman's model in two directions: (i) we allow the central bank to perform intramarginal interventions, that is, to use its monetary policy to affect the exchange rate before it hits the limits of the band and, (ii) we introduce lack of credibility of the target zone, that is, the perception by market participants of the possibility that the central bank will not defend the band when it has to. Several authors (see Bertola and Caballero [6] and Bertola and Svensson [7]) have previously proposed the introduction of both extensions to improve the empirical fit of target zone models. However, to the best of our knowledge no such experiment has been conducted so far. We show that both features are sufficient to reconcile the model with data.[1] As a second issue, we use the model to rationalize the choice of the degree of flexibility of the exchange rate. It is shown that, by imposing a band of fluctuation for the exchange rate, either explicitly or implicitly, the monetary authority may gain efficiency through reducing volatility of both the exchange rate and the interest rate simultaneously.

The paper is organized as follows. Section 2 sets up the general model. Subsections 2.2 to 2.4 show how different exchange rate regimes can be seen as extreme cases of a target zone. Section 2.5 develops the target zone solution. Section 3 deals with the two questions outlined in the previous paragraph. The last section concludes.

2 Set up of the Model

2.1 General assumptions

The model represents a highly stylized dynamic stochastic general equilibrium economy. It can be thought of as a reduced form version of more complicated fully optimizing models. Time is discrete. First, consider an equation specifying

[1] Models with perfectly credible target zones have introduced intramarginal interventions by allowing some degree of mean reversion in the stochastic motion governing fundamentals (see Froot and Obstfeld [18] as well as Lindberg and Söderlind [28]). Mean reversion is then associated with *leaning against the wind* exchange practices. Bertola and Caballero [6] introduce the assumption of imperfect credibility but do not consider intramarginal interventions.

equilibrium in the money market:

$$m_t - p_t = \varphi y_t - \gamma i_t + \xi_t, \tag{1}$$

where m_t is money supply, p_t is the domestic price level of y_t, a tradable good, i_t is the domestic interest rate of a one-period bond, and ξ_t is some shock to money demand. These variables are all expressed in logs, with the exception of the interest rate. The parameters φ and γ are both positive: money demand increases with output because of a transaction motive and there is an implicit liquidity preference behavior, meaning that money can be a substitute for a bond that returns a nominal interest i_t.

Let x_t be the log of the nominal exchange rate, expressing the price of one unit of foreign currency in terms of domestic currency. The (log) real exchange rate is given by

$$q_t = x_t - p_t + p_t^*, \tag{2}$$

where p_t^* is the foreign price (variables with star will denote the foreign analogue).

Call $d_t \equiv i_t - i^*$, the interest rate differential, where i^* is the foreign (constant) rate, and assume perfect capital mobility, risk aversion and the uncovered interest rate parity condition (UIP)

$$d_t = E_t \left\{ x_{t+1} - x_t \right\} + r_t, \tag{3}$$

where E_t is the expectation operator conditional on the information available at time t. Thus, the expected rate of depreciation must compensate for the interest rate differential plus the foreign premium, r_t. We assume the variable r_t to be exogenous and governed by a first order Markov process

$$r_t = r_{t-1} + \varepsilon_t. \tag{4}$$

The white noise $\{\varepsilon_t\}$ is supposed to be Gaussian, $\varepsilon_t \sim N\left(0, \sigma^2\right)$, for convenience.[2]

Using (2), (3) and (1) one obtains

$$\begin{aligned} x_t &= f_t + \gamma E_t \left\{ x_{t+1} - x_t \right\}, \\ f_t &= m_t + v_t, \\ v_t &= \theta_t + \gamma r_t, \\ \theta_t &= q_t - p_t^* - \varphi y_t + \gamma i^* - \xi_t. \end{aligned} \tag{5}$$

[2] The foreign risk premium plays a key role in the current paper. In general, the UIP does not hold (see Ayuso and Restoy [2]). Conventional target zone models consider that deviations from UIP are negligible in target zones (see Svensson [37]). However, Bekaert and Gray [5] find that the risk premia in a target zone are sizable and should not be ignored. They argue that this might be the reason of why the credibility tests run on EMS at the beginning of the nineties failed to anticipate the 1992-93 turbulences. Alvarez, et al. [1] show from data on interest rates and exchange rates that variations in the interest rate differential are driven almost entirely by variations in the risk premium.

Thus, the exchange rate, as the price of any asset, depends linearly on fundamental variables affecting its value, f_t and on its own expected rate of change. The fundamental determinants of the exchange rate, f_t, amount to the policy variable, m_t, plus an exogenous process, v_t, which include variables like domestic output, prices, the risk premium, etc. Controlling the money supply, the central bank has control over total fundamentals and, therefore, the exchange rate. By iterating forward on x_t we have that

$$x_t = (1-\nu)\sum_{\tau=0}^{\infty}\nu^{\tau}E_t f_{t+\tau} = (1-\nu)\sum_{\tau=0}^{\infty}\nu^{\tau}E_t\left\{m_{t+\tau}+v_{t+\tau}\right\}, \tag{6}$$

with $\nu \equiv \gamma(1+\gamma)^{-1}$. The forward looking behavior from UIP implies that the value of asset x_t is the present discounted value of the future stream of fundamentals. Notice that $(1-\nu)\sum_{\tau=0}^{\infty}\nu^{\tau}=1$, thus, if $m_{t+\tau}$ were orthogonal to $v_{t+\tau}$ for all t and τ, the long run effect of an increase in m_t is to impulse x_t by an equal amount. We assume that both the central bank and traders can observe the realization of θ_t and r_t.

The central bank (henceforth, CB) preferences are modeled to evaluate the trade-off between interest rate variability *versus* exchange rate variability

$$J = \frac{1}{2}E_t\left\{\sum_{\tau=0}^{\infty}\beta^{\tau}\left[d_{t+\tau}^2+\lambda\left(x_{t+\tau}-c_t\right)^2\right]\right\}. \tag{7}$$

This objective function is intended to induce the fear of floating behavior described in the Introduction. The intuition is as follows. We think of a very short maturity term for the bond in the UIP, say a few days, a week or a month at most. The idea is that the CB controls some monetary aggregate $\{m_t\}$ to target the pair $\{d_t, x_t\}$. Output realizations and real fluctuations are observed with some delay, and not available by the time monetary policy is decided so the only available information at any period is $\{\theta_t, r_t\}$. From (3) and (5) it is easy to show that m_t will respond one to one to the shock θ_t and the problem reduces to deciding how to split the shock r_t between d_t and x_t. Thus, the CB just needs to choose d_t and x_t every period given the value of r_t to minimize (7) subject to (3), (4) and the exchange regime which restricts the policies available to the CB. Because of the nature of the problem, the central bank will smooth fluctuations in the exchange rate by channeling part of the variation in the risk premium to interest rates. Thus, the parameter λ in the function (7) will reflect how much the central bank fears the fluctuations in the exchange rate.[3]

In what follows, any of the exchange rate regimes can be characterized by the triple $\{\lambda, w, \alpha\}$. The first element is related to the preferences. The number w is the band width. The last term α is the probability that the CB defends the currency when it is outside the band and measures the credibility of the exchange

[3] Alvarez, Atkenson and Kehoe [1] show that time-varying risk premia is the primary force driving the link between interest rate differentials and exchange rates. Because one of the objectives of the paper is to reproduce the distribution of these two variables, we concentrate on risk premia as the driving exogenous force in the model.

regime. This triple will adopt particular values for any of the exchange regimes considered.

Our model shares some features with Svensson's [38]. For example, the objective function (7) can be seen as a simplified version to the one used by Svensson. However, Svensson does not impose the band restriction (*i.e.* the width w in our context) and approximates the nominal exchange rate solution with a linear function. On the contrary, we will show that the band restriction is a source of non linearity that should not be neglected. In combination with the degree of credibility and intramarginal interventions, the non linear solution is relevant to analyze how smooth is the response of the exchange rate to the fundamental at any point within the band. In addition, this width allows us to approximate the degree of flexibility allowed to the exchange rate.

2.2 The fixed exchange rate

Consider first a CB that can credibly commit to fix the exchange rate to $c_t = c$. This regime represents the extreme situation in which no flexibility is allowed to the exchange rate. It appears as a particular case of the target zone when $w = 0$ and $\alpha = 1$. Given the forward looking restriction of (3) and the pure random walk exhibited by r_t, the solution for d_t and x_t is

$$d_t = r_t \text{ and } x_t = c. \tag{8}$$

That is, under a fixed exchange rate regime the nominal interest rate absorbs the whole variability of r_t. The expected value of the game under this regime is

$$J^c(r_t) = \frac{1}{2}\left(\frac{1}{1-\beta}\right)\left[r_t^2 + \frac{\beta}{1-\beta}\sigma^2\right]. \tag{9}$$

The variability of the interest rate is a source of time inconsistency. If the CB could commit to a rule like (8), it should respond to the risk premium by moving the interest rate in the same magnitude so as to maintain the exchange rate at the central parity. However, there arises the possibility of deviating from this simple rule in order to reduce interest rate variability. If this temptation is captured by market participants, the simple rule will no longer be credible. Hence, (8) would be time inconsistent.

2.3 The pure free float

Consider now the case where the CB announces that the risk premium shocks will not be dampened over the exchange rate, whatever the preference parameter λ is. That means

$$x_t = c + r_t. \tag{10}$$

This is a *laissez faire* solution. The slope of the exchange rate function with respect to the fundamental is one. According to the UIP (3), the interest rate differential is again given by

$$d_t = x_t - c = r_t.$$

As with the fixed rate, the CB is not deciding its monetary policy rule to optimize (7) and this may be a source of time inconsistency. In general, the market will perceive that CB's incentives to trade volatility of the exchange rate for volatility of the interest rate are different than 1 and the announcement of the free float will be no longer credible.

