Social Security and the search behaviour of workers approaching retirement

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Abstract

This paper explores the links between unemployment, retirement and their associated public insurance programs. It is a contribution to a growing body of literature focused on a better understanding of the labor behavior of advanced-age workers, which has gained importance as the pension crisis looms. It also contributes to the literature of optimal unemployment insurance by exploring the interaction of unemployment benefits and retirement pensions. The analysis combines the development of a new theoretical model and a detailed exploration of the empirical regularities using the Spanish Muestra Continua de Vidas Laborales (MCVL) dataset. The model is an extension of the standard search model, designed to reproduce the non-stationary environment faced by workers of advanced ages (in the age range 50/65).

Via calibrated simulations we show that the basic empirical re-employment and retirement patterns can be considered as rational responses to both the labor market conditions and the institutional incentives. Generous Unemployment Benefits (for durations of up to two years) together with very significant early retirement penalties, make optimal to stay unemployed without searching for large groups of unemployed workers. This moral hazard problem can be substantially alleviated through institutional reform. We explore several potential reforms and find that changing the details of early retirement pensions seems more promising than changing the Unemployment Benefit system.

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1 Introduction

In the last three decades, a large majority of OECD countries has seen a widespread tendency towards lower employment rates among its older workforce. At the same time, declining birth rates and rising life expectancies have started to have an impact on the size and age structure of the population in OECD countries. These changes are expected to produce dramatic effects in the near future. For example, the OECD predicts that the working-age population (15–64 years) will be 18% smaller than the current one by 2050, and the numbers of those aged over 65 years will increase by 60%. These changes will have profound implications for the European economy and its capacity to finance its welfare and health care systems. This gloomy prospect has triggered an intense academic and political debate, whose first practical implications are starting to become apparent. The recent delays in the normal retirement age in the US or Germany or the higher participation rates targeted in the Lisbon-2000 objective are good examples.

From an academic point of view, these changes have resulted in renewed efforts to understand the labor supply behavior of older workers. Up to now, the main target of this research has been exploring the labor decisions of the employed workers. This focus is understandable, as this group represent a large majority of the labor force. However, in countries with substantial unemployment rates, like Spain, the unemployed constitute a potentially significant alternative source of extra labor supply. However, attracting unemployed workers back to employment seems, at first inspection, really challenging. Firstly, their reemployment rates are very low, with independence of the workers characteristics (education, gender, sector, ...). Furthermore, according to their answers to specific questions in labor surveys, their job-search effort is very low. For example, less than 17% of the unemployed aged 50-54 reply that they are actively looking for a new job in the full sample of the European Community Household Panel (1994-2001). This number falls to less than 3% for those aged more than 60. In Spain, the values are a slightly bigger: 17.7% and 6.7% respectively according to the Spanish Labor Force Survey in 2005-2006).

The traditional interpretation of these facts emphasizes the low demand existing for workers in such an advanced age interval, and the high personal costs imposed by searching and re-training at those ages. Alternatively (and without denning the importance of the more traditional factors) in this paper we explore whether low participation rates may also reflect a rational answer to “perverse” incentives provided by public institutions. On the one hand, this group frequently enjoy special Unemployment Benefits schemes, that may make them more reluctant to take low job offers. On the other hand, early retirement is an attractive possibility for large groups of workers of this age. This makes the relevant institutional environment substantially

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1 There is, however, some evidence that in the last few years this tendency has slowed down and in some countries reversed itself. See eg Benitez-Silva et al. (2006) for recent evidence for the US.
2 There has been widespread delays in both the Early and Normal retirement ages in OECD countries. See Casey et al. (2003) for a detailed enumeration. For the EU members, the policy action has revolved around the Lisbon-2000 objectives. They set a explicit target (50%) for the employment rate of workers of more than 50 years of age in 2010.
3 An exception is Stiglitz and Yun (2005), a pioneer work advocating the integration of the retirement and unemployment schemes.
4 See García Pérez and Sánchez-Martín (2007) for a detail discussion of the Spanish case.
more complex than the one for younger workers. In particular, some unemployed may find it advantageous to use the unemployment benefit to stay in the labor force without searching, with the only purpose of reducing the penalties associated with early retirement. Note that the quantitative importance of these penalties can be important (In Spain, the replacement rate of the final pension over the accrual pension rights is reduced by 7.5% for each year retirement is brought forward). Exploring the rationality of these alternatives is our main target in this paper.

Viewed in this light, the paper is an extension of the optimal unemployment benefit literature, in a world of imperfect information and hidden actions (as exemplified by eg. Hopenhayn and Nicolini (1997), Kocherlakota (2004) or Shimer and Werning (2003)): our final target is to improve the institutional design to make it more robust to the moral hazard problem associated with the provision of public insurance. This task is more complicated for workers approaching retirement due to the interactions between Unemployment Benefits and pension regulations. And, needless to say, the opportunistic use of the unemployment benefit may result in a significant increased of the costs implied by these workers to the Social Security System (ie, the combined pension/unemployment insurance systems).

