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JEL Codes: H21, H3, H41, H43
Keywords: Second Best, Excess Burden, Public Input.

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Abstract

This paper studies the optimal level of public inputs under two different tax settings: with lump-sum taxes and with labor taxes. With this aim, we adapt the approach by Gronberg and Liu (2001) to the case of productivity-enhancing public spending. On this basis, it is not analytically clear whether or not the first-best level of public spending is higher than the second-best level. A numerical simulation has been carried out to shed some light on this issue. After taking account the type of public input (firm or factor-augmenting), we achieve the conclusion that the second-best level is below the first-best level.

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1 Introduction

A part of the current debate regarding the provision of public goods deals with their optimal levels. Indeed, the controversy concerns more the quantity of public goods than the optimality rules derived from the first order conditions. Papers such as Wilson (1991), Chang (2000) and Gaube (2000, 2005a, 2005b) highlight this topic, in many cases using numerical examples (and counterexamples). The underlying idea is that using distortionary taxation leads to an optimal level of public goods below its first-best level; this is based on the argument that the optimal extent of public spending should be inversely related to the welfare cost derived from distorting taxation. As Pigou (1947) argued, the total welfare cost of public spending must include not only its marginal cost of production but also the deadweight loss caused by distortionary taxation. Government should thus provide less public spending in a second-best scenario than in a situation with lump-sum. However, Gaube (2000, 2005b) shows that this statement is not as straightforward as it might seem. Leaving aside distributional concerns, if some assumptions on the demands of taxed private commodities are not made, Pigou's intuition may go wrong.

All these issues have received very little attention in terms of public inputs. However, we believe that the particular features of productive public spending deserve a specific treatment, as Feehan and Matsumoto (2000, 2002) has recently shown. In this paper, we use a simple model where public spending yields productive services to firms. Different tax settings are available for government: a lump-sum tax, a tax on labour, and taxes on economic profits.

In order to discuss the level of public spending, we follow the approach suggested by Gronberg and Liu (2001), which is based on the sign of marginal excess burden. Particularly, we have adapted their approach to an economy with public inputs and confirmed that the bulk of their results hold. In parallel to their nonseparable public good case, given the assumptions of our model, we cannot determine analytically whether or not the first-best level will exceed the second-best level.

Anyway, as is usual in the literature, we have studied several particular cases in which the standard results found for the public goods remain when public inputs are considered. The nature of returns to scale in the production function with public inputs appears as relevant to make sense the controversy on the optimal level of public spending. In fact, only when the so-called factor-augmenting public input is considered, the optimal policy requires taxes different to those levying rents. Moreover, this particular case implies increasing returns to scale in all the production factors and, consequently, problems related to nonconvexities arise. Accordingly, we have used an appropriate numerical methodology -the Rational Iterative Multisection Procedure (RIMP), see Sánchez and Martínez (2006)- which avoids this difficulty.

Our numerical results show that the first-best level of public input always exceeds the second-best level. This is in line with the mainstream of above references dealing with public goods in which the level reversal appears as unusual. And this happens despite the feedback effect that public inputs exert on tax revenues, which might encourage the level of public spending in a second-best environment by decreasing the marginal cost of provision. We also offer other results concerning the impact of changes in the output elasticity with respect to public input and in the number of households on the level of public input.

The structure of the paper is as follows. Section 2 presents the model and how the Gronberg and Liu's methodology can be applied to our case. Section 3 carries out some numerical simulations in order to obtain the optimal level of public input. Finally, section 4 concludes.

2 The model

We shall set up a theoretical framework as close as possible to that of Gronberg and Liu (2001) in order to make easier the translation of their results to our environment with public inputs. We assume an economy of n identical households whose utility function is expressed as $u(x, l)$ where x is a private good used as a numeraire and l the labor supply[1]. Let Y be the total endowment of time such that $h = Y - l$ is the leisure. Output in the economy is produced using labour services and a public input g according to the aggregate production function $F(nl, g)$. This function satisfies the usual assumptions: increasing in its arguments and strictly concave. The type of returns to scale does not matter at the moment, and consequently using the Feehan's (1989) nomenclature, the public input can be treated as firm-augmenting (constant returns to scale in the private factor and the public input combined, creating rents) or as factor-augmenting (constant retuns to the private factor, and therefore scale economies in all inputs). Output can be costlessly used as x or g.

