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JEL Codes: H21, H3, H41, H43
Keywords: Second Best, Excess Burden, Public Input.

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Abstract

This paper studies the optimal level of public inputs under two different tax settings. With this aim, we adapt the approach by Gronberg and Liu (2001) to the case of productivity-enhancing public spending. We find that it is not analytically clear whether the first-best level of public spending exceeds the second-best level. After taking account the type of public input, a wide numerical simulation has been carried out. We obtain that the second-best level is always below the first-best level but the criterion by Gronberg and Liu has to be qualified.

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1 Introduction

A part of the current debate regarding the provision of public goods deals with their optimal levels. Indeed, the controversy concerns more the quantity of public goods than the optimality rules derived from the first order conditions. Papers such as Wilson (1991a,b), Chang (2000) and Gaube (2000, 2005a, 2005b) highlight this topic, in many cases using numerical examples (and counterexamples). The underlying idea is that using distortionary taxation leads to an optimal level of public goods below its first-best level; this is based on the argument that the optimal extent of public spending should be inversely related to the welfare cost derived from distorting taxation. As Pigou (1947) argued, the total welfare cost of public spending must include not only its marginal cost of production but also the deadweight loss caused by distortionary taxation. Government should thus provide less public spending in a second-best scenario than in a situation with lump-sum. However, Gaube (2000, 2005b) shows that this statement is not as straightforward as it might seem. Leaving aside distributional concerns, Pigou's intuition may go wrong under certain assumptions on the demand of taxed private commodities.

All these issues have received very little attention in terms of public inputs. However, we believe that the particular features of productive public spending deserve a specific treatment, as Feehan and Matsumoto (2000, 2002) have recently shown. In this paper, we use a simple model where public spending yields productive services to firms. Different tax settings are available for government: a lump-sum tax, a tax on labour, and taxes on economic profits.

In order to discuss the level of public spending, we follow the approach suggested by Gronberg and Liu (2001), which is based on the sign of marginal excess burden (MEB). Particularly, we adapt their approach to an economy with public inputs and confirmed that the bulk of their results hold. In parallel to their nonseparable public good case, given the assumptions of our model, we cannot determine analytically whether or not the first-best level will exceed the second-best level.

With the aim of going beyond this ambiguity, as is usual in the literature, we have studied several particular cases in which the standard results found for the public goods remain when public inputs are considered. The nature of returns to scale in the production function with public inputs appears as crucial to make sense the controversy on the optimal level of public spending. In fact, only when the so-called factor-augmenting public input is considered, the optimal policy requires taxes different to those levying rents.

Our numerical results show that the first-best level of public input always exceeds the second-best level. This is in line with the mainstream of above references dealing with public goods in which the level reversal appears as unusual. And this happens despite the feedback effect that public inputs exert on tax revenues, which might encourage the level of public spending in a second-best environment by decreasing the marginal cost of provision. Additionally, our paper qualifies the contribution of Gronberg and Liu (2001) by using the criterion of the sign of MEB. Indeed, they establish that a positive MEB is a sufficient and necessary condition for having a normal level relation when preferences are homothetic. However, we obtain that the first-best level of public input is higher than the second-best level despite of having a negative MEB and with a Cobb-Douglas utility function. We also offer other results concerning the impact of changes in the output elasticity with respect to public input and in the number of households on the levels of public input and utility.

The structure of the paper is as follows. Section 2 presents the model. Section 3 presents how the Gronberg and Liu's methodology can be applied to our case and carries out some numerical simulations in order to obtain the optimal level of public input. Finally, section 4 concludes.

2 The model

We shall set up a theoretical framework as close as possible to that of Gronberg and Liu (2001) in order to make easier the translation of their approach to our model with public inputs. We assume an economy of n identical households whose utility function is expressed as $u(x,l)$ where x is a private good used as a numeraire and l the labor supply[1]. Let Y be the total endowment of time such that $h = Y - l$ is the leisure. Output in the economy is produced using labour services and a public input g according to the aggregate production function $F(nl, g)$. This function satisfies the usual assumptions: increasing in its arguments and strictly concave. Output can be costlessly used as x or g.

Labour market is perfectly competitive so that the wage rate is given by the marginal productivity of labour:

$$\omega = F_L(nl, g), \tag{1}$$

where firms take g as exogenous. Profits may arise and defined as:

$$\pi = F(nl, g) - nl\omega, \tag{2}$$

[1]The properties of $u(x,l)$ are the standard ones to ensure a well-behaved function: strictly monotone, quasiconcave and twice differentiable.

which will be completely taxed away by government[2].