The theoretical framework implemented in the paper extends the traditional search model to include the alternatives of retirement and non-participation (ie. staying in the labor force without searching). The model contributes to the existing retirement literature (exemplified by eg. Rust and Phelan (1997), French (2005) or Van der Klaauw and Wolpin (2005)) by formally exploring the unemployment path into retirement. The paper also contributes to the literature on search models by considering non-participation decisions in a non-stationary environment including the risk of dismissal. The possibility of non-participation in an otherwise standard search model was first analyzed in Van den Berg (1990). More recently, Frijters and Van der Klaauw (2006) estimates an structural, non-stationary search model with non-participation, where the state of inactivity (considered as an absorbing one) is unrelated to the economic conditions. Our analysis improves upon the former by considering the fundamental non-stationarity induced by age considerations, and upon the latter by providing a full economic description of the non-participation state (ie, retirement).

We explore the basic theoretical predictions of this model but, due to the impossibility of obtaining closed-form expressions for the optimal decisions in the most general cases, we also rely on calibrated simulations. To this end, we select the values of the main parameters of the model so as to approximately replicate the empirical retirement and reemployment hazards, and the average accepted wages by the unemployed that reenter the labor force. After calibrating the model, we are able to quantify the incidence of voluntary non-participation. We also explore

5Including frictions in the re-employment process is probably more important at the ages considered in this paper that for younger workers. This is due to a number of elements: to the (potential) labor market bias against older workers; to the uncertainty about the cost and usefulness of the (re-training and) search process; and to the existence of generous public income-maintenance programs, that make it feasible to stay out of the labor force for long periods (even forever, if the workers opts for the retirement option).

6We have also conducted an extensive sensitivity analysis on the basic parameters of the model. The results confirms that the basic qualitative predictions of the model are pretty robust to the particular set of parameters chosen.
some institutional changes designed to improve on the labor-incentives generated by the current system.

A basic finding of our analysis is that the empirical search behavior at advanced ages can be rationalized as the response of rational individuals to both the labor market conditions and the institutional incentives. We show that a model that combines those two elements goes a long way in reproducing the re-employment and retirement patterns observed in the Spanish data. Both elements matter: we show via simulation that even in presence of a much larger arrival rate of job offers, the predicted re-employment rate in the current institutional setting would be rather small. The reason lies in the combination of generous Unemployment Benefits (for durations of up to two years), and substantial penalties for early retirement. Together, they make staying unemployed without searching an optimal strategy for a very large part (around 50%) of the unemployed in the age range 55/65. Although the Spanish authorities have largely turned a blind eye on this “alternative” exit route into retirement, it seems a clear abuse of the original mandate of the unemployment insurance scheme (namely, support workers in their search process). Changing this pattern would demand a substantial reform of the present institutions. Our simulations show that reducing the generosity of Unemployment Benefits would not be particularly successful, as it pushes workers mainly into retirement (in the end, to search for new jobs, workers must be provided with a decent income in the process). A more promising (although rather ruthless) avenue focuses on redesigning the early retirement penalties of the unemployed. If the penalty were fixed according to the age when the individual effectively withdraws from the labor force (rather than when he/she claims the pension for the first time), the incentives to stay idle would be much smaller. Our simulation indicate that such a measure would be very effective in reducing non-participation and increasing labor supply, specially after the early retirement age.

The structure of the paper is the following. Firstly (section 2) we present the basic empirical regularities concerning re-employment and retirement behavior in our sample of Spanish workers in the age range 55/65. These regularities provide the stylized facts that guide the specification of our theoretical model, which is discussed at length in section 3. In section 4 we present the main predictions of the model. We review both the theoretical results (section 4.1), the simulation design (section 4.2), its calibration (section 4.3) and its main results, including our policy simulation experiments (section 4.4). Finally, section 5 concludes.

2 Basic Stylized Facts

To analyze the evidence on the labor supply patterns of Spanish workers of advanced age we have used the latest release (January 2007) of the Muestra Continua de Vidas Laborales, MCVL05 hereafter, an administrative dataset based on a random draw from the Spanish Social Security archives. The database (along with some reduced-form econometric analysis) is described in detail in the companion paper García Pérez and Sánchez-Martín (2007). Here we only refer to the most fundamental findings.\footnote{The regularities reported correspond to a relatively narrow sub-sample of the MCVL05, selected to guarantee that the economic incentives of the individuals are clearly identified. We focus, then, on providing good empirical}
We have explored the transitions of unemployed workers out of the labor force (into retirement) or back into work (and their associated accepted wages) paying special attention to the role of age, duration in unemployment, pension rights (“Bases reguladoras”, defined precisely in section 3.1) and wages in the immediately preceding job. This has resulted in a relatively small set of empirical patterns, that we summarize (and enumerate for later reference) in the next two (sub)sections.

2.1 Empirical evidence on search behavior

The most significant findings regarding the job-acceptance performance of unemployed workers can be summarized as follows:

S1 The re-employment hazard decreases with age in the range 55/60 and remains constant thereafter.

S2 In the age range 55/60, the re-employment hazard decreases with duration (the length of the unemployment spell in years, represented by \( h \)). After 60, the pattern is more erratic.