Labour market is perfectly competitive so that the wage rate is linked to the marginal productivity of labour:

$$\omega = F_L(nl, g), \tag{1}$$

where firms take g as given. Profits may arise and defined as:

$$\pi = F(nl, g) - nl\omega, \tag{2}$$

which will be completely taxed away by government given their inelastic supply[2].

We distinguish two different tax settings. First, we consider a lump-sum tax T so that the representative household faces the following problem:

$$\begin{aligned} &Max \quad u(x,l) \\ s.t. \quad : \quad & x = \omega l - T, \end{aligned} \tag{3}$$

which yields the labour supply $l(\omega, \omega Y - T)$ and the indirect utility function $V(\omega, \omega Y - T)$. It is to be assumed that $l_\omega \geq 0$.

For later use, we describe some comparative statics of $\omega(g, T, n, Y)$ and $\pi(g, T, n, Y)$[3]:

$$\omega_g = \frac{F_{Lg}}{1 - nF_{LL}l_\omega} > 0 \tag{4}$$

$$\omega_T = \frac{nF_{LL}l_T}{1 - nF_{LL}l_\omega} > 0 \tag{5}$$

$$\pi_T = -\frac{nlF_{LL}l_T}{1 - nF_{LL}l_\omega} < 0 \tag{6}$$

An alternative scenario is that using a specific tax on labour τ. Under this tax setting squeme, the consumer's optimization problem could be expressed as:

$$\begin{aligned} &Max \quad u(x,l) \\ s.t. \quad : \quad & x = (\omega - \tau) l \end{aligned} \tag{7}$$

obtaining $l(\omega_N, Y)$ and $V(\omega_N, Y)$, where $\omega_N = \omega - \tau$ is the net wage rate. Again for future reference we derive the following results:

$$\omega_\tau = \frac{-nF_{LL}l_\omega}{1 - nF_{LL}l_\omega} > 0 \tag{8}$$

$$\pi_g = F_g - (nF_{LL}l_\omega + 1) nlF_{Lg} \gtreqless 0 \tag{9}$$

$$\pi_\tau = (1 - \omega_\tau) n^2 lF_{LL}l_\omega < 0 \tag{10}$$

The optimization problem of government in the first-best scenario is as follows:

$$\begin{aligned} &\underset{R}{Max} \quad V(\omega(g), \omega Y - R) \\ s.t. \quad : \quad & g = nR = nT + \pi(g, T), \end{aligned} \tag{11}$$

where $R = T + \pi(g, T)/n$ is the renevue per person[4].

In the second-best scenario, the optimization problem of government is given by:

$$\begin{aligned} & \underset{R}{Max} \quad V(\omega(g), \omega Y - TEB - R) \\ s.t. \quad : \quad & g = nR = n\tau l + \pi(g, \tau), \end{aligned} \tag{12}$$

with $R = \tau l + \pi(g, \tau)/n$ and TEB denoting the total excess burden we define next.

With both tax settings and after some manipulations involving the FOCs of both problems and expressions (4)-(6) and (8)-(10), an important condition for the optimal provision of public inputs is achieved:

$$F_g = 1 \tag{13}$$

It means that the production effects of public input must equal its marginal production cost at optimum. This result is consistent with the production-efficiency theorem by Diamond and Mirrlees (1971).

On the basis of optimality rules, Gaube (2000) and Chang (2000) have suggested several criteria for comparisons between the first and second-best levels of public goods. The support for their approaches is related to the complementarity or substitutability relationships among private goods, and between these and public goods as well. Unfortunately, this procedure has a limitation in our case: the public input does not enter directly the utility function as argument, and consequently cannot be defined *strictu sensu* as a substitute or complement to the (taxed) private goods.