We distinguish two different tax settings. First, we consider a lump-sum tax T so that the representative household faces the following problem:

$$\begin{aligned} &Max \quad u(x,l) \\ s.t. \quad : \quad &x = \omega l - T, \end{aligned} \tag{3}$$

which yields the labour supply $l(\omega, \omega Y - T)$ and the indirect utility function $V(\omega, \omega Y - T)$. It is to be assumed that $l_\omega \geq 0$.

For later use, we describe some comparative statics of $\omega(g, T, n, Y)$ and $\pi(g, T, n, Y)$[3]:

$$\omega_g = \frac{F_{Lg}}{1 - nF_{LL}l_\omega} > 0 \tag{4}$$

$$\omega_T = \frac{nF_{LL}l_T}{1 - nF_{LL}l_\omega} > 0 \tag{5}$$

$$\pi_T = -\frac{nlF_{LL}l_T}{1 - nF_{LL}l_\omega} < 0 \tag{6}$$

An alternative scenario is that using a specific tax on labour τ. Under this tax setting squeme, the consumer's optimization problem could be expressed as:

$$\begin{aligned} &Max \quad u(x,l) \\ s.t. \quad : \quad &x = (\omega - \tau) l \end{aligned} \tag{7}$$

obtaining $l(\omega_N, \omega Y)$ and $V(\omega_N, \omega Y)$ where $\omega_N = \omega - \tau$ is the net wage rate. Again for future reference we derive the following results:

$$\omega_\tau = \frac{-nF_{LL}l_\omega}{1 - nF_{LL}l_\omega} > 0 \tag{8}$$

$$\pi_g = F_g - (nF_{LL}l_\omega + 1) nlF_{Lg} \gtreqless 0 \tag{9}$$

$$\pi_\tau = (1 - \omega_\tau) n^2 lF_{LL}l_\omega < 0 \tag{10}$$

The optimization problem of government in the first-best scenario is as follows:

$$\begin{aligned} &\underset{R}{Max} \quad V(\omega(g), \omega Y - R) \\ s.t. \quad : \quad &g = nR = nT + \pi(g, T), \end{aligned} \tag{11}$$

[2]Pestieau (1976) analyzed how the optimal rule for the provision of public inputs has to be modified when these rents are not taxed away.

[3]Note that variables n and Y are exogenously determined. For the sake of simplicity, we will drop them hereafter as arguments in these functions.

where $R = T + \pi(g, T)/n$ is the renevue per person[4].

In the second-best scenario, the optimization problem of government is given by:

$$\begin{aligned} &\underset{R}{Max} \quad V(\omega(g), \omega Y - TEB - R) \\ s.t. \ : \ & g = nR = n\tau l + \pi(g, \tau), \end{aligned} \tag{12}$$

with $R = \tau l + \pi(g, \tau)/n$ and TEB denoting the total excess burden defined as the difference between the equivalent variation measure (absolute value) of the loss in utility due to taxation and the revenue collected[5]. Algebraically, TEB can be implicitly given in our case by

$$V(\omega(g), \omega Y - TEB - R) = V(\omega(g, \tau), \omega Y), \tag{13}$$

or explicitly:

$$TEB = -e(\omega(g), V(\omega(g, \tau) - \tau)) + \omega Y - R, \tag{14}$$

where $e(.)$ is the expenditure function and R is the revenue per capita. Hence, the MEB can be defined as $MEB = \dfrac{dTEB}{dR}$.

With both tax settings and after some manipulations involving the FOCs of both problems and expressions (4)-(6) and (8)-(10), an important condition for the optimal provision of public inputs is achieved:

$$F_g = 1 \tag{15}$$

It means that the production effects of public input must equal its marginal production cost at optimum. This result is consistent with the production-efficiency theorem by Diamond and Mirrlees (1971).

3 Level comparisons of public input provision

3.1 The Marginal Excess Burden approach

Different approaches have been used in the literature for comparing the optimal level of public input under different tax settings. Based upon optimality rules, Gaube (2000) and Chang (2000) have suggested several criteria for

[4]Here it is useful to consider that rents accrue to consumers before being taxing away by government.

[5]This definition based on equivalent variations is superior to other measures, for instance those based on compensating variations (Pazner and Sadka, 1980; Kay, 1980).

comparisons between the first and second-best levels of public goods. The support for their approaches is related to the complementarity or substitutability relationships among private goods, and between these and public goods as well. Unfortunately, this procedure has a limitation in our case: the public input does not enter directly the utility function as argument, and consequently cannot be defined *strictu sensu* as a substitute or complement to the (taxed) private goods.