S3 The re-employment hazard decreases with the size of the pension rights (with one exception: before the early retirement age, 60, the unemployed with average pension rights have higher re-entry hazards that those with low pension rights).

S4 Accepted re-employment wages are roughly constant with age.

Regularity S1 is illustrated in Figure 2: the quarterly re-employment hazard (ie. the conditional probability of making a transition from unemployment to employment) goes down from a value around 7% for workers of 55 years of age, to only 2% at 60. This small number confirms the general impression that finding jobs is a very difficult task at ages close to the “standard” retirement ages for the pension system. Regularity S2 (displayed in the top left panel of Figure 4) is a negative duration dependence of the type usually described in mainstream search literature. In our data, the reemployment hazard before 60 is clearly lower for the long-term unemployed (those that receive benefits for more than 2 years, ie \( h=3 \)). The pattern is not as clear cut at lower durations (\( h=1 \) or \( h=2 \)). After the early retirement age, the ranking of hazards changes and the long-term unemployed seem to reenter in larger proportion. This may be pointing to a composition effect: workers with lower re-employment chances probably retire in larger numbers at the early retirement age. The composition of this group after 60 may, then, be different from that before 60. We must bear in mind, in any case, that the sample size after 60 is not very big. S3 is illustrated in the top-right panel of Figure 4. For that graph we split the sample in three groups according with the percentiles 1/3 and 2/3 of the sample distribution of pension rights. At early ages, the reemployment hazard of workers with high pension rights counterparts to the stylized individuals in our theoretical model rather than in generality. Consequently, we consider low skilled males of 55 years of age or older, affiliated with the General Regime of the Spanish Social Security system, who are entitled to receive Unemployment Benefits and an old-age pension at retirement. We exclude individuals who sign special agreements with the Social Security, and individuals with missing information that prevent us from computing their pension rights.
Figure 1: **Transition from Unemployment to Employment**: Quarterly re-employment hazard by age in MCVL05

Figure 2: **Accepted wages**: Annual average accepted wage (in thousand of Euros of 2002) by unemployed workers that return to employment in MCVL05.

Figure 3: **Transition from Unemployment to Retirement**: Annual retirement hazard by age in MCVL05
Figure 4: quarterly reemployment hazard (top row) and annual retirement hazard (bottom row), by age; duration (h=1 is plotted with a blue, continuous line, h=2 with a green, dashed line, and h=3 with a red, dotted line); and pension rights (those below the percentile 33% are plotted with a blue, continuous line, those between 33% and 67% with a green, dashed line and those above 67% with a red, dotted line)
is very low, while that of workers with low pension rights is clearly higher, specially after 58. Finally, S4 is presented in Figure 2. Average annual accepted wages by unemployed workers re-entering the labor force show little variation. Most of them stay within the range from 11.5 to 12.5 thousand Euros (of 2004). It is also possible to detect some selection effects after 60, as the general pattern seem to be slightly decreasing before the Normal Retirement Age (NRA).

2.2 Empirical evidence on retirement behavior

Regarding the transition from unemployment to permanent retirement, we highlight the following empirical regularities:

R1 The retirement hazard varies strongly with age. It is higher at the early retirement age (60) and, more clearly, at the normal retirement age of 65. In the intermediate ages, the hazard remains roughly constant at a lower level.

R2 The retirement hazard is strongly increasing with duration in unemployment (h).

R3 The retirement hazard is larger the higher the size of accrued pension rights, with the exception of the early retirement age (at 60, the hazard is slightly higher for the unemployed with low pension rights).

The basic pattern of the retirement hazard by age, R1, is reported in Figure 3. The spikes at 60 and 65 are similar to those reported in the previous literature based on the behavior of employees (See, for example Jiménez Martín and Sánchez Martín (2005) or Rust and Phelan (1997)). The main difference is the substantially larger hazard out of the labor force at the intermediate ages. The bottom panels of Figure 4 clearly illustrate regularities R2 and R3. The evidence regarding duration is a new and apparently very strong new stylized fact. R3 is better known from the previous literature. It is well known that minimum pensions tend to foster the early retirement of workers with low pension rights. This seems to be also the case in our sample of unemployed workers. After that age the higher disincentive effect to stay active created by higher pension rights prevail.

3 The model

We model the search, participation and retirement behavior of unemployed workers in the age range \( a \in \{55, 65\} \) at one particular point in time. Time is assumed to be discrete, with one period in the model standing for one year of calendar time. All agents in the model face the same survival uncertainty, represented by the (age-conditional) survival function \( S_a \). Labor market uncertainty depends on the state of the individual.