To gain an insight into whether the second-best level may exceed the first-best level, we shall follow the approach suggested by Gronberg and Liu (2001), which is better suited to our environment. The crucial point is the concept of marginal excess burden (MEB). Previously, we define the total excess burden (TEB) of a tax system as the difference between the equivalent variation measure (absolute value) of the loss in utility due to taxation and the revenue collected. Algebraically, TEB can be implicitly given in our case by

$$V(\omega(g), \omega Y - TEB - R) = V(\omega(g, \tau), Y), \tag{14}$$

or explicitly:

$$TEB = -e(\omega(g), V(\omega(g, \tau) - \tau)) + \omega Y - R, \tag{15}$$

where $e(.)$ is the expenditure function and R is the revenue per capita. Hence, the MEB can be defined as $MEB = \dfrac{dTEB}{dR}$.

Gronberg and Liu (2001) in their Propositions 1 and 3 claim that if the utility function is strictly quasi-concave and the $MEB > 0$ for all R, then the second-best public good level lies below the first-best level (sufficient condition). The key issue here is then: can the Gronberg and Liu's approach be used for elucidating the same question but applied to public inputs? In principle, no special problems should arise on this but at least two circumstances may bring new elements to debate. First, public inputs have a clear impact on tax revenues. When public goods are considered, this feedback effect on government revenues only occurs under the assumptions of the nonseparable case, and the reasoning based on the sign of MEB becomes more difficult. But a common feature of public inputs is that they *always* affect government revenues by mean of the feedback effect.

Second, precisely regarding the way through which the public inputs affect tax revenues, this type of public spending has an impact on prices, in particular the wage rate. This prevents the immediate and direct application of the Gronberg and Liu's approach to our framework as long as new relationships between variables have to be taken account.

Next, we will adapt their methodology to the case with public inputs. Particularly, we will prove that the associated indirect utility function is also strictly quasi-concave in (Y, g) and that $\frac{V_Y}{V_g}$ is non-increasing in Y. With these two ingredients we shall arrive at the same conclusion: if the MEB is positive for all R, then the level of public inputs in a second-best economy is less than the first-best level.

Lemma 1 *If the utility function $u(x, l)$ is strictly quasi-concave in x and l, then the associated indirect utility function $V(\omega(g), Y)$ is also strictly quasi-concave in Y and g.*

Proof. Let (x_i, l_i) be the solution to consumer's optimization problem when $Y = Y_i$ and $g = g_i$ $(i = 1, 2)$. That means $u(x_i, l_i) = V(\omega_i(g_i), Y_i)$.

For any $0 < \theta < 1$,

$u(\theta x_1 + (1-\theta)x_2, \theta l_1 + (1-\theta)l_2) \leq$

$V(\theta\omega_1(g_1) + (1-\theta)\omega_2(g_2), \theta Y_1 + (1-\theta)Y_2)$.

As $u(x, l)$ is strictly quasi-concave, then $u(\theta x_1 + (1-\theta)x_2, \theta l_1 + (1-\theta)l_2) >$ $\min\{u(x_1, l_1), u(x_2, l_2)\} = \min\{V(\omega_1(g_1), Y_1), V(\omega_2(g_2), Y_2)\}$.

Therefore, $V(\theta\omega_1(g_1) + (1-\theta)\omega_2(g_2), \theta Y_1 + (1-\theta)Y_2) >$

$\min\{V(\omega_1(g_1), Y_1), V(\omega_2(g_2), Y_2)\}$, which means that $V(\omega(g), Y)$ is strictly quasi-concave in Y and g. ■

Lemma 2 *If the indirect utility function $V(\omega(g), Y)$ is strictly quasi-concave in Y and g, then $\frac{V_Y}{V_g}$ is non-increasing in Y for all g.*

Proof. First, we need to have that $V_g > 0$. Since $V_g = \frac{\partial V}{\partial \omega}\frac{d\omega}{dg}$, we can say that $\frac{\partial V}{\partial \omega} > 0$ by Roy's identity and that $\frac{d\omega}{dg} > 0$ by the expression (4). Consequently, $V_g > 0$.