We can compute the indirect value function for the free float:

$$J^{ff}(r_t) = \frac{1}{2}\left(\frac{1+\lambda}{1-\beta}\right)\left[r_t^2 + \frac{\beta\sigma^2}{1-\beta}\right]. \tag{11}$$

2.4 Intervention with an infinite band

This regime is a particular case of the target zone when $\{\lambda > 0, w \to \infty, \alpha = 1\}$. With this regime the CB intervenes to exploit the trade-off between the variability of the exchange rate and the interest rate every period. After the shock r_t is realized and agents have formed their expectations on future values for the exchange rate, the CB must set the two target variables $\{d_{t+\tau}, x_{t+\tau}\}_{\tau=0}^{\infty}$ with one instrument $\{m_{t+\tau}\}_{\tau=0}^{\infty}$, to minimize the loss (7) subject to the arbitrage condition (3) and the relation in (4). Notice the exchange regime does not impose additional restrictions. We analyze this system because it is a limiting case of the target zone regime described next and because it has an analytical solution that serves as a reference to compare with the target zone solution.

The optimality condition is given by the difference equation

$$x_t = c + \frac{r_t}{\lambda} + \frac{1}{\lambda}E_t\{x_{t+1} - x_t\}. \tag{12}$$

The central parity is constant here since, with an infinitely-wide band, there will be no realignments. From (4), a linear closed form solution of (12) is

$$x_t = c + \frac{r_t}{\lambda}. \tag{13}$$

The discretionary rate is then equal to the fixed rate plus a depreciation bias. Forming a one-period ahead expectation yields

$$E_t x_{t+1} = c + \frac{r_t}{\lambda} = x_t, \tag{14}$$

so, from (3), the target variable d_t is

$$d_t = r_t. \tag{15}$$

A shock to the risk premium impulses exchange and interest rates in the same direction. Combining (13) and (15), a linear control condition is given by

$$d_t = \lambda(x_t - c), \tag{16}$$

meaning that the interest rate differential is proportional to the depreciation (or appreciation) bias with respect to the central parity. This is just a *leaning* *against the wind* policy, where the interest rate differential is positive when the currency is above the target and negative below. Under the uncovered interest rate parity, conditional on x_t, the risk premium has a positive relation with d_t. Similarly, conditional on d_t, r_t has also a positive relation with x_t. Therefore, a central bank that is concerned about the volatility of interest rate differentials and of exchange rates will optimally split the variation of r_t between d_t and x_t. This results in a regime of intramarginal interventions or managed float. The higher the value of the exchange rate weight in preferences, λ, the smaller the variance for the exchange rate is.[4]

The explicit form for the expected value of the game under this regime is calculated as

$$J^{\infty}(r_t) = \frac{1+\lambda}{2\lambda}\left(\frac{1}{1-\beta}\right)\left[r_t^2 + \frac{\beta}{1-\beta}\sigma^2\right]. \tag{17}$$

The relation between the values for the fixed rate, the free float and this regime can be written as

$$J^{ff} = \lambda J^{\infty} = (1+\lambda) J^c,$$

for any r_t. Differences among regimes will have to be found in the exchange rate dynamics since they imply the same process for the interest rate differential. Clearly, the fixed rate is preferred over the two other regimes, although it is time inconsistent. In the intervention regime with an infinite band, the attempt of the CB in giving the interest rate a lower volatility by trading with the exchange rate variance is a vain effort. It results in a discretionary time consistent solution, where the volatility of the interest rate is unaffected and the exchange rate begins to float. The CB is worse off. The ordering of the losses for the free float and the band with an infinite amplitude will depend upon the value of λ. If $\lambda < 1$, the CB is better off with the free float. Otherwise, for $\lambda \geq 1$, intervening the exchange rate is a better regime.

2.5 The target zone

In this case, the regime is characterized by $\{\lambda \geq 0, w \geq 0, \alpha \in [0,1]\}$. The first element is related to the preferences in (7). The number w is the band width. The last term α is the probability that the CB defends the currency when it is outside the band and measures the credibility of the exchange regime. Here we assume that the band is imposed either explicitly or implicitly. Either way, markets participants know of its existence. Furthermore, we suppose that the CB defends the currency with probability 1 within the bands.

From the previous discussion, both the exchange rate and the interest rate differential will be functions of the shock r_t. So, the timing of events at any time t will be as follows:

1. The shock r_t is realized.

[4] Expression (16) can also be considered a nominal monetary conditions index (MCI). The optimal policy is thus to move interest rates and exchange rates so as to leave the MCI constant to movements in the risk premium. Appendix A includes an interpretation of the preferences of the central bank along the lines of the MCI. For further information on MCI see Ball [3].

2. If the realization of the shock makes the exchange rate be within the target zone, the CB solves a minimization problem. The process is as follows:

   - The central parity from last period is left unaltered: $c_t = c_{t-1}$.
   - Forward looking agents form expectations $E_t(x_{t+1})$.
   - For given $\{E_t(x_{t+1}), r_t, c_t\}$, the CB chooses $\{x_t, d_t\}$ by minimizing the loss (7) subject to the arbitrage condition (3), the relation in (4) and the additional restriction imposed by the target zone system.

3. If for that value of the shock the exchange rate should be outside a band $[c_t - w, c_t + w]$, for instance, say it is above the upper limit $c_t + w$, the CB can do any of two actions:

   - It defends the currency with probability α. This means that the exchange rate is pegged at the edge of the band, $x_t = c_t + w$, and the central parity is not altered, $c_t = c_{t-1}$.
   - It realigns the currency with probability $(1 - \alpha)$. In this case, the central bank devalues the central parity by $\mu \geq w$, *i.e.* $c_t = c_{t-1} + \mu$, and situates the exchange rate at $x_t = c_t$.

4. Finally, for a given shock θ_t in the money demand equation (5), the CB supplies the optimal quantity of money m_t supporting the pair $\{x_t, d_t\}$.

To understand the way this economy works consider the following. Assume the risk premium starts from $r_0 = 0$. The CB can set the exchange rate at $x_0 = 0$ and a target zone of width w around $c_0 = 0$. Because of the symmetry of the forcing process, the interest rate differential is $d_0 = 0$, which equals the target value for that variable. As time progresses, the risk premium wanders around and the exchange rate moves away from its center $c_0 = 0$. Given that the exchange rate is a function of the shock r_t, the foreign risk premium must also be fluctuating within a symmetric zone with center $\rho_0 = 0$. This zone is denoted as $[\rho_0 - \overline{r}, \rho_0 + \overline{r}]$. How far the exchange rate wanders from the center of its band should be a function of the distance between the risk premium and ρ_0. So, for the periods before the first realignment, we could write

$$x_t = c_0 + u(r_t - \rho_0),$$

where $u(r_t - \rho_0)$ is the function linking the exchange rate to the fundamental process r_t *within the band*.

Imagine that at time $t = \tau$, one of the limits of the exchange rate band is reached. This means that $r_\tau = \rho_0 \pm \overline{r}$ two level conditions must be satisfied

$$u(-\overline{r}) = -w, \tag{18}$$

and

$$u(\overline{r}) = w. \tag{19}$$

When the exchange rate is at the boundaries of the target zone, the CB could keep x_t at the edge of its band. In such a case, all movements in r_t will be transferred to the interest rate differential. The other possibility implies the central bank realigning the target zone. In such a case, the exchange rate jumps to $c_\tau = c_{\tau-1} + \mu$, and the center of the band for the risk premium moves to $\rho_\tau = r_\tau$. After the realignment takes place, the behavior of the exchange rate band within the target zone is again governed by the function u so we can write in general

$$x_t = c_t + u(r_t - \rho_t).$$

Let $x(r_t)$ be the function relating the exchange rate to the risk premium at any time, that is, within as well as outside the target zone. This function satisfies

$$x(r_t) = c_t \begin{cases} +\mu & \text{if } r_t > \rho_t + \overline{r} \text{ with probability } 1-\alpha \\ +w & \text{if } r_t > \rho_t + \overline{r} \text{ with probability } \alpha \\ +u(r_t - \rho_t) & \text{if } r_t \in [\rho_t - \overline{r}, \rho_t + \overline{r}] \\ -w & \text{if } r_t < \rho_t - \overline{r} \text{ with probability } \alpha \\ -\mu & \text{if } r_t < \rho_t - \overline{r} \text{ with probability } 1-\alpha. \end{cases} \tag{20}$$

The unknown continuous function $u(r_t - \rho_t)$ represents the CB's best response when the risk premium lies within its band $[\rho_t - \overline{r}, \rho_t + \overline{r}]$. To find it, we use the first order condition for values of $r_t \in [\rho_t - \overline{r}, \rho_t + \overline{r}]$,

$$(1+\lambda)\, u(r_t - \rho_t) + c_t = r_t + E_t[x(r_{t+1})]. \tag{21}$$

Outside of the band, condition (21) does no longer hold and the trade-off is not optimal. We use numerical methods to approximate the function $u(r_t - \rho_t)$ from conditions (21) and (20). Once the function $u(r_t - \rho_t)$ is computed, we use (20) and (3) to find the exchange rate and the interest rate differential, respectively.

3 Results

Before we start with the computations, let us first remind the reader about the implications derived from the main features of Krugman's model: the *honeymoon* and the *smooth pasting* effects (see Svensson [36] and Krugman and Miller [24]). The first means that the response of the exchange rate to changes in fundamentals is smaller than the response under the free floating regime. That is, as compared to a free floating, the band works as an exchange rate stabilizer. The smooth pasting implies that the response of the exchange rate tends to zero as it approaches the edges of the band. This is because agents are forward looking and anticipate the intervention by the central bank when the exchange rate gets close to the limit of the band.

Svensson [36] has summarized the testable implications of Krugman's model. First, the distributions of both the exchange rate and the interest rate are U-shaped. That is, the exchange rate tends to live close to the limits of the band.

Second, there is a negative and deterministic relation between the exchange rate and the interest rate differential. This relation is given by the uncovered interest rate parity. Finally, the exchange rate exhibits a non-linearity in its univariate forecasting equation, which is a consequence of the smooth pasting effect. However, empirical tests have challenged these predictions. The distribution of the exchange rate has been observed to be *hump*-shaped rather than U-shaped, so the exchange rate accumulates probability around the center of the band. Secondly, the relationship between the exchange rate and the interest rate is positive rather than negative. Finally, it seems that the effects of the smooth pasting condition are not as relevant as predicted by theory and exchange rates are linear functions of the fundamentals.