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8When we refer to a peak in the hazard at 60 we are stressing the drop in the conditional retirement probability observed after that age. Note that, as we identify retirement with the collection of pension benefits, the absence of retirement before the Early Retirement Age is entirely a matter of construction. In the graphs, however, we see a non-negative hazard at earlier ages, due to measurement error—probably related to an erroneous recording as old-aged pensions of other type of pensions that can be enjoyed at earlier ages.
At the beginning of any period individuals of all ages are classified in one of three mutually exclusive labor states: employed, unemployed or retired. Retirement is a purely passive state, associated with the perception of the pension benefit, $B$, and with a permanent withdrawal from the labor force. Employed workers have a time invariant real wage, $w$, and face a constant probability $\delta$ of being fired at the end of the current period. After the Early Retirement Age, ERA, they have the option of voluntarily leaving the labor force and start collecting the pension benefit.\footnote{Although we do consider the optimal retirement decisions of employees in the following analysis, we do not report the findings here due to our focal interest on the behavior of the unemployed.} In this paper we focus our attention on the behavior of the unemployed. Figure 5 summarizes in a visual way the different options opened at different times to each unemployed worker. Immediately after the dismissal, three courses of action are possible: to retire immediately (R in the graph); to actively search for a new job (S) or to stay inactive (N). Those that opt for an active search strategy incur the associated search costs immediately, and receive one job offer at the beginning of the next period with probability $\lambda_h$ (conditional on the duration of the unemployment spell, $h$). The offer is fully characterized by the wage, which is a random draw from the invariant distribution $F(w)$. If the offer is rejected, the unemployed can either stay unemployed for another period or retire immediately (again, after ERA). Unemployed workers who opt for non-participation do not incur in any search cost in the present, but do not receive a job offer in the next period either. In theory, it should not be possible to receive the normal unemployment benefit without searching for a job at the same time, but we assume the public administration do not enforce this prohibition. This is in line with the empirical evidence.\footnote{For example, by the end of 2006, only 13 of those aged more than 45 and receiving Unemployment Benefits declared to be searching (according to the Spanish Labor Force Survey).} Therefore, in both states (search and non participation), the individual receives the same unemployment benefit, $b$. We assume nobody can stay active after 70 years of age.

### 3.1 Institutions and market arrangements

**Unemployment benefits**

The general unemployment benefit scheme pays a proportion $b_h$ of the wages enjoyed in the immediately preceding job, although the wage/benefit proportionality is broken by a floor $b_{min}$ and a ceiling $b_{max}$ on the final benefits. The replacement rates $b_h$ decrease with the duration, in
years, of the current unemployment spell, \( h \). In the population at large, individuals can receive the (contributive) Unemployment Benefits for a maximum of two years. People older than 52, in contrast, are entitled to a specific program granting the subsidy \( b_{\text{min}}^s \) (75\% of the minimum wage) till their retirement. Equation (1) shows how all these elements feature in our model:

\[
\begin{align*}
 b(\pi, h) = & \begin{cases} 
  \text{Max}\{b_{\text{min}}, \text{Min}\{b_1\pi, b_{\text{max}}\}\} & \text{if } h = 1 \\
  \text{Max}\{b_{\text{min}}, \text{Min}\{b_2\pi, b_{\text{max}}\}\} & \text{if } h = 2 \\
  b_{\text{min}}^s & \text{if } h \geq 3 
\end{cases} 
\end{align*}
\]

(1)

The public employment agency (INEM) not only provides immediate income to the unemployed, it also protects their future pension income by paying their pay-roll taxes to the Social Security system. These social contributions are a fixed proportion of the “pensionable wage” of the individual. In the current system, INEM contributes the full previous wages of those unemployed with a duration of less than two years, and the minimum contribution in case of a longer duration. More generally, we represent the “pensionable wage” (i.e. the labor income considered in the pension formula described below) with a duration-dependent replacement rate of the previous wage, \( \kappa_h \pi \). Finally, we do not consider in this paper workers covered by special agreements with the pension administration.

**The pension system**

The public pension can be claimed at any age after the Early Retirement age, ERA (\( N_m \) in the following equations), conditional on a complete withdrawal from labor market activities. The pension benefit of each worker is computed in two steps. First, an individual-specific component related to the worker’s previous earnings is calculated. Next, this benefit is subsequently compared with the legal minimum and maximum pensions prevailing at each year to determine the final effective payment.

The individual component (\( \tilde{B} \)) depends on age and on the retiree’s accrued pension rights, \( \tilde{w} \):

\[
\tilde{B}(\tilde{w}, a) = \mu(a) \tilde{w}_a 
\]

(2)

The age-dependant replacement rate \( \mu(a) \) reflects an annual \( \mu_1 \) \% penalty for early retirement before the Normal Retirement Age (\( N \)):

\[
\mu(a) = \begin{cases} 
  \mu_0 & \text{if } a < N_m \\
  \mu_0 + \mu_1(a - \tau_m) < 1 & \text{if } a \in \{ N_m, \ldots, N - 1 \} \\
  1 & \text{if } \tau \geq N 
\end{cases} 
\]

(3)

Accrued pension rights (called “Base Reguladora”) are, in the real world, a moving average of total labor earnings in the \( D \) years immediately before retirement \( \tilde{w} \):

\[
\tilde{w}_a = \frac{1}{D} \sum_{i=1}^{D} w_{a-i} 
\]
Unfortunately, implementing this formula in the model implies large computational costs. Instead, we simplify the dynamics of \( \hat{w} \) by assuming that, for an individual with current labor income \( w \), next period pension rights will be:\(^{11}\)

\[
\hat{w}_{a+1} = \hat{w} + \frac{w - \hat{w}}{D}
\]  

(4)