Second, given that $V\left(\omega\left(g\right), Y\right)$ is strictly quasi-concave in Y and g and the above result on the positive sign of V_g, $\frac{V_Y}{V_g}$ has to be non-increasing in Y for all g. ■

On the basis of these two intermediate results, we can replicate the Gronberg and Liu's (2001) result:

Proposition 3 *Assume that $u(x, l)$ is strictly quasi-concave in x and l. If the $MEB(R) > 0$ for all R, then the second-best public input level is less than the first-best level.*

Proof. Consider the government problem (11). First order condition is as follows:

$$n\left(\frac{\partial V\left(\omega(g), \omega Y - R\right)}{\partial \omega}\frac{d\omega}{dg}\right) - \frac{\partial V\left(\omega(g), \omega Y - R\right)}{\partial Y} = 0. \tag{16}$$

Manipulation gives:

$$\frac{n}{\frac{\partial V(\omega(g), \omega Y - R)/\partial Y}{\left(\frac{\partial V(\omega(g), \omega Y - R)}{\partial \omega}\frac{d\omega}{dg}\right)}} = 1. \tag{17}$$

In turn, the government optimization problem in a second-best scenario is given by (12), which yields the next expression:

$$\frac{n}{\frac{\partial V(\omega(g), \omega Y - TEB - R)/\partial Y}{\left(\frac{\partial V(\omega(g), \omega Y - TEB - R)}{\partial \omega}\frac{d\omega}{dg}\right)}} = 1 + MEB(R). \tag{18}$$

As the $TEB(R) \geqslant 0$ for all R and we have proved that $\frac{V_Y}{V_g}$ is non-increasing in Y, the LHS of (17) is no smaller than the LHS of (18) for any R. Consequently, if $MEB(R) > 0$ for all R, we need that the second-best government revenue R^{SB} to be smaller than the first-best R^{FB} to guarantee that both expressions (17) and (18) hold, and therefore $g^{SB} < g^{FB}$. ■

The question now is how can we elucidate the sign of MEB. Things are more difficult than in the case of public goods because the influence of productive public spending on the relevant functions such as the indirect utility function or the expenditure function is now indirect. Consequently, no clearcut conclusion on the sign of the MEB can be drawn. Particularly, on the basis of expression (15), we have:

$$MEB(R) = \frac{dTEB}{dR} = -1 - \frac{de}{dR} + Y\frac{\partial\omega}{\partial g}\frac{dg}{dR}. \tag{19}$$

The second term can be dissagregated in turn as:

$$\frac{de}{dR} = \frac{\partial e}{\partial\omega}\frac{\partial\omega}{\partial g}\frac{dg}{dR} + \frac{\partial e}{\partial V}\frac{\partial V}{\partial\omega_N}(\frac{\partial\omega_N}{\partial g}\frac{dg}{dR} + \frac{d\omega_N}{dR}). \tag{20}$$

Given the assumptions of our model, all the terms of (20) are positive, except $\frac{d\omega_N}{dR}$ which sign is unclear. Indeed, applying the implicit function theorem on the government budget constraint of problem (12), taking account that $\tau = \omega - \omega_N$, and solving for $\frac{d\omega_N}{dR}$, we have:

$$\frac{d\omega_N}{dR} = \frac{1 - (\omega - \omega_N)\left(\frac{\partial l}{\partial\omega_N}\frac{\partial\omega_N}{\partial g}\frac{dg}{dR}\right) + \left(\frac{\partial\pi}{\partial g} + \frac{\partial\omega}{\partial g}\left(\frac{\partial\pi}{\partial\omega} + l\right)\right)\frac{dg}{dR}}{(\omega - \omega_N)\frac{\partial l}{\partial\omega_N} - l - \frac{\partial\pi}{\partial\omega_N}}, \tag{21}$$

whose sign is indeterminate. Additionally, expression (19) includes a positive term ($Y\frac{\partial\omega}{\partial g}\frac{dg}{dR}$) which adds more doubts on the final sign of MEB.