It is easy to show that lack of credibility together with intramarginal interventions contribute separately to reproduce these stylized facts. First, the smooth-pasting condition is no longer valid without credibility which decreases the probability of the exchange rate to be close to the edges of the band. Furthermore, intramarginal interventions smooth exchange rates which also contribute to push the exchange rate towards the center of the band. Second, the leaning against the wind policy derived from intramarginal interventions produces a positive relation between the interest rate differential and the exchange rate and makes the function linking the exchange rate to fundamentals closer to be linear. However, these results are far from being guaranteed once the two extensions considered in this paper are included at the same time. On the one hand, lack of credibility also reduces the honeymoon effect (see Bertola and Caballero [6]) which counteracts the exchange rate smoothing associated with intramarginal interventions. On the other hand, realignment risks can make the relation between the exchange rate and the interest rate differential being either positive, negative or zero (see Bertola and Svensson [7]).

In what follows we calibrate our model and compare its predictions with the data to evaluate whether the interaction of the two extensions considered in this paper can reproduce the styled facts mentioned above. Furthermore, we compute the losses (7) under different regimes and provide a rationale as of why limiting the fluctuations of the exchange rate has been widely used.

3.1 Parameters

There are 6 parameters in the model: the discount factor β, the standard deviation of the risk premium, σ, the width of the band, w, the realignment rate, μ, the probability of defense, α, and the weight of the exchange rate in the preferences of the central bank, λ. We think of a week as the time frequency. As in Svensson [38], the time discount factor is set to $\beta = 0.90^{\frac{1}{52}}$. We use this value to ease the calculations but the main results of the paper do not hinge on it.

The selection of a standard deviation for the shock to the risk premium, σ, is a troublesome task. Bekaert [4] reports an unconditional variance for a time invariant risk premium of 10.622^2, for the Dollar/Yen rate. This figure yields a weekly standard deviation of about $\sigma = 0.002$ basis points per week.

We also use the ARCH-in-mean estimation by Domowitz and Hakkio [13], for monthly observations 1973:6-1982:8, of the British Pound, the French Franc, the Deustche Mark, the Japanese Yen and the Swiss Franc, all against the US Dollar. A Montecarlo simulation has been run in order to calculate this moment. Table 1 reports the standard deviations for a first difference of r_t. The Swiss Franc presents a time invariant risk premium. Hence, it seems reasonable to assume an *a priori* value $\sigma = 0.002$, as in Bekaert [4].

Two band widths are chosen, $w = \pm 2.25\%$ and $\pm 6.00\%$, the ones experienced in the ERM. The value of the constant μ is estimated depending on the band width w. Tables 2 collects data of realignment rates for the currencies participating in the ERM of the EMS, except for the Dutch Guilder. From this table it seems that realignment rates of $\pm 4.5\%$ and $\pm 6.3\%$ for widths of ± 2.25 and $\pm 6\%$, respectively, are consistent with the EMS history

Since there is no prior for the relative weight of the exchange rate variability in the preferences of the CB, we use a wide range of values within which the parameter λ is thought to be about. These values are $\lambda = 0.2$, 0.5, 1.0, 2.0 and 5.0. We also compute results for different values of α, the probability of defense.

3.2 Nonlinearities in the exchange rate function

Let us start examining the degree of nonlinearities in the exchange rate function predicted by our model. Figures 1 to 3 plot the function $u(r_t)$ for probabilities of defense $\alpha = 0$, 0.5, and 1, together with the infinite band and the free float. The three figures differ on the value of λ which are 0.5, 1, and 2, respectively. The 45 degree line corresponds to the free float and the other linear solution is the infinite band with interventions. The relative slopes of these two solutions depend on the value of λ. Among the nonlinear solutions, the flattest function corresponds to $\alpha = 1$, the target zone regime under perfect credibility. On the other extreme, the most sloping curve corresponds to the case where the currency is realigned for sure at margins, that is, $\alpha = 0$.

According to our model, the degree of non linearity increases with (i) a tightening of the fluctuation band, that is, a reduction in w, (ii) an increase in the preferences for exchange rate smoothing, that is, an increase in λ, and (iii) changes in credibility, that is, in the probability α. With respect to the amplitude, if w is increased, the exchange regime naturally tends to some form of floating. As sections 2.3 and 2.4 show floating regimes make the exchange rate a linear function of fundamentals. Second, the stronger the preferences for exchange rate smoothing, that is, the larger λ is, the more homogeneous the response of the exchange rate is to fundamentals which increases the linearity of the function u. Finally, if credibility is low, the exchange rate displays an inverted S form as in Bertola and Caballero [6]. As α approaches 1, the full credibility case, the slope curve flattens and becomes S-shaped.

A considerable body of the empirical literature found that the effects of the nonlinearities from the honey moon/smooth pasting condition are negligible. See, for example, Meese and Rose [29] for three alternative regimes, the ± 1 percent band of the Bretton Woods system, the gold standard for the British

Pound, the French Franc and the Deustche Mark (versus the US Dollar), and third, the EMS regime for the Dutch Guilder and the French Franc cases (versus the Deustche Mark). The null hypothesis of nonlinearities is rejected for the three regimes. Lewis [26] runs a variety of tests for the G-3 case (US, Germany and Japan) in order to check for possible non linearities arising from *implicit* target bands and from the intervention policy. In all the cases, the exchange rate seemed to be a linear function of the estimated fundamentals. Evidence on the rejection of the nonlinear hypothesis is also reported in Lindberg and Söderlind [27] for Swedish data. Flood, Rose and Mathieson [15] do not find relevant evidence of the smooth pasting for three alternative testing methods. Iannizzotto and Taylor [22] for the Belgian Franc, Danish Krona, and the French Franc against the Deustche Mark and Taylor and Iannizzotto [40] for the French Franc against the Deustche Mark find nonlinearities to be negligible using the method of simulated moments to estimate Krugman's model. Opposite to these results, and also using a maximum Likelihood Method of Simulated Moments estimator applied to Krugman's model, De Jong [11] found that the exchange rates of the Belgian Franc, Danish Krona, and the French Franc against the Deustche Mark presented significant nonlinearities.

Evidence of nonlinearities appears once intramarginal interventions are allowed. This is the case of Bessec [8] and Crespo *et al.* [10]. These authors assume the exchange rate fluctuates freely within a band of inaction. However, when the exchange rate moves outside this interval, the authorities intervene to drive back the exchange rate to the central parity. The estimation from a SETAR model confirms this type of behavior for the Belgian Franc, the French Franc, the Irish Punt and the Dutch Guilder.

Another group of papers are able to corroborate nonlinear specifications for the exchange rate introducing problems of credibility in the target zone. In this sense, Vilasuso and Cunningham [41] test for nonlinearities using the bispectrum. They find that the Belgian Franc, the Danish Krona, the Dutch Guilder and the French Franc follow a linear process over the period 1979–1987, when several realignments took place. But from 1987–1992, these currencies follow a nonlinear process, as with the credible target zone model where an inherent nonlinearity stabilizes exchange rates. Also, Bekaert and Gray [5] specify the distribution of exchange rate changes conditioned on a latent jump variable where the probability and size of a jump is a function of financial and macroeconomic variables. With this modeling strategy they find significative evidence of the presence of nonlinearities in the French Franc/Deutsche Mark rate during the EMS. Finally, the relation between credibility and linearity is explicitly tested in Forbes and Kofman [16]. As in our theoretical model, these authors find that a reduction in the credibility of the target zone implies a linearization of the S-shape. Once they take this relation into account, strong evidence of a nonlinear structure for the French Franc-Deustche Mark exchange rate appears in the data.

3.3 U-shaped *versus* hump-shaped distributions

In the standard model, the U-shaped distribution is implied by the perfect credibility assumption. Since the exchange rate function flattens near the edges, a considerable mass of probability will be concentrated at the margins. However, the empirical literature has pointed out that the distribution of the exchange rate is *hump*-shaped rather than U-shaped.[5]

Figures 4, 5 and 6 plot the ergodic probabilities of different depreciation rates within the band for $\lambda = 2$ and for three different values of α: 0, 0.8 and 1. As suggested before, figure 4 shows that for the particular case of $\alpha = 1$, the exchange rate distribution displays a U-shaped distribution. On the contrary, figures 5 and 6 show that for α different than 1, the distribution is hump-shaped. This occurs even for values of α close to 1. The explanation is twofold. The first reason is related to the value of α. For low values of α, continuous realignments move the exchange rate to new central parities, that is, to new centers of new bands. Additionally, lack of credibility implies that the slope of exchange rate function increases at margins, as we have shown in the previous subsection. Thus, stability is higher around the centers. This helps to accumulate probability mass at the interior of the bands. This second reason has to do with the value of λ. If this parameter is high, the central bank will have incentives to further stabilize the exchange rate.

3.4 The relation between exchange rates and interest rate differentials

Empirical observations have shown the existence of a positive relationship between the exchange rate and the interest rate differential.[6] Bertola and Svensson [7] suggest that incorporating a realignment risk premium may alter the sign of the covariance between these two variables from negative to positive.

In our model, this relation has always a positive sign, regardless the value of the parameters. This is a consequence of intramarginal interventions, summarized by the first order condition (16). Both the interest rate and the exchange rate are driven by the same variable, namely, the risk premium. The central bank decides at each period how to distribute the shock on r_t between x_t and d_t. Expression (16) tells us that it is always optimal to move both variables on the same direction with λ being the ratio between the two. This gives rise to a positive linear relationship between the two variables. Hence, the honeymoon effect is also transmitted to the interest rate smoothing.