Finally, the scheme is made progressive by the inclusion of floors and ceilings in the final payment. This breaks the strict proportionality between the effective benefit \( B(\hat{w}, a) \) and the (average) level of previous wages that would otherwise prevail:

\[
B(\hat{w}, a) = \begin{cases} 
B_{\text{min}} & \text{if } \hat{B}(w, \hat{w}, a) < B_{\text{min}} \\
\hat{B}(\hat{w}, a) & \text{if } B_{\text{min}} \leq \hat{B}(w, \hat{w}, a) \leq B_{\text{max}} \\
B_{\text{max}} & \text{if } B_{\text{max}} < \hat{B}(w, \hat{w}, a)
\end{cases}
\]  

(5)

The effective benefit is first computed when the individual retires and is kept constant in real terms throughout the rest of his/her life:\(^{12}\)

### 3.2 Optimal individual behavior

In every period (as described at the beginning of the section and represented in Figure 5, the unemployed who searched in the previous period take the decision of whether to accept or reject any wage offer they receive and (in case of rejection) whether to stay unemployed or retire from the labor force. The unemployed who did not search in the past may reconsider his decisions and search during the current period, stay unemployed without searching or retire. Finally, employees must decide on whether to keep working for the same wage or to retire. Note that we do not allow voluntary transitions from employment into unemployment. In all cases we assume individuals are expected utility maximizers, ie. they decide by comparing the expected discounted utility obtained from the associated flows of income and leisure in the different alternatives:

\[
e^*_a = \arg\max E \left[ \sum_{i=a}^{T} \beta^{i-a} u(y_i, e_i) \right]
\]

where \( e^*_a \) stands for the optimal sequence of present and future labor states from age \( a \) till the maximum possible age for labor participation, \( N \) (70 in the base calibration). \( T \) is the maximum longevity, \( \beta \) is a constant discount factor representing a pure preference for earlier consumption, and individual preferences at every age are represented by an (age-invariant) additively separable CES \textit{indirect} utility function:

\(^{11}\)The approximation is exact under our assumption of constant wages for the employees.\(^{12}\)We abstract from a number of relatively minor details of the pension and fiscal systems. In particular, the individual in the model does not take into account income taxes and the worker part of pay-roll taxes (much smaller than the employer part in Spain). We also do not include pension reductions due to an insufficient number of contributive years.
\begin{equation}
\nu(e) = \begin{cases} 
0 & e = E \\
-c & e = S \\
l & e = (R, N)
\end{cases}
\end{equation}

where $\eta$ measures the curvature of the objective function (which, in turn, determines both the degree of risk aversion and the willingness to substitute income intertemporally) and $\nu(e)$ represents the variation in the value attached to the different uses of time, according with the labor state. In our base specification, we normalize this value to cero for employees, assume a negative value ($-c$) for the unemployed that search and a positive one ($l$) for non participants and retirees. Finally $c$, represents the costs associated with the search process (including the cost of re-training and the possible ‘stigma’ cost of staying unemployed), while $l$ reflects the value of leisure enjoyed when time is not devoted to labor-market activities.

**Recursive Representation of the individual problem**

As usual, we work with a recursive representation of this discrete-time, optimal control problem. This means that we characterize its solution via the value functions associated with each labor state. These value functions are solved by backward induction starting at $N$. As everybody is assumed to retire at that age, the only relevant value function is that of the workers to leave the labor force at that age:

\begin{equation}
R_{\bar{w}}(\bar{w}) = \sum_{i=N}^{T} S(i) \beta^{i-N} u(B(\bar{w}, \bar{N}), R) = \left( \frac{[B(\bar{w}, \bar{N})(1 + l(R))]^\eta}{\eta} \right) \cdot A_{\bar{w}}^{T}
\end{equation}

where $S(i)$ stands for the survival probability to age $i \geq N$ conditional on survival to age $N$ and $A_{i}^{j}$ is the expected discounted value of a constant income flow of one unit starting at age $i$ and ending at age $j$. To simplify the notation, from here onwards we denote the (one period ahead) effective discounting at age $a$, $\beta S(a + 1)$, by $\beta_{a}$. $R$ is simply the expected discounted value of the utility derived from the enjoyment of pension $B(\bar{w}, \bar{N})$ and the full allocation of time to non-market activities.

At earlier ages, $a < N$, all relevant information is captured by the value functions corresponding to the four possible labor states: $W, S, N, R$ representing, respectively, employees, unemployed who search, non-participant unemployed and retirees. We review them in turn.