One could say that our case is essencially equivalent to that of nonseparable public goods by Gronberg and Liu (2001). They claim that it is more likely to have a negative MEB when the public good is a complement to the taxed good, and with public inputs we also have a positive impact of public spending on labor and wage rate, and consequently on tax revenues (feedback effect). In a similar way to the nonseparable public good case, the provision of public inputs makes the tax system less distorting and that may increase the likelihood of having a level reversal result (first-best below the second-best level). However, the complexity of relationships involved in (20) and (21) does not allow us to state whether there is an univocal link between both facts and we conclude that it is not possible to know analytically the sign of the MEB.

3 Level comparisons of public input provision

A standard way of giving an insight into the debate on the optimal level of public inputs is using numerical procedures to solve particular cases. With this aim, we consider three different utility functions in an attempt to achieve results as general as possible and related to previous references on public goods. Particularly, we have chosen the quasi-linear utility function (Gronberg and Liu, 2001); the Cobb-Douglas utility functions (Atkinson and Stern, 1974;

Wilson, 1991); and the CES utility function (Wilson, 1991b; Gaube, 2000). Specifically,

$$U(x,h) = x + 2h^{\frac{1}{2}} \tag{22}$$

$$U(x,h) = a \log x + (1-a) \log h \tag{23}$$

$$U(x,h) = \left(x^{\rho} + h^{\rho}\right)^{\frac{1}{\rho}}, \tag{24}$$

where $a \in (0,1)$ and $\rho = 0.5$. The relevant point in our case comes from the specification of the production function because the different alternatives by defining how the private and public factors enter the production function have notable implications on the debate. In particular, whether this function exhibits constant returns to scale in public and private inputs (firm-augmenting public input) or only constant returns to the private factors (factor-augmenting public input) have consequences on the controversy.

3.1 Firm-augmenting public input

We assume a Cobb-Douglas production function given by $F(nl,g) = (nl)^{\alpha} g^{1-\alpha}$, where $\alpha \in (0,1)$. This specification creates firm-specific rents. As Pestieau (1976) proved, if these rents are also an argument in the consumer's indirect utility function, the optimal spending condition is not the first-best one; however, recall that our model precisely establishes that all economic rents are taxed away by the government. Therefore, the production efficiency condition $F_g = 1$ applies here and, consequently, the numerical solution comes from solving the simultaneous equation system consisting of the production efficiency condition for g and the government budget constraint, with firms and households solving their respective optimization problems. We have used the Newton-Raphson's algorithm.

Tables 1-3 show the main results for several scenarios and each utility function. Particularly, we have taken $a \in \{0.1, \mathit{0.5}, 0.9\}$, $\alpha \in \{0.6, \mathit{0.7}, 0.8\}$ and $n \in \{1, \mathit{100}, 1000\}$ as the set of parameters to be used, where the benchmark values have been emphasized. Note that no distinction is done between distorting and lump-sum taxes. This is why the controversy between the first-best and second-best level of public spending has no sense when the firm-augmenting public input creates rents which are completely taxed by the government. Indeed, both the analytical solution of our model and its numerical resolution give the intuitive result that the optimal level of productive public spending must be exclusively financed with the economic rents.

In a sense, this situation can be compared to that of Feehan and Batina (2004), in which a (semi)public input is equivalent to a common property resource. A Lindahl pricing system is then the appropriate policy instrument, with a charge on firms (only one in our framework) for their utilization and according to the value of public input's marginal contribution to the firms' profits (Sandmo, 1972). All in all, the complete taxation of rents implies to solve the common problem arising when public input provision is involved.

INSERT TABLES 1-3 ABOUT HERE

Several comments can be drawn regarding Tables 1-3. First, the level of public input provision has a non-monotonic relationship with its output share $(1-\alpha)$ in the production function in the case of the quasilinear and CES utility functions, while the relationship is increasing in the case of the Cobb-Douglas utility function. This latter result is easy to be explained; it can be proved that F_g is decreasing in α. Since with a Cobb-Douglas utility the labor supply is fixed, the only way to hold $F_g = 1$ is to provide a lower level of public input. Things are more complex for the case of the other two utility functions. The use of both quasi-linear and CES functions implies a labor supply varying with respect to α, and hence the adjustments to hold $F_g = 1$ are not limited to the level of public inputs but also to the quantity of labor.