To gain an insight into whether the second-best level may exceed the first-best level, we shall follow the approach suggested by Gronberg and Liu (2001), which is better suited to our environment.The crucial point is the concept of marginal excess burden (MEB). Previously, we define the total excess burden (TEB) of a tax system as

Gronberg and Liu (2001) in their Propositions 1 and 3 claim that if the utility function is strictly quasi-concave and the $MEB > 0$ for all R, then the second-best public good level lies below the first-best level (sufficient condition). The key issue here is then: can the Gronberg and Liu's approach be used for elucidating the same question applied to public inputs? In principle, no special problems should arise on this but at least two circumstances may bring new elements to debate. First, public inputs have a clear impact on tax revenues. When public goods are considered, this feedback effect on government revenues only occurs under the assumptions of the nonseparable case. With public inputs this feedback effect *always* appear and the reasoning based on the sign of MEB becomes more difficult.

Second, precisely regarding the way through which the public inputs affect tax revenues, this type of public spending has an impact on prices, in particular on the wage rate. This prevents the immediate and direct application of the Gronberg and Liu's approach to our framework as long as new relationships between variables have to be taken account.

Next, we will adapt their methodology to the case with public inputs. Particularly, we will prove that the associated indirect utility function is also strictly quasi-concave in $(\omega Y, g)$ and that $\frac{V_{\omega Y}}{V_g}$ is non-increasing in ωY. With these two ingredients we shall arrive at the same conclusion: if the MEB is positive for all R, then the level of public inputs in a second-best economy is less than the first-best level.

Lemma 1 *If the utility function $u(x,l)$ is strictly quasi-concave in x and l, then the associated indirect utility function $V(\omega(g), \omega Y)$ is also strictly quasi-concave in g and ωY.*

Proof. Let (x_i, l_i) be the solution to consumer's optimization problem when $Y = Y_i$, $\omega = \omega_i$ and $g = g_i$ $(i = 1, 2)$. That means $u(x_i, l_i) = V(\omega_i(g_i), \omega_i Y_i)$.

For any $0 < \theta < 1$,
$u\left(\theta x_1 + (1-\theta)x_2, \theta l_1 + (1-\theta)l_2\right) \leq$
$V\left(\theta\omega_1\left(g_1\right) + (1-\theta)\omega_2\left(g_2\right), \theta\omega_1 Y_1 + (1-\theta)\omega_2 Y_2\right).$
As $u(x,l)$ is strictly quasi-concave, then $u\left(\theta x_1 + (1-\theta)x_2, \theta l_1 + (1-\theta)l_2\right) >$ $\min\{u\left(x_1, l_1\right), u\left(x_2, l_2\right)\} = \min\{V\left(\omega_1\left(g_1\right), \omega_1 Y_1\right), V\left(\omega_2\left(g_2\right), \omega_2 Y_2\right)\}$.
Therefore, $V\left(\theta\omega_1\left(g_1\right) + (1-\theta)\omega_2\left(g_2\right), \theta\omega_1 Y_1 + (1-\theta)\omega_2 Y_2\right) >$ $\min\{V\left(\omega_1\left(g_1\right), \omega_1 Y_1\right), V\left(\omega_2\left(g_2\right), \omega_2 Y_2\right)\}$, which means that $V\left(\omega\left(g\right), \omega Y\right)$ is strictly quasi-concave in g and ωY. ■

Lemma 2 *If the indirect utility function $V\left(\omega\left(g\right), \omega Y\right)$ is strictly quasi-concave in g and ωY, then $\frac{V_{\omega Y}}{V_g}$ is non-increasing in ωY for all g.*

Proof. First, we need to have that $V_g > 0$. Since $V_g = \frac{\partial V}{\partial \omega}\frac{d\omega}{dg}$, we can say that $\frac{\partial V}{\partial \omega} > 0$ by Roy's identity and that $\frac{d\omega}{dg} > 0$ by the expression (4). Consequently, $V_g > 0$.