### 3.2.1 Employed workers

Currently employed workers have the option to retire immediately or to retain their current status for one more period. In the latter case (denoted by a small $e$ as a superscript of $W$), workers face, on top of survival uncertainty, the risk of being fired and start next period as unemployed. This is easily reflected in the corresponding value function:

\begin{equation}
W_{a}^{e}(w, \bar{w}) = u(w, E) + \beta_{a} \left[ (1 - \delta) W_{a+1}(w, \bar{w'}) + \delta U_{a+1}(w, \bar{w'}, 1) \right]
\end{equation}

For any variable, a prime denote the value of the same variable in the next period. Note that we assume constant wages and we update the *pension rights* as in (4). The value functions $W$
and $U$ are defined below. For the employees who find it optimal to retire at age $a \in [N_m, N]$, things do not change in any fundamental way with respect to what we described above for the case $a = N$. In particular, the value function takes a completely analogous form to that in (6). Finally, the total value of being employed, including the option of retirement, is simply characterized by:

\[
W_a(w, \hat{w}) = \max\{ W_a^e(w, \hat{w}), R_a(\hat{w}) \}
\]

### 3.2.2 Unemployed workers

For the unemployed we consider three different value functions. On the one hand, we represent with $S_a(\pi, \hat{w}, h)$ the value associated with engaging in an active search process while unemployed, and with $N_a(\pi, \hat{w}, h)$ the value of avoiding search cost, at the price of giving up the chance of receiving job offers in the next period. In both cases, a vector of four state variables (age $a$, previous wage $\pi$, pension rights $\hat{w}$ and duration in unemployment $h$) is needed to fully characterize the economic situation of those unemployed. On the other hand, the unemployed at the beginning of the period have the option of leaving the labor force and retire, with value $R_a(\hat{w})$. Consequently, the total value attached to be unemployed at the beginning of the period, $U_a(\pi, \hat{w}, h)$, is:

\[
U_a(\pi, \hat{w}, h) = \max\{ S_a(\pi, \hat{w}, h), N_a(\pi, \hat{w}, h), R_a(\hat{w}) \}
\]

We review each component next.

The value of getting involved in an active search process, $S_a(\pi, \hat{w}, h)$, is made up of two elements, a (presumably modest) current value $u(b(\pi, h), S)$ of searching, and a expected future value of searching ($EV^S$):

\[
EV^S = \beta_a \left\{ \lambda_{h+1} E_w [ \max\{ U_{a+1}(\pi, \hat{w}', h + 1), W_{a+1}(w, \hat{w}') \} ] + (1 - \lambda_{h+1}) U_{a+1}(\pi, \hat{w}', h + 1) \right\}
\]

where $\lambda_{h+1}$ represents the conditional offer arrival rate in the next period and pension rights are updated as in (4), ie: $\hat{w}' = \hat{w} + \frac{\kappa_h \pi - \hat{w}}{D}$. In words, the expected future value reflects two elements:

- If an offer of size $w$ arrives, the individual must decide whether to accept or reject it. The optimal decision is obtained by comparing $W_{a+1}(w, \hat{w}')$ to $U_{a+1}(\pi, \hat{w}', h + 1)$. Of course, at $t$, the size of the wage offer is uncertain. Consequently, the individual operates by taking conditional expectations, which accounts for the $E_w[.]$ in (8).

As usual in the literature, the job-acceptance decision is summarized (for each possible value of the state variables) by the corresponding Reservation Wage, $w_a(\pi, \hat{w}, h)$: the wage offer that makes the unemployed indifferent between taking the job or staying unemployed. Formally:

\[
w_a(w_a(\pi, \hat{w}, h), \hat{w}) = U_a(\pi, \hat{w}, h)
\]
• If no offer arrives or if the offer received is unacceptable, the associated value is that of staying unemployed one more period, i.e. $U_{a+1}(\pi, \hat{w}', h + 1)$. Note the different updating of the pension rights with respect to the previous case (in presence of an acceptable offer). The probability of this case is $1 - \lambda_{h+1} (1 - F(\overline{w}'))$, where $\overline{w}'$ stands for the next period reservation wage, $\overline{w}_{a+1}(\pi, \hat{w}', h + 1)$. We slightly simplify this expression by representing $1 - F(x)$ by $\overline{F}(x)$. In what follows, then, the probability of this case reduces to $1 - \lambda_{h+1} \overline{F}(\overline{w}')$.

Overall, the expected future value of searching is composed of two elements: a Stop Value of finishing the current unemployment spell by accepting an offer immediately and an Option Value derived from staying unemployed and so keeping the chances of getting acceptable job offers in the future.

The value of Non-Participant unemployment, $N_a(\pi, \hat{w}, h)$, is much simpler to formulate than the option with active search, as it does not involve any uncertainty (apart from survival risk):

$$N_a(\pi, \hat{w}, h) = u(b(\pi, h), l) + \beta_{a+1} U_{a+1}(\pi, \hat{w}', h + 1)$$

with pension rights updated as in the immediately preceding case.

Finally, the retirement option is similar to what has been previously discussed and does not demand any special additional comment.

4 Results

4.1 Theoretical Findings

Due to the lack of analytical solutions, it is difficult to characterize the properties of the optimal individual behavior in a completely general way. Fortunately, it is relatively easy to show the main trade-offs faced by the individuals at different ages. This is the task we tackle in this section. To streamline the discussion, the economic environment faced by the unemployed here is simpler than the general one presented in Section 3. We assume a linear and additive separable period utility $u = w + l$ and forced retirement at 65; we abstract from floor/ceilings in the public regulations (like minimum pensions or maximum Unemployment Benefits) and assume the employees face no risk of being fired. In this setting the value functions are linear, allowing for a very illustrative discussion of the main forces involved. Once equipped with the intuitions obtained in this simple setting, it would be much easier to interpret the findings of the simulation of the full model in section 4.2.