Second, there is a closely link between the number of households and the level of public inputs for all the utility specifications. Particularly, the quantity of public input increases at the same rate than the size of population. Since the production function exhibits constant return to scale (i. e., it is homogeneous of degree 1), it can be claimed that the function $\pi(.)$ is also homogeneous of degree 1. Accordingly, increases in the number of households are followed by increases in profits at the same rate, and consequently by identical increases in the provision of public inputs.

Third, as a comment referred to the Cobb-Douglas case exclusively, it can be seen how the level of public input increases when the preference for the private good goes up (parameter a). The higher the preference for the private good, the smaller the preference for leisure, and consequently more time is devoted to work. Given the assumptions of the model, this leads to increase the production and decrease the wages, which jointly determine higher economic rents[5].

3.2 Factor-augmenting public input

The main difference between the above environment and this of factor-augmenting lies in the assumptions on the returns to scale in the production function. Particularly, we assume again a Cobb-Douglas production function but exhibiting increasing returns in all the inputs (constant returns in labor): $F(nl, g) = nlg^{\beta}$, where $\beta \in (0, 1)$. Under this framework, the debate on the level of public spending in alternative tax settings is reborn. Indeed, the use of lump-sum or distorting taxes is necessary here as long as rents are null. Here we have considered $a \in \{0.1, \mathit{0.5}, 0.9\}$, $\beta \in \{0.1, \mathit{0.2}, 0.3\}$ and $n \in \{1, \mathit{100}, 1000\}$ as the set of parameters to be taken account, where the benchmark values have been again emphasized.

Solving the government optimization problem with factor-augmenting public inputs is not as straightforward as before. In fact, the Newton-Raphson algorithm presents some caveats when non-convex sets of constraints are involved. Note that this is our case because we have increasing returns in the production side of the model. Consequently, in order to avoid the problems derived from multiple equilibria and corner solutions, we have adopted an alternative algorithm: the Rational Iterative Multisection Procedure (RIMP), which has relative advantages with respect to the standard Newton-Raphson method under these conditions.

The RIMP consists of an iterative subdivision of the initial decision variables set. On this basis, we select the points of the grid that satisfy the constraints of the problem with a determined precision in each stage. This process continues until the maximum previously fixed precision is achieved. Therefore, the different stages can be interpreted as different requirements of precision by obtaining the optimal solution. Additionally, we have checked that the Newton-Raphson algorithm and the RIMP obtain the same solution when the optimization problem has enough regularity properties (for instance, the firm-augmenting case). For a detailed explanation of RIMP, see Sanchez and Martinez (2006).

Tables 4-6 report the levels of public inputs, labor supply and the tax rates depending on the tax setting for the three utility functions. The most important issue is that the second-best level of public input is always below the first-best level[6]. Therefore we are here in line with the mainstream of previous literature in which the level reversal is unusual. Although the presence of a feedback effect on tax revenues might increase the likelihood of having a negative MEB, we confirm that this situation is certainly an atypical case.

INSERT TABLES 4-6 ABOUT HERE

Other interesting issues are as follows. First, the level of provision in the second-best scenario is always non-decreasing in the output share of public inputs β for the three utility functions. This contrasts with the firm-augmenting case, in which this fact only occurred with the Cobb-Douglas utility. Recall that the labor supply increased with the firm-augmenting public input for CES and quasi-linear utility functions when the output share of public inputs $(1-\alpha)$ decreased. That fact allowed to hold $F_g = 1$ with a smaller output share for the public input without reducing the level provision of g. The relationship between the level of public input and $(1-\alpha)$ was non-monotonic for these utility functions.

However, things are different when a factor-augmenting public input is considered. An increase in the parameter β leads to a higher F_g.[7] As the labor supply is always increasing in β and given that F_g is increasing in nl, the only way to hold the production efficiency condition is going up the level of public input.

Second, the level of public input is not just linearly increasing in the number of households, but a direct relationship remains. Again the explanation comes from holding the production efficiency condition. As we have already pointed out, our model gives that F_g is increasing in nl and consequently when the population is growing, *ceteris paribus*, the level of public input has to increase in order to diminish its productivity.