[5] See, for example, Bertola and Caballero [6] for the French Franc against the Deustche Mark exchange rate case during 1979-87; and Lindberg and Söderlind [27], [28], for the Swedish unilateral target zone with a vast set of daily data covering from 1982 to 90. The work of Flood, Rose and Mathieson [15] also finds *hump*-shaped histograms for EMS exchange rates. They use daily data for eight EMS currencies and the British Pound (during its pre EMS membership) over 1979-90, weekly data from the classical gold standard (UK, US, France and Germany), and monthly data from the Bretton Woods regime.

[6] See Svensson [35], Flood, Rose and Mathieson [15], Lindberg and Söderlind [27], and Bertola and Caballero [6].

3.5 On the choice of the degree of exchange rate flexibility

In this subsection, we estimate the loss functions for different degrees of exchange rate flexibility using the set of parameters stated before. Losses for the fixed rate, the infinite band and the free float are given by expressions (9), (17) and (11). The target zone loss is numerically computed. We consider a starting point $r_0 = 0$, and a time horizon of 2000 periods. This time length accumulates 91.2% of the total accruing value and does not alter the ordinality of values for the exchange regimes. Calculations of the indirect costs are reported in table 3 for $w = \pm 2.25\%$ and table 4 for $w = \pm 6\%$. In each table, the first row refers to the cost of the fixed rate regime, J^c, and the following five rows represent the costs of a target zone for $\alpha = 0$, $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, and 1, respectively. The seventh and eighth rows collect the infinite band costs, J^{∞}, and the free float costs, J^{ff}. From these tables we highlight the following results.

First, as it was mentioned above, the fixed rate is the regime with the lowest value for the loss function.

Second, with respect to the target zone regime, the closer α is to 1, the smaller the losses are. This represents the gain from credibility of commitment to a zone. The slopes of both the exchange rate function and the interest rate differential flatten for α approaching to 1. When market traders perceive that the CB will defend the fluctuation band with a high probability, the realignment risk will be low and the exchange rate responses to changes in the foreign risk premium will be soft. In turn, this perception will contribute to smooth the expected rate of depreciation. Hence, the outcome is that the interest rate differential will be more stable as well.

Third, for target zones very close to being credible (α close to 1), the present value of the costs of a target zone are smaller than the costs of intramarginal interventions with an infinite band. Notice that, unlike the previous literature that used the free floating as an alternative, the right regime to be compared with is the infinite band with interventions. As with the second result, forward looking agents help stabilize the rate without pressuring the interest rate, due to the honey moon effect.

To make this point clearer, figures 7, 8 and 9 represent the contributions to the loss functions due to the variability of the exchange rate and the interest rate for all the regimes and values of λ equal to 0.5, 1, and 2, respectively. The circle corresponds to the fixed rate, the square to the infinite band and the triangles are the target zones for different values of α. It is clear that there must be an interval for α close to 1 that reduces the volatility of both the exchange rate and the interest rate with respect to the managed float.

4 Conclusions

This paper presents a target zone model based on two extensions of Krugman's [23] model. These extensions are the introduction of intramarginal interventions and the lack of credibility of the target zone. These two features had been analyzed separately by the literature. Here, we show that both extensions are sufficient to reconcile the model with the data. The leaning against the wind policy derived from the time-consistent intramarginal intervention produces a positive relation between exchange rates and interest rates differentials as we observe in the data as well as is behind the almost linear relation between exchange rates and fundamentals. Furthermore, lack of credibility contributes to the hump-shape distribution of exchange rates.

As a second application, the model provides a framework in which to evaluate why imposing bands of fluctuation seem to be the dominant exchange rate regime in contemporary history. We show that even not perfectly credible bands of fluctuation may improve with respect to infinite bands in terms of reducing simultaneously the volatility of both exchange rates and interest rates differentials. In this way, we operationalize the common view that fluctuation bands are preferred because they reap the benefits of both flexible and fixed exchange rates, by stabilizing the exchange rate without loosing monetary independence. These two benefits are explicitly included in the model and are a result of the interaction between the preferences of the central bank and the credibility of its monetary policy.

One important assumption in the model is that the width of the band is known by market participants. This assumption is crucial for including in the analysis regimes where the central bank does not publicly announce a target zone but it is in fact restricting the fluctuation of the exchange rate around a particular level. It has been argued that the uncertainty about the exact location of this target zone may be more important for the behavior of the exchange rate than the credibility of the band. In [33] we tackle this issue. Using a model along the lines of the one presented in this paper, we endogenize the width of the band. To capture the notion of implicit bands, we solve the model for a subgame perfect equilibrium where the central bank is given a menu of alternative band widths. Given the deep parameters of the model (i.e., those of the central bank preferences, λ and β, and the exogenous process, σ) any of these bands induces an endogenous probability of realignment. Backward induction then reveals market traders the minimum cost regime, which in turn signals the optimal band to be credibly committed by the central bank. Thus, once we allow for the band width to be endogenous, market participants should be able to infer it from information on the central bank preferences and the exogenous shock.

On the other hand, the empirical literature has pointed out that the credibility of a target zone erodes as time goes by. In the present paper, however, we have assumed that the parameters of the model are all constant across time. This means that preferences and the credibility perceived by market traders remain constant forever. To the extent that the credibility of exchange rate bands decreases over time, any recommendation regarding the virtues of limiting the fluctuations of the exchange rate should be moderated. By endogenizing the probability of the band, [33] can also help to understand why the credibility of target zones has been decreasing over time.

A A General Equilibrium Model

The central bank preferences in the paper, included in expression (2.7), can be interpreted in two ways. First, it could represent the fear of floating of a central bank, that is, the fear that the exchange rate would deviate from its long run equilibrium or have excess volatility. In this situation, central banks try to smooth exchange rates by means of interest rate policy or interventions. This is captured by losses that combine volatility of interest rate together with volatility of the exchange rate with respect to its long-run equilibrium value.

The second interpretation deals with the first order condition in (2.14). This expression can be interpreted as a monetary conditions index which implies that the variation of the risk premium should be split optimally between changes in the exchange rate and the interest rate. To motivate such a form for a monetary conditions index, consider the following model in the spirit of Detken and Gaspar [12], Walsh [42] or West [43]:

$$\pi_t = \eta z_t + \beta E_t(\pi_{t+1}) + u_t, \tag{22}$$

$$z_t = -\vartheta[i_t - E_t(\pi_{t+1})] + E_t(z_{t+1}) + \psi(x_t - p_t) + g_t, \tag{23}$$

$$i_t = E_t(x_{t+1}) - x_t + r_t, \tag{24}$$

where π_t is the inflation rate, z_t is the deviation of output from its natural rate, i_t is the nominal interest rate, x_t is the nominal exchange rate, and p_t is the price level. The variables u_t, g_t are disturbances. Equation (22) is a Phillips curve while (23) is an open economy version of an IS curve. Expression (24) represents deviations from the uncovered interest rate parity due to the risk premium r_t.[7] The central bank is assumed to modify the interest rate i_t to minimize the standard objective function

$$L = E_t\left[\sum_{\tau=0}^{\infty} \beta^{\tau}\left(\pi_{t+\tau}^2 + \xi z_{t+\tau}^2\right)\right].$$

which represent a second order Taylor expansion of the expected utility of the representative consumer as derived in Woodford [44] or Rotemberg and Woodford [34] for a closed economy.

The disturbances u_t and g_t are assumed *iid* and not known at the time of the policy decision.[8] They serve the purpose of preventing the central bank from knowing the current (and possibly the near past) values of output and inflation. The risk premium is assumed to follow a AR(1) process with coefficient equal to h. The only information the monetary authority has at the time of its policy decision is the interest rate, the exchange rate and, by (24), the risk premium.

The first order condition reads

$$z_t = -\frac{\eta}{\xi}\pi_t.$$

[7] Here the foreign interest rate is assumed equal to zero for simplicity.

[8] The results below hold under more general processes for u_t and g_t.

Using this expression, in (22) and (23) allows us to write

$$\vartheta i_t - \psi x_t = \left[\frac{\eta}{\xi} - \psi\right] \Phi u_t + g_t - \psi p_{t-1}, \tag{25}$$

with

$$\Phi = \frac{\xi}{\xi + \eta^2}.$$

In terms of x_t and i_t, the solution is

$$x_t = \left[1 - \frac{\eta}{\xi\left(\vartheta + \psi\right)}\right] \Phi u_t - \frac{1}{\vartheta + \psi} g_t + \frac{\vartheta}{\vartheta(1-h) + \psi} r_t + p_{t-1} \tag{26}$$

and

$$i_t = \left[\frac{\eta}{\xi\left(\vartheta + \psi\right)}\right] \Phi u_t + \frac{1}{\vartheta + \psi} g_t + \frac{\psi}{\vartheta(1-h) + \psi} r_t. \tag{27}$$

The left hand-side of (25) can interpreted as a nominal monetary conditions index (MCI). This expression implies that this MCI should not be affected by the risk premium. Thus, as (26) and (27) indicate, in order to manage the changes in the output gap and inflation optimally, the variation in the risk premium r_t should be split between the interest rate i_t and the exchange rate x_t so as to leave the MCI constant. Notice the degree to which the risk premium affects the interest rate and the exchange rate (governed by the parameter λ in the preferences (2.7)) depends on the parameter ψ which measures the openness of the economy, the parameter ϑ, which measures the response of output to the interest rate, and the persistence of the risk premium shock as measured by the parameter μ. In general, as the value of ψ (ϑ) increases (decreases), the central bank should channel a larger proportion of the risk premium shock to interest rates and a smaller fraction to the exchange rate. Also notice that for $\mu = 1$, that is, when the risk premium follows a random walk, as we have assumed in the paper, the interest rate should move one to one with the risk premium. Furthermore, the risk premium impact on the exchange rate is in that case ϑ/ψ, which could be interpreted as the relative importance of the internal transmission channel (i.e. that of the interest rate) to the external transmission channel (i.e. that of the exchange rate).