4.1.1 Retirement vs. Non participation at the age of 64

Consider first the options available to an unemployed worker of 64 years of age. Given our assumption of retirement with certainty at 65, it is clear that the only relevant alternatives are
immediate retirement or staying in the labor force for one more year without searching (ie, to non-participate). The value of the latter option, is:

\[ N_{64} = (b_{64} + l) + \beta_{64} A_{65} (B_{65} + l) = (b_h \pi + l) + \beta_{64} A_{65} \left( \hat{w} + \frac{\kappa_h \pi - \hat{w}}{D} + l \right) \tag{10} \]

with \( \beta_{64} \) standing for the one-period discount factor at age 64; \( b_{64} \) is the unemployment benefit currently available to the individual, \( B_{65} \) is the pension available in one year time when retiring at 65 (updated according to equation (2)); \( A_{65} \) is a short for \( A_{65}^T \) in section 3.2, ie the present discounted value of one unit of income at every period after 65: \( A_{65} = \sum_{i=65}^{T} \beta^{i-65} S(i | 65) \). In case of immediate retirement, the value function takes the form:

\[ R_{64} = (B_{64} + l) + \beta_{64} A_{65} (B_{64} + l) = (\mu_{64} \hat{w} + l) + \beta_{64} A_{65} (\mu_{64} \hat{w} + l) \tag{11} \]

These two expressions make clear that the non-participation decision is based on purely financial considerations (the value of leisure is similar in both pathways):

\[ N_{64} > R_{64} \iff \beta_{64} A_{65} (B_{65} - B_{64}) > B_{64} - b_{64} \]

\[ \beta_{64} A_{65} \left( \hat{w} + \frac{\kappa_h \pi - \hat{w}}{D} - \mu_{64} \hat{w} \right) > \mu_{64} \hat{w} - b_h \pi \tag{12} \]

Expression (12) makes apparent the rationality of choosing to non-participate by a large part of the Spanish unemployed of advanced age. In most common situations, current pension payments would exceed the unemployment benefits (the difference is the RHS of (12)) while the opposite would be true of future pensions (the term in brackets in the LHS of (12)). This is because the gain from lower early-retirement penalties is normally larger than the loss implied by the dynamic updating inside \( \hat{w} \). If that is the case, postponing retirement can lead to significant utility gains, due to the fact that the pension increase is enjoyed during the whole remaining lifespan of the individual (reflected by the factor \( A_{64} \) in the LHS). Overall, equation (12) formalizes that it is optimal to stay in the labor force without searching if the discounted value of the future gains from a larger pension exceeds the associated losses in current income.\(^{13}\)

With the current Unemployment Benefit rules, only the long-term unemployed (\( h \geq 3 \)) suffer significant reductions in current income and future pension rights. Consequently, it seems clear that (for moderate values of the discount factor), delaying retirement at 64 may be optimal for a majority of the unemployed with a duration of less than two years.\(^{14}\)

A second important insight from expression (12) is the essential role played by the pension rules in fostering non-participation. In particular, delaying retirement is more favorable in utility

\(^{13}\)With a non linear utility, elasticity/risk aversion considerations also have a role in the decision. Individuals with higher intertemporal elasticity of substitution (or a smaller degree of relative risk aversion) would not mind temporary drops in income in exchange for larger income gains in the future. Therefore, these workers would tend to non-participate rather than to early retire. Minimum/maximum pensions will also have a clear (and well known) impact, fostering earlier retirement. The absence of firing cost, in contrast, is neutral in this case and has a marginal impact in the cases analyzed below.

\(^{14}\)Expression (12) can also be used to predict the differences in behavior with different pension rights and previous wages. For a fixed \( b_h \), we should expect more non-participation among those with lower pension rights and with higher previous wages, if the following parametric restrictions apply: \( \beta_{64} A_{65} [1 - \mu_{64} - (1/D)] < \mu_{64} \) (which guarantees \( \partial (N_{64} - R_{64})/\partial \hat{w} < 0 \)) and \( \beta_{64} A_{65} \kappa_h / D > b_h \) (which guarantees \( \partial (N_{64} - R_{64})/\partial \pi > 0 \))
terms the *smaller* the value of $\mu_{64}$, i.e. the higher the early retirement penalty.\(^{15}\) Large early retirement penalties make sense to stop the employees from leaving the labor force early. The fact that they also push the unemployed into non-participation (by preventing their retirement) is, most certainly, an important unintended consequence of the design of this policy.