Third, although their values are not reported here, the level of utility achieved is increasing in the output share of public inputs β. In other words, the higher the elasticity of output with respect to the public input, the bigger the utility of the representative agent. This result has a clear policy implication: government must be aware of the productivity impact of public input because this is welfare-enhancing.

4 Concluding remarks

This paper has dealt with an issue to which the existing literature has not paid much attention: the optimal *level* of public inputs under different tax settings. Previous contributions have focused on the case of consumption public goods or have discussed the optimal *rules* of productive public spending. However, both the welfare implications of taxation and the characterization of public inputs as a growth-enhacing tool make this issue highly relevant for policy-makers.

We have built a simple model where public inputs provide productive services to firms. Different tax settings are available for government: a lump-sum tax, a tax on labour, and taxes on economic profits. In order to shed some light on the controversy on level comparisons, we have adopted the Gronberg and Liu (2001)'s strategy. This consists of studying the sign of MEB: if positive, the first-best is higher than the second-best level; otherwise if negative.

On the basis of the MEB's approach, we conclude that it is not possible to elucidate analytically the sign of the MEB. In principle, the feedback effect arising as the public input encourages the tax base might reduce the welfare loss in a distorting tax setting, and consequently increasing the likelihood of having a negative MEB. However, in a similar way to the nonseparable case by Gronberg and Liu (2001), the complexity of relationships which are involved in the expression of MEB makes impossible to know analytically whether or not the first-best level is higher. In fact, the expression we obtain for the MEB is much more cumbersome than that of public goods, even under the nonseparable case. This is due to the fact that public inputs have a direct effect on prices (wages) and this yields that things become more complicated.

Given the caveats of this analytical approach for the case of public inputs, we have implemented a numerical simulation to give an insight into the controversy. With this aim, we have taken three standard utility functions and used a more suitable new algorithm to solve non-convex optimization problems. The first battery of results refer to the case of firm-augmenting public inputs. This situation creates rents which are taxed away by the government, and precisely they suffice to finance the optimal public spending. Therefore, the debate on level comparisons under different tax settings has no scope in this case. Anyway, we have obtained that the level of public inputs is linearly proportionate to the number of households and the relationship between the output share of public inputs and their levels of provision is positive in the case of the Cobb-Douglas utility function but non-monotonic for the other two.

A second group of results provides information in the case of factor-augmenting public inputs. Here the controversy on the optimal level of public spending matters because government financing is not based on levying rents but labor or lump-sum taxes have to be used. Our numerical results are clear: the level of public input in the first-best scenario always exceeds that of the second-best. Additionally, we find that the optimal level of public inputs is non-decreasing in the output share of public inputs and in the number of households.

Our analysis based on the MEB could be translated to the cost-benefit analysis (CBA). Triest (1990) and Browning *et al.* (2000) have developed a discussion on the relationship between the MEB and the marginal cost of public funds, which is the key variable in the cost-benefit analysis. The challenge for a further development of this paper could be extending the

Gronberg and Liu's methodology to the CBA.

Finally, some policy implications could be extracted from this paper. One of them is that it is essential to identify the nature of returns to scale in a production function with public inputs. In fact, only when the so-called factor-augmenting public input is considered, the optimal policy requires taxes different to those levying rents, such as income or factor taxes. Alternatively, when a "firm-augmenting" public input is considered, the complete taxation of rents is the best policy option to solve the commons (or second-best) problem we face. Moreover, the policy-makers have to be aware of the welfare implications stemming from the provision of public inputs. These are not only public expenditures affecting the production side of economy, but also they have a great significance in terms of households' utility levels.

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Tables

Table 1: Firm-augmenting. Quasi-linear utility.

	Benchmark	$n = 100$		$\alpha = 0.7$	
		$\alpha = 0.6$	$\alpha = 0.8$	$n = 1$	$n = 1000$
Public Input	327,2103	316,4995	274,2657	3,2721	3272,1030
Labour Supply	18,2723	14,5749	20,5061	18,2723	18,2723
Profits	327,2080	316,4996	274,2657	3,2721	3272,0800

Source: Benchmark ($n = 100, \alpha = 0.7$)

Table 2: Firm-augmenting. Cobb-Douglas utility.