B Solution of the model

In this appendix we do not assume that the realignment rate must be higher than the width. Consider a center $\rho_t = 0$ and parity $c_t = 0$. Then, the function $x\left(r_t\right)$ must be rewritten as

$$x\left(r_t\right) = c_t \begin{cases} +\max\left\{w, \mu\right\} & \text{if } r_t > \overline{r} \text{ with probability } 1-\alpha \\ +w & \text{if } r_t > \overline{r} \text{ with probability } \alpha \\ +u\left(r_{t+1}\right) & \text{if } r_t \in \left[-\overline{r}, \overline{r}\right] \\ -w & \text{if } r_t < -\overline{r} \text{ with probability } \alpha \\ -\max\left\{w, \mu\right\} & \text{if } r_t < -\overline{r} \text{ with probability } 1-\alpha. \end{cases} \tag{28}$$

We show how to make the computations for the solution of the target zone regime. Forward recursion in the first order condition (21) is used to solve for $u(r_t)$ subject to (28). Let the expected rate of depreciation out of the band be given by

$$\begin{aligned} \delta_{\tau,\tau-1} &= [\alpha w + (1-\alpha)\max\{w,\mu\}] \\ &\quad \times [\Pr(r_{t+\tau} > \overline{r} | r_{t+\tau-1}) - \Pr(r_{t+\tau} < -\overline{r} | r_{t+\tau-1})] \end{aligned} \tag{29}$$

Forward iteration of (21) leads to the following general solution

$$u(r_t) = \frac{r_t}{1+\lambda} + \frac{1}{1+\lambda}\sum_{\tau=1}^{\infty}\frac{F[r_{t+\tau}|\mathcal{F}_\tau]}{(1+\lambda)^{-\tau}} + \sum_{\tau=1}^{\infty}\frac{F[\delta_{\tau,\tau-1}|\mathcal{F}_{\tau-1}]}{(1+\lambda)^{-\tau}}, \tag{30}$$

where the sequence $\mathcal{F}_{\tau\geq 0}$ represents filtered information sets of the next form:

$$\mathcal{F}_0 = \{r_t\}, \tag{31}$$

$$\mathcal{F}_{\tau\geq 1} = \left\{\{r_{t+n} \in [-\overline{r},\overline{r}]\}_{n=1}^{\tau}, r_t\right\}. \tag{32}$$

On the other hand, the components in (30) are given by:

$$\begin{aligned} F[\delta_{1,0}|\mathcal{F}_0] &= [\alpha w + (1-\alpha)\max\{w,\mu\}] \\ &\quad \times [\Pr(r_{t+1} > \overline{r}|r_t) - \Pr(r_{t+1} < -\overline{r}|r_t)] \end{aligned} \tag{33}$$

$$F[r_{t+1}|\mathcal{F}_1] = \int_{-\overline{r}}^{\overline{r}} r_{t+1}\phi(r_{t+1}|r_t)\,dr_{t+1}, \tag{34}$$

for $\tau = 1$, and

$$F[\delta_{\tau,\tau-1}|\mathcal{F}_{\tau-1}] = \int_{-\overline{r}}^{\overline{r}}\cdots\int_{-\overline{r}}^{\overline{r}} \delta_{\tau,\tau-1}\phi(r_{t+\tau-1},...,r_{t+1}|r_t)\,dr_{t+\tau-1}...dr_{t+1}, \tag{35}$$

$$F[r_{t+\tau}|\mathcal{F}_\tau] = \int_{-\overline{r}}^{\overline{r}}\cdots\int_{-\overline{r}}^{\overline{r}} r_{t+\tau}\phi(r_{t+\tau},...,r_{t+1}|r_t)\,dr_{t+\tau}...dr_{t+1}, \tag{36}$$

for $\tau = 2,3,...$, where

$$\phi(r_{t+\tau},...,r_{t+1}|r_t) = (2\pi\sigma^2)^{-\tau/2}\exp\left[\frac{-1}{2\sigma^2}\sum_{n=1}^{\tau}(r_{t+n} - r_{t+n-1})^2\right]. \tag{37}$$

This general solution is consistent with (21) and (28). In order to determine a particular solution, it is necessary to identify the value of $\overline{r}$ for which $u(\overline{r}) = w$.

B.1 A numerical approximation

The general solution (30) involves a collection of integrals where only (33) and (34) enjoy an explicit form. Numerical solutions are requested for the remaining ones. Here, we propose a method that discretizes variable r_t on K values ($K \geq 3$ odd) within the interval $[-\overline{r}, \overline{r}]$ as

$$
\begin{aligned}
r &= r_1, r_2, ..., r_K, && (38)\\
r_1 &= -\overline{r}, \\
r_k &= r_{k-1} + h, && (39)\\
h &= 2\overline{r}/(K-1) > 0, && (40)\\
r_K &= \overline{r}, \\
r_{\frac{K+1}{2}} &= 0,
\end{aligned}
$$

In practice, we have used $K = 101$.

Let P, Q, R, S and T be row vectors $(1 \times K)$, adopting the following form

$$
\begin{aligned}
P_1 &= \Phi\left(\frac{-\overline{r} + h/2 - r_t}{\sigma}\right) - \Phi\left(\frac{-\overline{r} - r_t}{\sigma}\right) && (41)\\
P_k &= \Phi\left(\frac{r_k + h/2 - r_t}{\sigma}\right) - \Phi\left(\frac{r_k - h/2 - r_t}{\sigma}\right)\\
P_K &= \Phi\left(\frac{\overline{r} - r_t}{\sigma}\right) - \Phi\left(\frac{\overline{r} - h/2 - r_t}{\sigma}\right)
\end{aligned}
$$

with r_t given and

$$
\begin{aligned}
Q_k &= 1 - \Phi\left(\frac{\overline{r} - r_k}{\sigma}\right) - \Phi\left(\frac{-\overline{r} - r_k}{\sigma}\right), && (42)\\
R_k &= \left[\Phi\left(\frac{\overline{r} - r_k}{\sigma}\right) - \Phi\left(\frac{-\overline{r} - r_k}{\sigma}\right)\right] r_k + && (43)\\
&\quad \sigma\left[\phi\left(\frac{-\overline{r} - r_k}{\sigma}\right) - \phi\left(\frac{\overline{r} - r_k}{\sigma}\right)\right],
\end{aligned}
$$

for $k = 1, 2, ..., K$, and

$$
T = [\alpha w + (1-\alpha)\max\{w, \mu\}]\, Q \tag{44}
$$

where ϕ and Φ represent, respectively, the Gaussian pdf and cdf.

For the *first* period ahead, and only for this period, integrals (33) and (34) have explicit form:

$$
E[\delta_{1,0} | \mathcal{F}_0] = [\alpha w + (1-\alpha)\max\{w, \mu\}]\left[1 - \Phi\left(\frac{\overline{r} - r_t}{\sigma}\right) - \Phi\left(\frac{-\overline{r} - r_t}{\sigma}\right)\right]
$$

$$
\begin{aligned}
E[r_{t+1} | \mathcal{F}_1] &= \left[\Phi\left(\frac{\overline{r} - r_t}{\sigma}\right) - \Phi\left(\frac{-\overline{r} - r_t}{\sigma}\right)\right] r_t\\
&\quad +\sigma\left[\phi\left(\frac{-\overline{r} - r_t}{\sigma}\right) - \phi\left(\frac{\overline{r} - r_t}{\sigma}\right)\right].
\end{aligned}
$$

For the *second* period ahead, a numerical approximation is given by

$$
\begin{aligned}
E\left[\delta_{2,1}\left|\mathcal{F}_1\right.\right] &= TP', \\
E\left[r_{t+2}\left|\mathcal{F}_2\right.\right] &= RP'.
\end{aligned}
$$

This implies that the value of the integrals at $t+2$ is determined for any possible mean at $t+1$, $r_{t+1} \in [-\overline{r}, \overline{r}]$, and any of these means is weighted by a probability P, given a starting value r_t at t.

The loop becomes harder as the period ahead increases over three. In order to solve this problem, we develope a backward recursion algorithm, for which the last period integral is firstly solved and then proceed backward up to the first one. Thereby, consider the following *non negative* matrix $M \in \mathbb{R}^{K\times K}$

$$
\begin{aligned}
M_{1,l} &= \Phi\left(\frac{-\overline{r}+h/2-r_l}{\sigma}\right) - \Phi\left(\frac{-\overline{r}-r_l}{\sigma}\right), \\
M_{k,l} &= \Phi\left(\frac{r_k+h/2-r_l}{\sigma}\right) - \Phi\left(\frac{r_k-h/2-r_l}{\sigma}\right), \\
M_{K,l} &= \Phi\left(\frac{\overline{r}-r_l}{\sigma}\right) - \Phi\left(\frac{\overline{r}-h/2-r_l}{\sigma}\right),
\end{aligned}
$$

for $k, l = 1, 2, ...K$, with the following properties: the sum over each column gives a row vector $m_c \in \mathbb{R}^{1\times K}$, with all its components lying within the $(0,1)$ interval

$$
m_c\left(l\right) = \sum_{k=1}^{K} M_{kl} = \int_{-\overline{r}}^{\overline{r}} \phi\left(s|r_l\right) ds \in (0,1) \tag{45}
$$

for $l = 1, 2, ..., K$. The proof follows trivially. This property is sufficient to verify the Hawkins-Simon condition (Brauer-Solow Theorem).

Matrix M contains the transition probabilities in the intermediate periods from τ up to $\tau+1$, within the interval $[-\overline{r}, \overline{r}]$ and for any conditional mean belonging to $[-\overline{r}, \overline{r}]$. Thus, the solution for the third period is given by

$$
\begin{aligned}
E\left[\delta_{3,2}\left|\mathcal{F}_2\right.\right] &= TMP', \\
E\left[r_{t+3}\left|\mathcal{F}_3\right.\right] &= RMP'.
\end{aligned}
$$

Again, the value of the integrals are first determined at $t+3$ for any possible mean at $t+2$, $r_{t+2} \in [-\overline{r}, \overline{r}]$, given by the columns of M. The vector P closes the calculation for any possible mean at $t+1$, $r_{t+1} \in [-\overline{r}, \overline{r}]$, for given a starting value r_t.