Second, given that $V\left(\omega\left(g\right), \omega Y\right)$ is strictly quasi-concave in g and ωY and the above result on the positive sign of V_g, $\frac{V_{\omega Y}}{V_g}$ has to be non-increasing in ωY for all g. ■

On the basis of these two intermediate results, we can replicate the Gronberg and Liu's (2001) result:

Proposition 1 *Assume that $u(x,l)$ is strictly quasi-concave in x and l. If the $MEB(R) > 0$ for all R, then the second-best public input level is less than the first-best level.*

Proof. Consider the government problem (11). First order condition is as follows:

$$n\left(\frac{\partial V\left(\omega(g), \omega Y - R\right)}{\partial \omega}\frac{d\omega}{dg}\right) - \frac{\partial V\left(\omega(g), \omega Y - R\right)}{\partial\left(\omega Y\right)} = 0. \qquad (16)$$

Manipulation gives:

$$\frac{n}{\frac{\partial V(\omega(g), \omega Y - R)/\partial(\omega Y)}{\left(\frac{\partial V(\omega(g), \omega Y - R)}{\partial \omega}\frac{d\omega}{dg}\right)}} = 1. \qquad (17)$$

In turn, the government optimization problem in a second-best scenario is given by (12), which yields the next expression:

$$\frac{n}{\frac{\partial V(\omega(g), \omega Y - TEB - R)/\partial(\omega Y)}{\left(\frac{\partial V(\omega(g), \omega Y - TEB - R)}{\partial \omega}\frac{d\omega}{dg}\right)}} = 1 + MEB(R). \qquad (18)$$

As the $TEB(R) \geqslant 0$ for all R and we have proved that $\frac{V_{\omega Y}}{V_g}$ is non-increasing in ωY, the LHS of (17) is no smaller than the LHS of (18) for any R. Consequently, if $MEB(R) > 0$ for all R, we need that the second-best government revenue R^{SB} to be smaller than the first-best R^{FB} to guarantee that both expressions (17) and (18) hold, and therefore $g^{SB} < g^{FB}$. ■

The question now is how can we elucidate the sign of MEB. Things are more difficult than in the case of public goods because the influence of productive public spending on the relevant functions such as the utility function or the expenditure function is now indirect. Consequently, no clearcut conclusion on the sign of the MEB can be drawn. Particularly, on the basis of expression (15), we have:

$$MEB(R) = \frac{dTEB}{dR} = -1 - \frac{de}{dR} + Y\frac{\partial \omega}{\partial g}\frac{dg}{dR}. \tag{19}$$

The second term can be dissagregated in turn as:

$$\frac{de}{dR} = \frac{\partial e}{\partial \omega}\frac{\partial \omega}{\partial g}\frac{dg}{dR} + \frac{\partial e}{\partial V}\frac{\partial V}{\partial \omega_N}\left(\frac{\partial \omega_N}{\partial g}\frac{dg}{dR} + \frac{d\omega_N}{dR}\right). \tag{20}$$

Given the assumptions of our model, all the terms of (20) are positive, except $\frac{\partial e}{\partial \omega}$, which is negative and $\frac{d\omega_N}{dR}$, which sign is unclear. Indeed, applying the implicit function theorem on the government budget constraint of problem (12), taking account that $\tau = \omega - \omega_N$, and solving for $\frac{d\omega_N}{dR}$, we have:

$$\frac{d\omega_N}{dR} = \frac{1 - (\omega - \omega_N)\left(\frac{\partial l}{\partial \omega_N}\frac{\partial \omega_N}{\partial g}\frac{dg}{dR}\right) + \left(\frac{\partial \pi}{\partial g} + \frac{\partial \omega}{\partial g}\left(\frac{\partial \pi}{\partial \omega} + l\right)\right)\frac{dg}{dR}}{(\omega - \omega_N)\frac{\partial l}{\partial \omega_N} - l - \frac{\partial \pi}{\partial \omega_N}}, \tag{21}$$

whose sign is indeterminate. Additionally, expression (19) includes a positive term $\left(Y\frac{\partial \omega}{\partial g}\frac{dg}{dR}\right)$ which adds more doubts on the final sign of MEB.

One could say that our case is essencially equivalent to that of nonseparable public goods by Gronberg and Liu (2001) but some of their results have to be reconsidered. They claim that it is more likely to have a negative MEB when the public good is a complement to the taxed good, and with public inputs we also have a positive impact of public spending on labor and wage rate, and consequently on tax revenues (feedback effect). In a similar way to the nonseparable public good case, the provision of public inputs makes the tax system less distorting and that may increase the likelihood of having a level reversal result (first-best level below the second-best level). However, the complexity of relationships involved in (20) and (21) does not allow us to state what the sign of the MEB is. What we do next is to carry out a numerical simulation with several specifications of the relevant functions with the aim of going beyond this ambiguity and determine whether the sign of the MEB becomes a crucial criterion for solving the controversy.