	Benchmark	$n = 100, \alpha = 0.7$		$n = 100, a = 0.5$		$a = 0.5, \alpha = 0.7$	
		$a = 0.1$	$a = 0.9$	$\alpha = 0.6$	$\alpha = 0.8$	$n = 1$	$n = 1000$
Public Input	214,8877	42,9775	386,7978	260,5851	160,5007	2,1489	2148,8770
Labour Supply	12,0000	2,4000	21,6000	12,0000	12,0000	12,0000	12,0000
Profits	214,8877	42,9775	386,7979	260,5845	160,4983	2,1489	2148,8770

Source: Benchmark ($n = 100, \alpha = 0.7, a = 0.5$)

Table 3: Firm-augmenting. CES utility ($\rho = 0.5$).

	Benchmark	$n = 100$		$\alpha = 0.7$	
		$\alpha = 0.6$	$\alpha = 0.8$	$n = 1$	$n = 1000$
Public Input	126,6551	128,0503	111,8768	1,2666	1266,5510
Labour Supply	7,0728	5,8977	8,3647	7,0728	7,0728
Profits	126,6550	128,0503	111,8768	1,2666	1266,5500

Source: Benchmark ($n = 100, \alpha = 0.7$)

Table 4: Factor-augmenting. Quasi-linear utility.

	Benchmark	$n = 100$		$\beta = 0.2$	
		$\beta = 0.1$	$\beta = 0.3$	$n = 1$	$n = 1000$
Lump-sum					
Tax rate	1,4300	1,4300	1,4300	6,9500	1,4100
Public Input	143,0040	143,0100	142,9940	6,9600	1409,8300
Labour Supply	23,8626	23,6294	23,9491	23,5398	23,9450
Distorsionary					
Tax rate	0,0100	0,0100	0,0100	0,2900	0,0100
Public Input	23,7680	23,4910	23,8580	6,7600	238,9000
Labour Supply	23,7154	23,4603	23,8498	23,2763	23,8874

Source: Benchmark ($n = 100, \beta = 0.2$)

Table 5: Factor-augmenting. Cobb-Douglas utility.

	Benchmark	$n = 100, a = 0.5$		$\beta = 0.2, a = 0.5$		$n = 100, \beta = 0.2$	
		$\beta = 0.1$	$\beta = 0.3$	$n = 1$	$n = 1000$	$a = 0.1$	$a = 0.9$
Lump-sum							
Tax rate	1,4300	1,4300	1,4300	3,4100	1,4100	1,4300	1,4300
Public Input	143,0040	143,0400	143,0120	3,4100	1410,0300	143,0100	143,0080
Labour Supply	12,2650	12,4353	12,1613	13,3340	12,1653	2,8770	21,6530
Distorsionary							
Tax rate	0,1100	0,1100	0,1100	0,2480	0,1100	0,5100	0,0100
Public Input	132,0100	132,0100	132,0100	2,9860	1320,0100	122,4100	21,6100
Labour Supply	12,0000	12,0000	12,0000	12,0000	12,0000	2,4000	21,0000

Source: Benchmark ($n = 100, \beta = 0.2, a = 0.5$)

Table 6: Factor-augmenting. CES utility ($\rho 0.5$).

	Benchmark	$n = 100$		$\beta = 0.2$	
		$\beta = 0.1$	$\beta = 0.3$	$n = 1$	$n = 1000$
Lump-sum					
Tax rate	1,4280	1,4300	1,4300	3,8900	1,4100
Public Input	142,8880	143,0340	143,0360	3,9000	1409,9860
Labour Supply	17,6528	15,2476	19,6414	14,9043	19,5038
Distorsionary					
Tax rate	0,0100	0,0100	0,0100	0,2480	0,0100
Public Input	15,1500	13,5010	16,8200	2,9800	176,9030
Labour Supply	15,1511	13,5073	16,7764	11,9764	17,6932

Source: Benchmark ($n = 100, \beta = 0.2$)