For $\tau = 2, 3, ...$, further generalization gives a sequence:

$$
\begin{aligned}
E\left[\delta_{\tau,\tau-1}\left|\mathcal{F}_{\tau-1}\right.\right] &= TM^{\tau-2}P', \\
E\left[r_{t+\tau}\left|\mathcal{F}_\tau\right.\right] &= RM^{\tau-2}P'.
\end{aligned}
$$

Plugging these values into (30), one obtains

$$\begin{aligned} u(r_t) &= \frac{r_t}{1+\lambda} + \frac{E[r_{t+1}|\mathcal{F}_1]}{(1+\lambda)^2} + \frac{E[\delta_{1,0}|\mathcal{F}_0]}{1+\lambda} \\ &\quad + \frac{1}{1+\lambda}\sum_{\tau=2}^{\infty}\frac{RM^{\tau-2}P'}{(1+\lambda)^{\tau}} + \sum_{\tau=2}^{\infty}\frac{TM^{\tau-2}P'}{(1+\lambda)^{\tau}}. \end{aligned} \tag{46}$$

Let the vector of eigenvalues be given by $\eta(M)$, and call $\eta^*(M)$ the Frobenius root, *i.e.* the maximum eigenvalue. From (45) we know that $m_c(l) < 1$, this is a sufficient condition to verify the Hawkins-Simon condition (see Brauer-Solow Theorem). In turn, verification of the Hawkins-Simon condition implies that $\eta^*(M) < 1$, (see Hawkins and Simon [20] and [21]). Then, matrix $(I-M)^{-1}$ exists, it is non negative and can be written as

$$(I-M)^{-1} = \sum_{j=0}^{\infty} M^j$$

This gives rise to a convenient simplification

$$\overline{M} = \sum_{\tau=2}^{\infty}(1+\lambda)^{-\tau} M^{\tau-2} = \frac{1}{1+\lambda}[(1+\lambda)I - M]^{-1}.$$

The general solution becomes

$$u(r_t) = \frac{\Omega(r_t)}{1+\lambda} + \Delta(r_t), \tag{47}$$

with

$$\Omega(r_t) \equiv r_t + \frac{E[r_{t+1}|\mathcal{F}_1]}{1+\lambda} + R\overline{M}P',$$

$$\Delta(r_t) \equiv \frac{E[\delta_{1,0}|\mathcal{F}_0]}{1+\lambda} + T\overline{M}P'.$$

Application of level conditions (18) and (19) gives the particular solution

$$\frac{\Omega(\overline{r})}{1+\lambda} + \Delta(\overline{r}) = w.$$

Finally, the exchange rate expectation is

$$E_t x_{t+1} = \sum_{\tau=1}^{\infty}\frac{F[r_{t+\tau}|\mathcal{F}_\tau]}{(1+\lambda)^{\tau}} + (1+\lambda)\sum_{\tau=1}^{\infty}\frac{F[\delta_{\tau,\tau-1}|\mathcal{F}_{\tau-1}]}{(1+\lambda)^{\tau}},$$

or using the previous approximation

$$E_t x_{t+1} = \Omega(r_t) + (1+\lambda)\Delta(r_t) - r_t. \tag{48}$$

Once we know the expression for the exchange rate and the expectation, the interest rates differential is obtained from the uncovered interest parity condition as

$$d_t = \Omega(r_t) + (1+\lambda)\Delta(r_t) - x_t \tag{49}$$

C Tables and figures

Table 1
Estimated standard deviation (σ)

British Pound	0.0005
French Franc	0.0011
Deustche Mark	0.0025
Japanese Yen	0.0012
Swiss Franc	0.0000

Table 2
Realignment rates in the ERM

± 2.25 bands						± 6.00 bands		
Date	FF	IRP	BF	DK	ITL	Date	SP	PE
24-Sep-1979	2.00	2.00	2.00	5.00	2.00	17-Sep-1992	5.26	
30-Nov-1979			0.14	5.00		23-Nov-1992	6.38	6.38
23-Mar-1981			-0.14		6.38	14-May-1993	8.70	6.95
5-Oct-1981	8.76	5.50	5.50	5.50	8.76			
22-Feb-1982			9.29	3.09				
14-Jun-1982	10.61	4.25	4.25	4.25	7.20			
21-Mar-1983	8.20	9.33	3.94	2.93	8.20			
25-Jul-1985					8.51			
6-Apr-1986	6.19	3.00	1.98	1.98	3.00			
2-Aug-1986		8.70						
12-Jan-1987	3.00	3.00	0.98	3.00	3.00			
8-Jan-1990				3.82				
14-Sep-1992					7.25			
Average	6.49	5.86	3.10	3.84	5.54	Average	6.78	6.67
Std. dev.	3.39	3.41	3.01	1.26	2.42	Std. dev.	1.75	0.40

Note: FF stands for French Franc, IRP for Irish Punt, BF for Belgium Franc, DK for Danish Krona, ITL for Italian Lira, SP for Spanish Peseta and PE for Portuguese escudo.

Table 3
Loss function for $w = 0.0225$

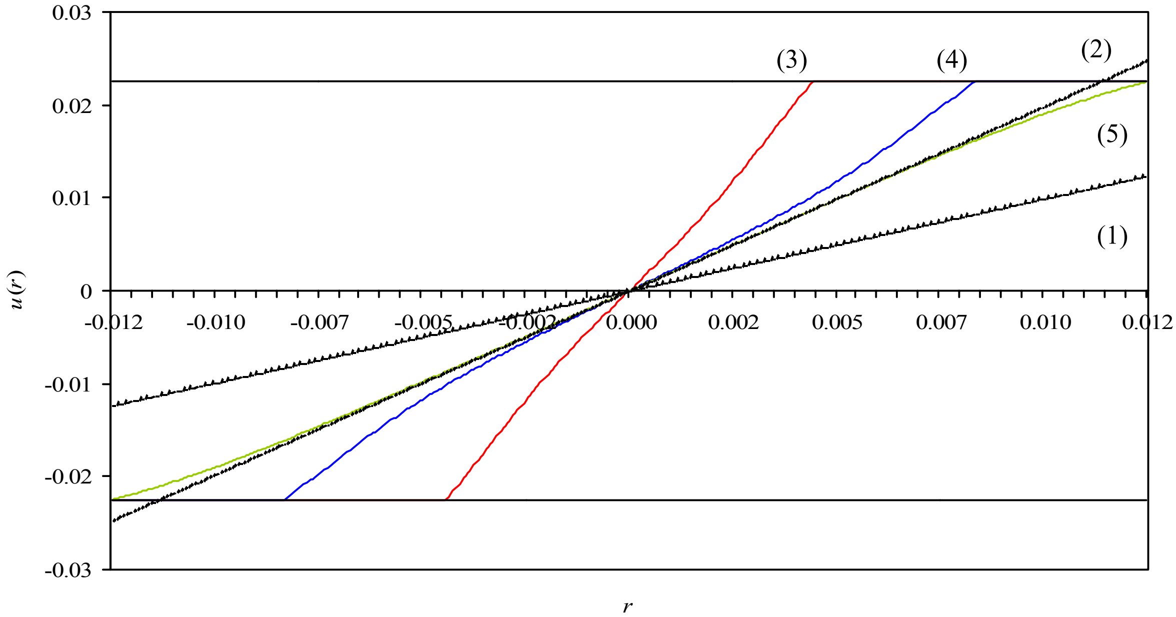
λ	0.2	0.5	1.0	2.0	5.0
J^c	0.4444	0.4444	0.4444	0.4444	0.4444
$J^{tz}\,(\alpha = 0)$	15.4674	15.1098	4.3379	1.6561	0.6280
$J^{tz}\left(\alpha = \frac{1}{4}\right)$	10.0236	8.3402	3.2813	1.4845	0.6123
$J^{tz}\left(\alpha = \frac{1}{2}\right)$	5.7592	4.9529	2.5509	1.3253	0.5999
$J^{tz}\left(\alpha = \frac{3}{4}\right)$	3.0643	3.0624	2.0009	1.1846	0.5864
$J^{tz}\,(\alpha = 1)$	0.4655	0.4919	0.5207	0.5430	0.5244
J^{mf}	2.6665	1.3333	0.8888	0.6666	0.5333
J^{ff}	0.5333	0.6666	0.8888	1.3333	2.6665

Table 4
Loss function for $w = 0.06$

λ	0.2	0.5	1.0	2.0	5.0
J^c	0.4444	0.4444	0.4444	0.4444	0.4444
$J^{tz}\,(\alpha = 0)$	2.0927	1.3081	0.9428	0.6559	0.5333
$J^{tz}\left(\alpha = \frac{1}{4}\right)$	1.9960	1.2758	0.9403	0.6559	0.5333
$J^{tz}\left(\alpha = \frac{1}{2}\right)$	1.8762	1.2399	0.9301	0.6547	0.5333
$J^{tz}\left(\alpha = \frac{3}{4}\right)$	1.6704	1.1951	0.9102	0.6544	0.5333
$J^{tz}\,(\alpha = 1)$	0.5761	0.6778	0.7106	0.6487	0.5333
J^{mf}	2.6665	1.3333	0.8888	0.6666	0.5333
J^{ff}	0.5333	0.6666	0.8888	1.3333	2.6665