3.2 Numerical level comparison

A standard way of giving an insight into this debate is considering specific situations which can be numerically solved looking for clear-cut conclusions. With this aim, we consider three different utility functions in an attempt to achieve results as general as possible and related to previous references on public goods. Particularly, we have chosen the quasi-linear utility (Gronberg and Liu, 2001); the Cobb-Douglas utility (Atkinson and Stern, 1974; Wilson, 1991a); and the CES utility functions (Wilson, 1991b; Gaube, 2000). Specifically,

$$U(x,h) = x + 2h^{\frac{1}{2}} \tag{22}$$

$$U(x,h) = a \log x + (1-a) \log h \tag{23}$$

$$U(x,h) = (x^{\rho} + h^{\rho})^{\frac{1}{\rho}}\,, \tag{24}$$

where $a \in (0,1)$ and $\rho = 0.5$. The crucial point in our case comes from the specification of the production function because the different alternatives by defining how the private and public factors enter the production function become a key issue. In particular, whether this function exhibits constant returns to scale in public and private inputs (firm-augmenting public input) or only constant returns to the private factors (factor-augmenting public input) has consequences on the controversy.

The numerical methods we have used for solving the above optimization problems depend on the particular features which are involved in each one.[6] The case of firm-augmenting public input has been solved using the well-known Newton-Raphson algorithm. By contrast, when a factor-augmenting public input is considered, we have to face non-convexities problems and, therefore, others alternative numerical methods must be employed. In this sense, the standard Nelder-Mead algorithm and the Rational Iterative Multisection Procedure (RIMP) have been the approaches we have chosen[7].

[6] With the aim of getting comparable solutions, the same level of precision, 10^{-4}, has been required for all of them.

[7] See Kelley (1999) and Mathews and Fink (2004) for further details on Nelder-Mead

Firm-augmenting public input

We assume a Cobb-Douglas production function given by $F(nl, g) = (nl)^{\alpha} g^{1-\alpha}$, where $\alpha \in (0, 1)$. This specification creates firm-specific rents. As Pestieau (1976) proved, if these rents are also an argument in the consumer's indirect utility function, the optimal spending condition is not the first-best one; however, recall that our model precisely establishes that all economic rents are taxed away by the government.

Regarding the set of parameters to be used, we have taken $a \in \{0.1, \mathit{0.5}, 0.9\}$ for the Cobb-Douglas utility function, $\alpha \in \{0.6, \mathit{0.7}, 0.8\}$ for the production function and $n \in \{1, \mathit{100}, 1000\}$ for population, where the benchmark values have been emphasized. Tables 1-3 show the main results for several scenarios and each utility function. Note that no distinction is done between distorting and lump-sum taxes. This is why the controversy between the first-best and second-best level of public spending has no sense when the firm-augmenting public input creates rents which are completely taxed by the government. Indeed, both the analytical solution of our model and its numerical resolution give the intuitive result that the optimal level of productive public spending must be exclusively financed with the economic rents.

In a sense, this situation can be compared to that of Feehan and Batina (2007), in which a (semi)public input is equivalent to a common property resource. A Lindahl pricing system is then the appropriate policy instrument, with a charge on firms for their utilization and according to the value of public input's marginal contribution to the firms' profits (Sandmo, 1972). All in all, the complete taxation of rents implies to solve the common problem arising when public input provision is involved.

INSERT TABLES 1-3 ABOUT HERE

Beyond the controversy on the optimal levels of public inputs, several comments can be drawn regarding Tables 1-3**.** First, the level of public input provision has a non-monotonic relationship with its output share $(1 - \alpha)$ in the production function in the case of the quasi-linear utility function, while the relationship is increasing in the case of the CES and Cobb-Douglas utility functions. This latter result is easy to be explained; it can be proved that F_g is decreasing in α. Since with a Cobb-Douglas utility the labor supply is fixed, the only way to hold $F_g = 1$ is to provide a lower level of public input.

algorithm. RIMP consists of a *selective* iterative subdivision of the initial decision variables set, in which the objective function is then evaluated. For a detailed explanation of it, see Sanchez and Martinez (2008). Matlab routines implementing the different methods are available upon request.

Second, there is a closely related link between the number of households and the level of public inputs for all the utility specifications. Particularly, the quantity of public input increases at the same rate than the size of population. Since the production function exhibits constant return to scale (i. e., it is homogeneous of degree 1), it can be claimed that the function $\pi(.)$ is also homogeneous of degree 1. Accordingly, increases in the number of households are followed by increases in profits at the same rate, and consequently by identical increases in the provision of public inputs.

Third, the output share of public inputs $(1-\alpha)$ and the level of utility are inversely related for all the utility functions. The higher the productivity of public input, the smaller the utility of the representative household. This fact comes from the decrease in the output share of labour as a result of constant returns to scale in public and private inputs.

Finally, as a comment exclusively referred to the Cobb-Douglas case, it can be seen how the level of public input increases when the preference for the private good goes up (parameter a). The higher the preference for the private good, the smaller the preference for leisure, and consequently more time is devoted to work. Given the assumptions of the model, this leads to increase the production and decrease the wages, which jointly determine higher economic rents[8].

Factor-augmenting public input

The main difference between the above environment and this of factor-augmenting lies in the assumptions on the returns to scale in the production function. Particularly, we assume again a Cobb-Douglas production function but exhibiting increasing returns in all the inputs (constant returns in labor): $F(nl, g) = nlg^{\beta}$, where $\beta \in (0, 1)$. Under this framework, the debate on the level of public spending in alternative tax settings is reborn. Indeed, the use of lump-sum or distorting taxes is necessary here as long as rents are null. Here we have considered $a \in \{0.1, \mathit{0.5}, 0.9\}$, $\beta \in \{0.1, \mathit{0.2}, 0.3\}$ and $n \in \{1, \mathit{100}, 1000\}$ as the set of parameters to be taken account, where the benchmark values have been again emphasized.

Regarding the comparison between the first and second-best level of pu-

[8] Given our Cobb-Douglas utility function, it can be written that $l(\omega_N) = Ya$. Since $\dfrac{\partial \pi}{\partial a} = -nYa\dfrac{\partial \omega}{\partial a}$ and n, Y and a are positive, then $sign(\dfrac{\partial \pi}{\partial a}) \neq sign(\dfrac{\partial \omega}{\partial a})$. As we know that $\dfrac{\partial \omega}{\partial a} = \alpha(1-\alpha)\left(\frac{g}{n}\right)^{1-\alpha}(Ya)^{\alpha}\frac{-1}{Ya^2} < 0$, therefore $\dfrac{\partial \pi}{\partial a}$ has to be positive.

blic inputs, we have also computed the MEB in each parametrization[9]. Previously, we have obtained the total excess burden using the familiar approach based on the concept of equivalent variation (see the expression 15), but taken account the special context of income taxation.

INSERT TABLES 4-6 ABOUT HERE

Tables 4-6 report the levels of utility, tax rates, public input, labor supply and MEB depending on the tax setting for the three utility functions. The most important issue is that the second-best level of public input is always below the first-best level. Therefore, we are here in line with the mainstream of previous literature in which the level reversal is unusual. Although the presence of a feedback effect on tax revenues might increase the likelihood of having a negative MEB, we confirm that this situation only occurs for the Cobb-Douglas case. Moreover, this result qualifies one of the findings of Gronberg and Liu, namely having a positive MEB is a necessary condition for obtaining a first-best level higher than the second-best level with homothetic preferences. This is not our case because we have that the public input provided under a lump-sum setting is bigger than that provided under distortionary taxation even with a negative value for the MEB and Cobb-Douglas preferences. In other words, the condition requiring a positive MEB is only sufficient (and not necessary) when public inputs are involved.

Other interesting issues are as follows. First, the level of provision in the second-best scenario is always increasing in the output share of public inputs β for the three utility functions. This contrasts with the firm-augmenting case, in which this fact occurred with two out of three utility functions. Second, the level of public input is not just linearly increasing in the number of households, but a direct relationship remains. Third, the level of utility achieved is increasing in the output share of public inputs β. In other words, the higher the elasticity of output with respect to the public input, the bigger the utility of the representative agent. This result has a clear policy implication: government must be aware of the productivity impact of factor-augmenting public input because this is welfare-enhancing.

4 Concluding remarks

This paper has dealt with an issue to which the existing literature has not paid much attention: the optimal *level* of public inputs under different tax

[9]For the sake of simplicity, we have actually computed the sign of $\frac{d(TEB)}{d\tau}$. A welfare maximizing government always sets tax rates in the increasing part of the Laffer curve, i. e. $\frac{dR}{d\tau} > 0$. In this context, the MEB has the same sign as $\frac{d(TEB)}{d\tau}$.

settings. Previous contributions have focused on the case of consumption public goods or have discussed the optimal *rules* of productive public spending. However, both the welfare implications of taxation and the characterization of public inputs as a growth-enhacing tool make this issue highly relevant for policy-makers.