FIGURE 1

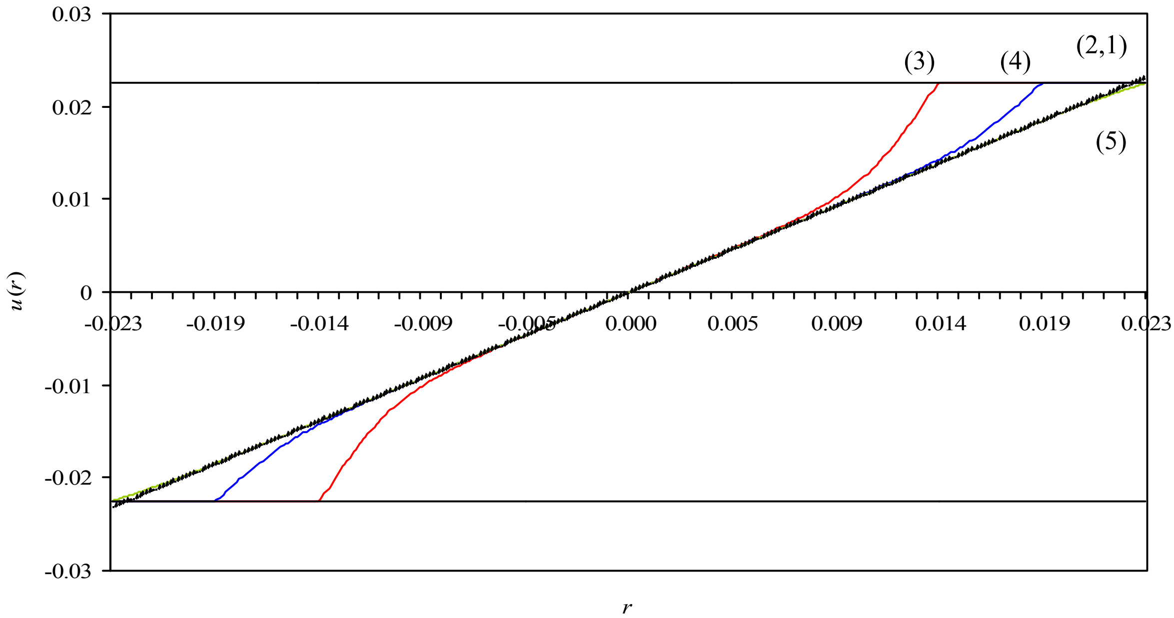
Function $u(r)$ for $\lambda = 0.5$



Note: This figure shows the function linking the exchange rate to the fundamental for different regimes. (1) is the free float, (2) is the managed float, (3) is the target zone with $\alpha = 0$, (4) is the target zone with $\alpha = 0.5$, and (5) is the target zone with $\alpha = 1$.

FIGURE 2

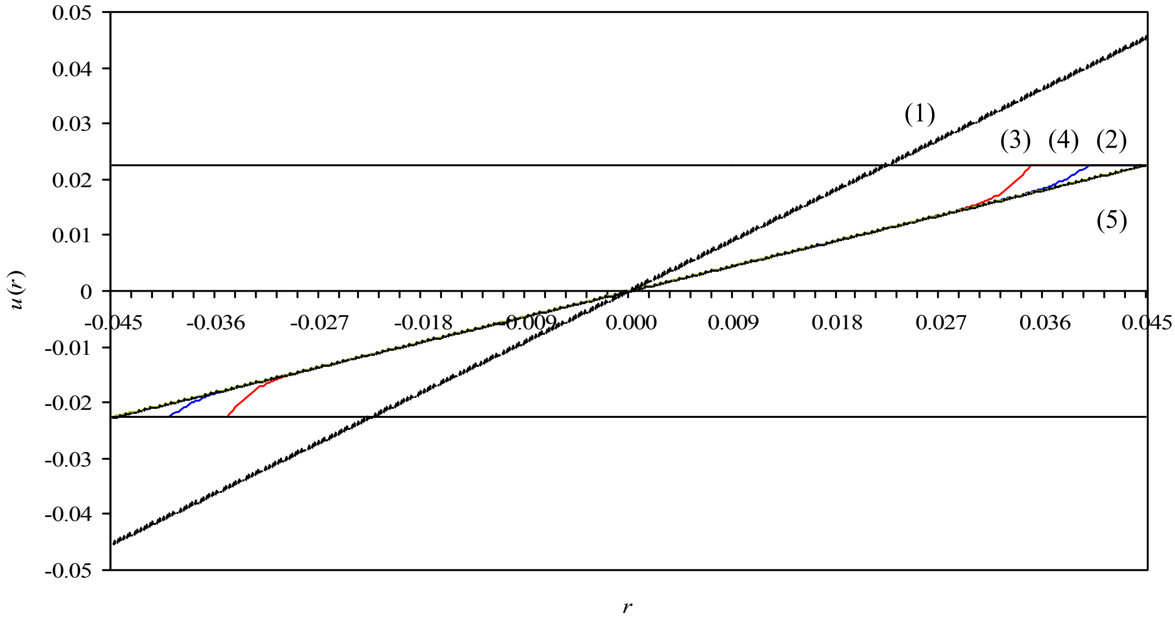
Function $u(r)$ for $\lambda = 1$



Note: See note to Figure 1.

FIGURE 3

Function $u(r)$ for $\lambda = 2$



Note: See note to Figure 1.