We have built a simple model where public inputs provide productive services to firms. Different tax settings are available for government: a lump-sum tax, a tax on labour, and taxes on economic profits. Moreover, two types of public inputs (firm and factor-augmenting) have been considered to study how the nature of returns to scale and rents affect our results.

In order to shed some light on the controversy on level comparisons, we have adopted the Gronberg and Liu (2001)'s strategy. This consists of studying the sign of MEB: if positive, the first-best is higher than the second-best level; otherwise if negative (with homothetic preferences). On the basis of the MEB's approach, we conclude that it is not possible to elucidate analytically the sign of the MEB. In principle, the feedback effect arising as the public input encourages the tax base might reduce the welfare loss in a distorting tax setting, and consequently increasing the likelihood of having a negative MEB. However, in a similar way to the nonseparable case by Gronberg and Liu (2001), the complexity of relationships which are involved in the expression of MEB makes impossible to know analytically what is the sign of MEB. In fact, the expression we obtain for the MEB is much more cumbersome than that of public goods, even under the nonseparable case. This is due to the fact that public inputs have a direct effect on prices (wages) and things become more complicated.

Given the caveats of this analytical approach for the case of public inputs, we have implemented a numerical simulation to give an insight into the controversy. With this aim, we have taken three standard utility functions and different values for exogenous parameters in order to get more general results as posible. Moreover, appropriate numerical methods have been used to deal with non-convex optimization problems.

The first battery of results refer to the case of firm-augmenting public inputs. This situation creates rents which are taxed away by the government, and precisely they suffice to finance the optimal public spending. Therefore, the debate on level comparisons under different tax settings has no scope in this case. Additionally, we have obtained that the level of public inputs is linearly proportionate to the number of households and the relationship between the output share of public inputs and their levels of provision is positive in the case of the Cobb-Douglas and CES utility functions but non-monotonic for quasi-linear preferences.

A second group of results provides information in the case of factor-augmenting public inputs. Here the controversy on the optimal level of public spending matters because government financing is not based on levying rents but labor or lump-sum taxes have to be used. Our numerical results are clear: the level of public input in the first-best scenario always exceeds that of the second-best. However, we qualified the Gronberg and Liu's finding which states that a positive MEB is a sufficient and necessary condition for having a normal level relation with homothetic preferences. By contrast, our paper shows that a negative MEB is compatible with a situation in which the first-best level is higher than the second-best one with Cobb-Douglas preferences. Additionally, we find that the optimal level of public inputs is increasing in the output share of public inputs and in the number of households. In terms of utility levels, we find different results for firm and factor-augmenting public inputs. Particularly, we observe that a higher elasticity of production with respect to the factor-augmenting public input is welfare-enhancing, while the opposite is true for the case of firm-augmenting public input.

Our analysis based on the MEB could be translated to the cost-benefit analysis (CBA). Triest (1990) and Browning *et al.* (2000) have developed a discussion on the relationship between the MEB and the marginal cost of public funds, which is the key variable in the cost-benefit analysis. A challenge for the further development of this paper could be extending the Gronberg and Liu's methodology to the CBA.

Finally, some policy implications could be extracted from this paper. One of them is that it is essential to identify the nature of returns to scale in a production function with public inputs. In fact, only when the so-called factor-augmenting public input is considered, the optimal policy requires taxes different to those levying rents, such as income or factor taxes. Alternatively, when a firm-augmenting public input is considered, the complete taxation of rents is the best policy option to solve the commons (or second-best) problem we face. Moreover, the policy-makers have to be aware of the welfare implications stemming from the provision of public inputs. These are not only public expenditures affecting the production side of economy, but also they have a great significance in terms of households' utility levels.

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Tables

Table 1: Firm-augmenting. Quasi-linear utility.

	Utility	Public Input	Labour Supply	Profits
Benchmark $n = 100, \alpha = 0.7$	*12.42137*	*327.20646*	*18.27223*	*327.20646*
$\alpha = 0.6$	10.88755	316.49977	14.57494	316.49977
$\alpha = 0.8$	14.70900	274.26570	20.50614	274.26570
$n = 10$	12.42137	32.72065	18.27223	32.72065
$n = 1000$	12.42137	3272.06465	18.27223	3272.06465

Table 2: Firm-augmenting. Cobb-Douglas utility.

	Utility	Public Input	Labour Supply	Profits
Benchmark $n = 100, \alpha = 0.7, a = 0.5$	*2.04858*	*214.8877*	*12*	*214.8877*
$a = 0.1$	2.76570	42.97755	2.4	42.97755
$a = 0.9$	2.06757	386.79790	21.6	386.79790
$\alpha = 0.6$	1.9241	260.58409	12	260.58409
$\alpha = 0.8$	2.17216	160.49767	12	160.49767
$n = 10$	2.04858	21.48877	12	21.48877
$n = 1000$	2.04858	2148.87725	12	2148.87725

Table 3: Firm-augmenting. CES utility ($\rho = 0.5$).

	Utility	Public Input	Labour Supply	Profits
Benchmark $n = 100, \alpha = 0.7$	*34.02809*	*126.65501*	*7.07281*	*126.65501*
$\alpha = 0.6$	31.81752	128.05032	5.89677	128.05032
$\alpha = 0.8$	36.83981	111.87680	8.36474	111.87680
$n = 10$	34.02809	12.66550	7.07281	12.66550
$n = 1000$	34.02809	1266.55010	7.07281	1266.55010

Table 4: Factor-augmenting. Quasi-linear utility.

		$n = 100$	$n = 100$		$\beta = 0.2$	
		$\beta = 0.2$	$\beta = 0.1$	$\beta = 0.3$	$n = 10$	$n = 1000$
Utility	LS	*90.0830*	40.2563	281.8203	50.9174	159.9329
	D	*90.0564*	40.2491	281.8022	50.8935	159.2948
Tax	LS	*22.4139*	4.3519	120.7290	12.5391	39.9232
rate	D	*0.8999*	0.1809	4.9566	0.5248	1.3531
Public	LS	*2241.3885*	435.1883	12072.9012	125.3912	39923.1558
Input	D	*2153.2517*	427.4710	11914.3832	124.7701	32443.8427
Labour	LS	*23.9543*	23.7033	23.9964	23.8552	23.9856
supply	D	*23.9286*	23.6335	23.9927	23.7734	23.9773
MEB		*23.9286*	23.6335	23.9927	23.7734	23.9773

Note: Benchmark scenario: $n = 100, \beta = 0.2$. LS=Lump-sum, D=Distorting.

Table 5: Factor-augmenting. Cobb-Douglas utility.

		$n = 100, a = 0.5$	$n = 100, \beta = 0.2$		$n = 100, a = 0.5$		$\beta = 0.2, a = 0.5$	
		$\beta = 0.2$	$a = 0.1$	$a = 0.9$	$\beta = 0.1$	$\beta = 0.3$	$n = 10$	$n = 1000$
Utility	LS	*3.0654*	2.9300	4.0200	2.6997	3.5869	2.7776	3.3532
	D	*3.0584*	2.9274	4.0175	2.6982	3.5679	2.7706	3.3462
Tax	LS	*10.7761*	1.6191	20.1986	2.1625	56.5836	6.0598	19.1629
rate	D	*0.7872*	0.5264	0.9118	0.1702	3.7383	0.4427	1.3999
Public	LS	*1077.6083*	161.9140	2019.8639	216.2484	5658.3633	60.5984	19162.8846
input	D	*944.6350*	126.3431	1969.4941	204.2685	4486.0149	53.1207	16798.2513
Labour	LS	*13.3333*	2.9268	22.0408	12.6316	14.1176	13.3333	13.3333
supply	D	*12*	2.4	21.6	12	12	12	12
MEB		*-12*	-2.4	-21.6	-12	-12	-12	-12

Note: Benchmark scenario: $n = 100, \beta = 0.2, a = 0.5$. LS=Lump-sum, D=Distorting.

Table 6: Factor-augmenting. CES utility ($\rho = 0.5$).

		$n = 100$	$n = 100$		$\beta = 0.2$	
		$\beta = 0.2$	$\beta = 0.1$	$\beta = 0.3$	$n = 10$	$n = 1000$
Utility	LS	*109.4471*	61.7688	298.7085	70.3698	179.2329
	D	*108.4150*	61.5920	295.7230	69.4549	178.1280
Tax	LS	*18.3059*	2.7861	113.7856	8.8778	35.5307
rate	D	*0.90038*	0.17532	4.84211	0.47327	1.57368
Public	LS	*1830.5908*	278.6116	11378.5635	88.7784	35530.7014
Input	D	*1682.5437*	256.8082	10677.9327	74.3357	32679.6423
Labour	LS	*20.3724*	15.8672	23.0218	18.0974	21.8501
supply	D	*18.6871*	14.6481	22.0522	15.7007	20.7664
MEB		*19.6099*	15.2789	22.6197	17.0035	21.3633

Note: Benchmark scenario: $n = 100, \beta = 0.2$. LS=Lump-sum, D=Distorting.