FIGURE 4

Stationary probability distribution for $\lambda = 0.5$ and $\alpha = 0$

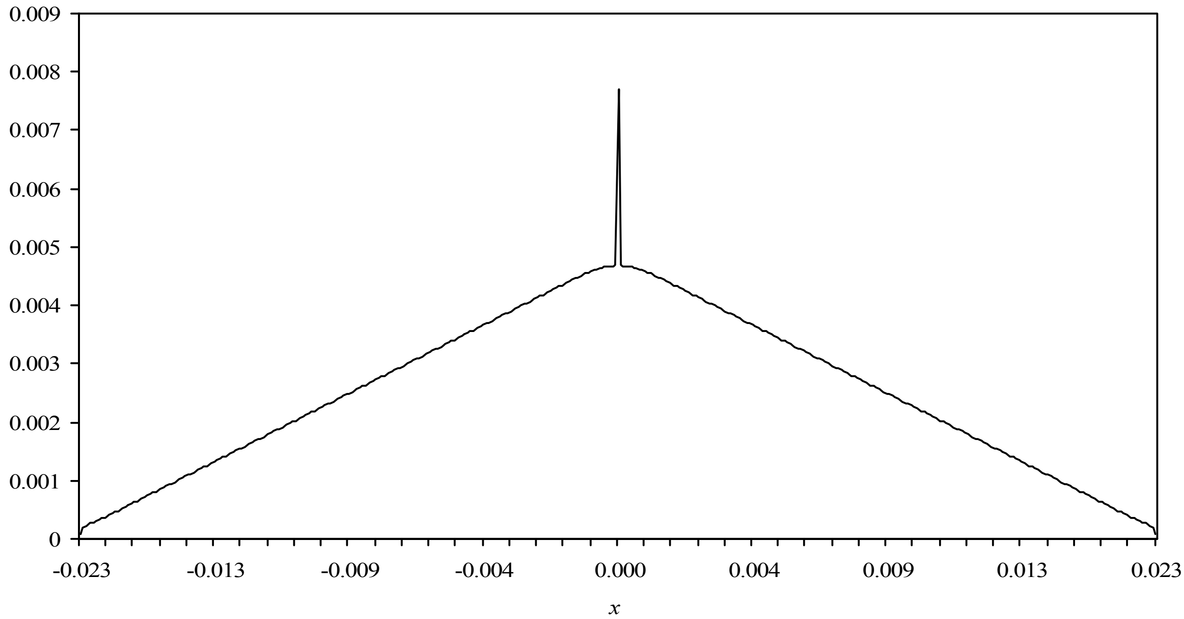


FIGURE 5

Stationary probability distribution for λ = 0.5 and α = 0.8

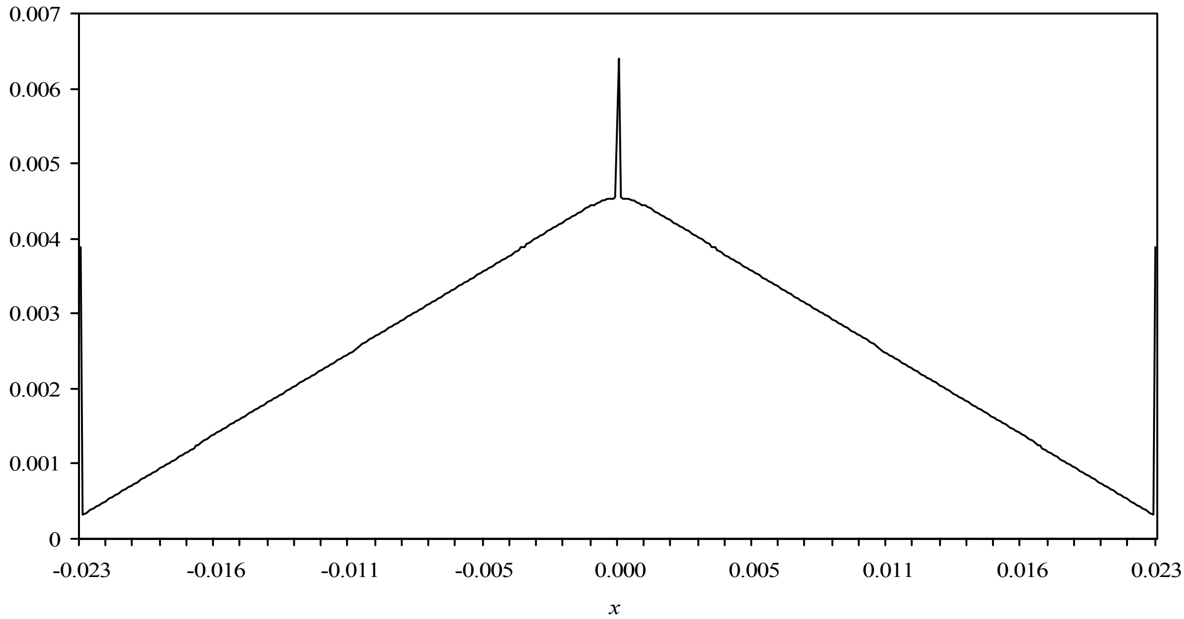


FIGURE 6

Stationary probability distribution for λ = 0.5 and α = 1

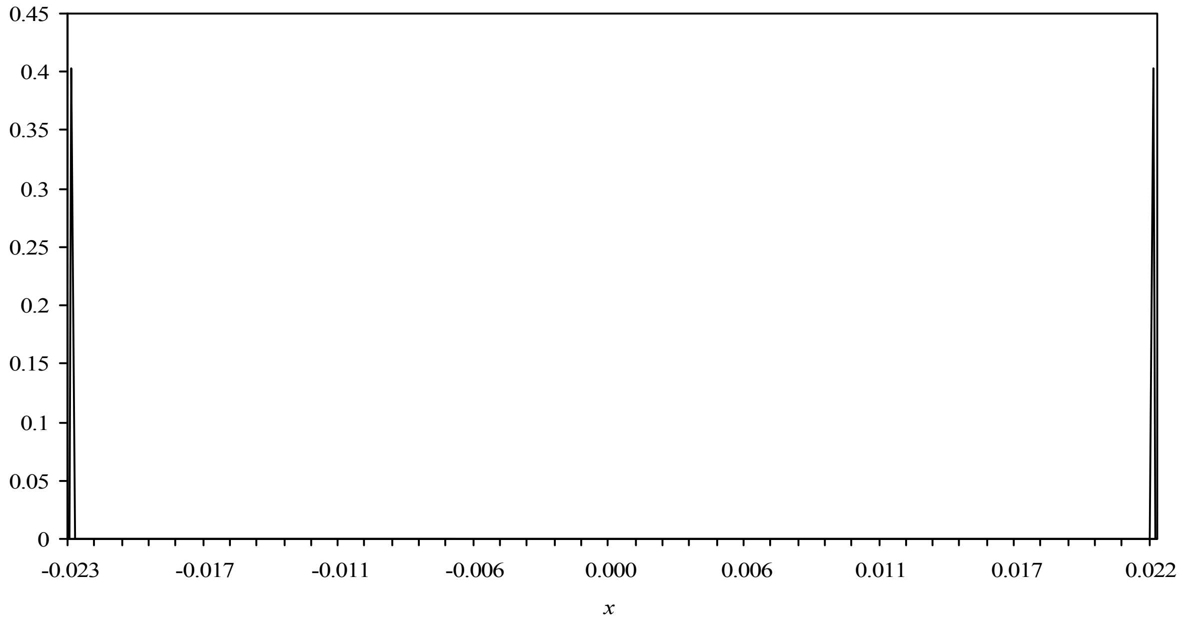
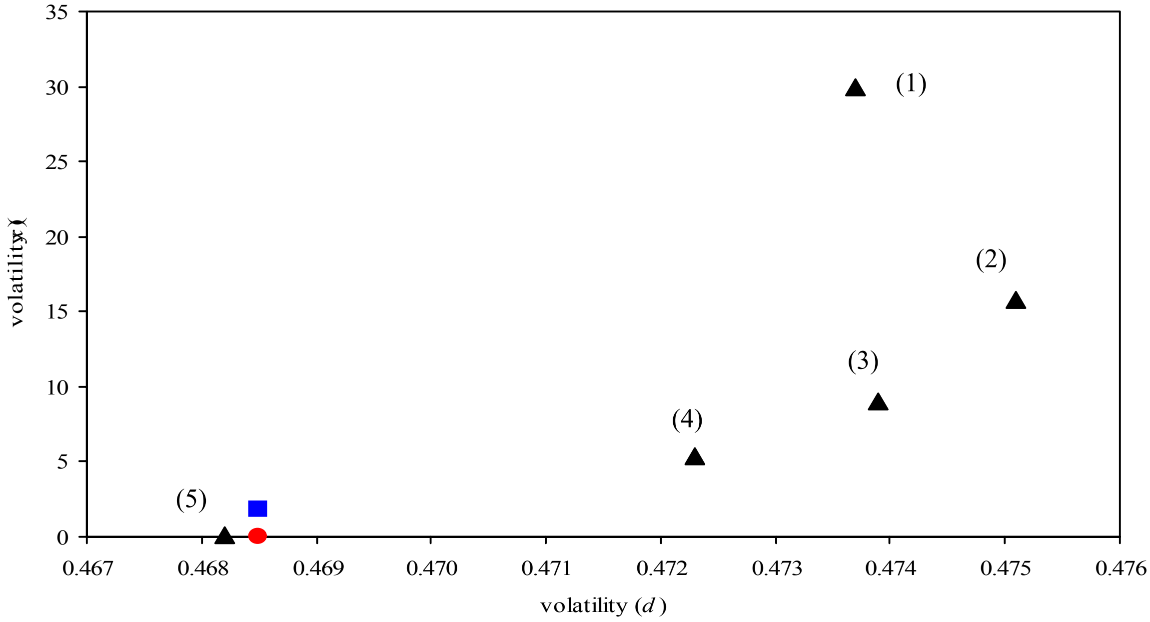


FIGURE 7

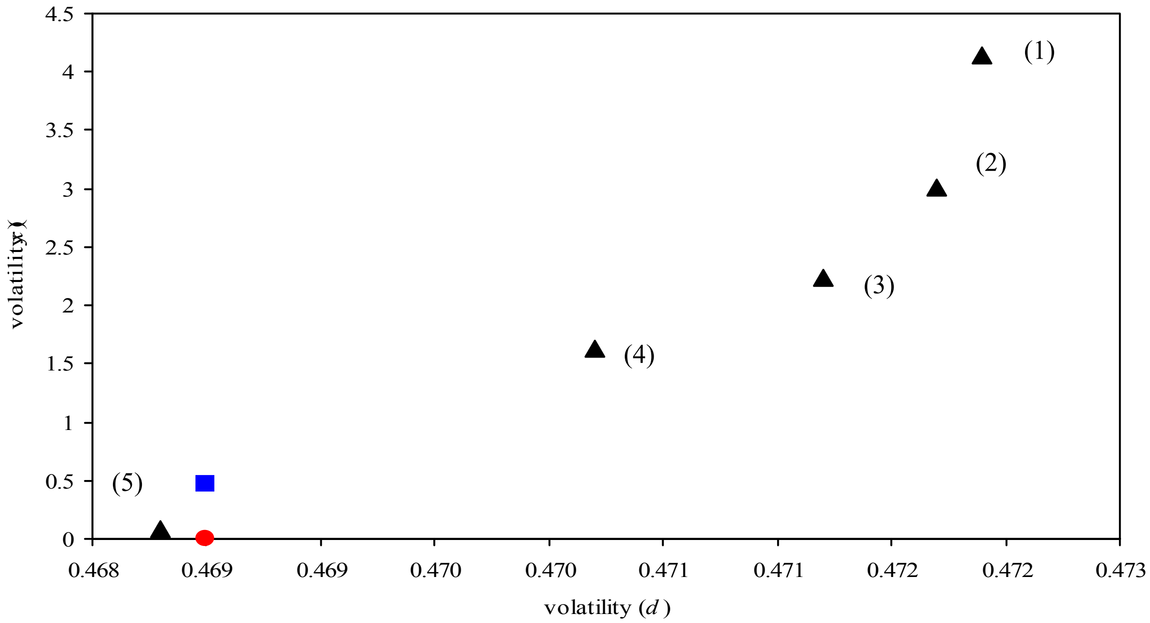
Contributions to the costs for λ = 0.2



Note: This figure shows the volatility of interest rate differentials (d) versus the volatility of the exchange rate (x) for different regimes. The circle is the free float, the square is the managed float. Among the triangles (1) is the target zone with $\alpha = 0$, (2) is the target zone with $\alpha = 0.25$, (3) is the target zone with $\alpha = 0.5$, (4) is the target zone with $\alpha = 0.5$, and (5) is the target zone with $\alpha = 1$.

FIGURE 8

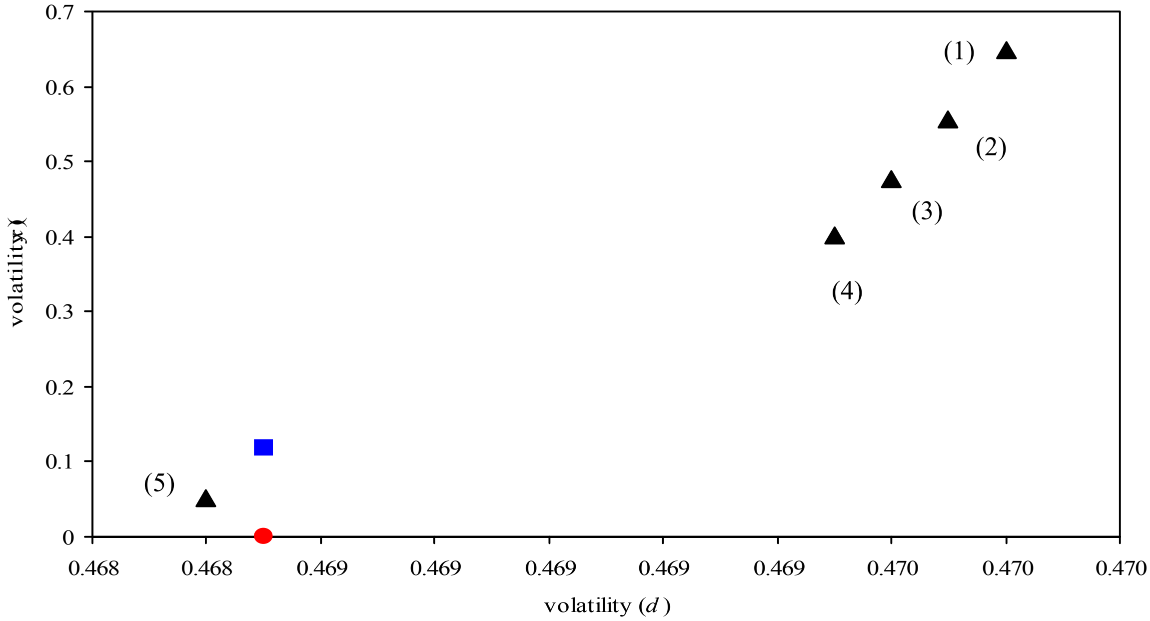
Contributions to the costs for λ = 1



Note: See note to Figure 7.

FIGURE 9

Contributions to the costs for λ = 2



Note: See note to Figure 7.

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