### 4.1.2 The job-acceptance decision of unemployed workers at 64

Unemployment rules and, more interestingly, pension regulations are also crucial for the job-acceptance decisions of the unemployed that try to return to employment. To illustrate this point we explore the reservation wages of the unemployed in the simplified environment discussed above. Firstly, it is clear that the value of accepting a wage $w$ at the age of 64, $W_{64}$, is simply:

$$W_{64} = w + \beta_{64} A_{65} \left( \hat{w} + \frac{w - \hat{w}}{D} + l \right)$$

The value of working at this age is linear in the wage and in the accrued pension rights. Direct comparison with the value functions of non-participant (10) and retirees (11) lead to the corresponding reservation wages (recall the definition in equation (9)):

$$w_{64} = \begin{cases} b h + \beta_{64} A_{65} \kappa h / D + \pi + l & \text{if Non-Participation is optimal} \\ \mu_{64} + \beta_{64} A_{65} \left( \mu_{64} - (1 - \frac{1}{D}) \right) \hat{w} + l & \text{if Retirement is optimal} \end{cases}$$

The impact of public regulations is clear. When the opportunity cost of accepting a job offer is given by the value of non-participation, the willingness to take offers clearly depends on the generosity of Unemployment Benefits. This generosity manifests in two forms; as current income (controlled by $\mu$) or as differed income materialized in bigger future pensions (controlled by $\kappa$). Higher $\mu$ or higher $\kappa$ will increase the LHS of (13), leading to larger reservation wages. When the best alternative to re-employment is immediate retirement (as in equation (14)), pension rules take over as the relevant public institutions. Again, more generous pensions will make unemployed workers more selective when accepting re-employment offers.

So far we have abstracted from the role played by labor market conditions in the optimal individual behavior. This can be changed just by iterating backwards one period to age 63, as the full set of labor states are potentially valuable options for workers at that age. The next sub-section deals with this more general case.

### 4.1.3 The role of labor market conditions in re-employment and retirement

At the age of 63, unemployed workers may search during one period to be entitled to the option of receiving a job offer in the next period. The value function in this case reflects a trade-off between the present cost of searching and the value derived from the chances of getting a good job offer in the next period:

\(^{15}\)Conditional on a fixed replacement rate at the normal retirement age of 65.
\[ S_{63} = b_{h} \pi - c + \beta_{63} \left[ U_{64}(\hat{w}', h') (1 - \lambda_{h} F_{\pi_{64}}) + \lambda_{h'} \int_{\pi_{64}}^{M} W_{64}(w, \hat{w}') dF_{w} \right] \] (15)

where \( h' = h + 1 \) and \( \hat{w}' \) is updated as in equation (4). Note that this expression is a particular case of the value of searching discussed in section 3.2.2. The alternatives to search are either staying in the labor force without searching or outright retirement. We review them in turn, starting with the former. The value of staying in the labor force without searching is:

\[ N_{63} = (b_{h} \pi + l) + \beta_{63} U_{64}(\hat{w}', h + 1) \]

With a little algebra, we can characterize the optimal searching behavior (vis a vis the alternative of non participation) by the following condition:

\[ S_{63} - N_{63} > 0 \iff \beta_{63} \lambda_{h'} \left[ 1 + \beta_{63} A_{65} \frac{1}{D} \right] \int_{\pi_{64}}^{M} (w - \pi_{64}) dF_{w} > c + l \] (16)

In words, it is optimal to search when the expected benefits (LHS) exceed the immediate costs (RHS) which include a direct income loss, \( c \), and the opportunity cost of forgone leisure, \( l \). The (potential) future benefits of returning to work also include two aspects: an improvement in immediate income at 64 and, through its effect on the pension rights, a boosts to future pension income. These two effects can be easily identified in the two terms inside the brackets of the expression in the left hand side of (16).

Condition (16) highlights the importance of labor market conditions on the self-selection of the unemployed into nonparticipation: it is rational not to engage in a search process for those unemployed workers with weak re-employment prospects (represented by the probability of receiving an acceptable offer). It also makes clear that the design of both the Unemployment Benefits and the Pension System can help to keep the unemployed away of the labor market, through its impact on the value of search. Note that this value is decreasing in the future reservation wage, whose value, in turn, is increasing in the generosity of the public institutions (as documented in section 4.1.2 above).

It is possible that the alternative of outright retirement were more valuable than nonparticipation. In this case, the trade-off is roughly similar to that discussed in section 4.1.1: a choice between the immediate income/leisure gains of retirement versus the prospect of better future income of searching. Here the higher future income comes from two sources: from the reduction in early retirement penalties and (in contrast with the situation at 64) from the possibility of receiving a good job offer, which would increase future labor income and pensions. The current costs of staying active, however, are increased by the need of incurring the search cost. The analytical expressions in this case become rather cumbersome, and had been confined to Appendix A. Overall, better market conditions would again make searching more attractive and would foster later retirement.

To conclude the review of the impact of the labor market conditions on individual behavior we must mention its effect on the job-acceptance decisions of the unemployed who opted for
searching at the age of 63. Note that the more general environment faced at the age of 63 adds nothing of substance to the discussion of Non-Participation versus Retirement in section 4.1.1 and to the discussion of reservation wages against the alternatives of retirement or non-participation in section 4.1.2. The value of reentering the labor force with wage $w$ at 63 is:

$$W_{63} = w + \beta_{63} w + \beta_{64} A_{65} \left[ \hat{w} \left( \frac{D-1}{D} \right)^2 + \frac{w}{D} \left( 1 + \frac{D-1}{D} \right) + l \right]$$

where we can appreciate the impact of the two extra working years (63 and 64) in the final value of pension rights. It is easy to obtain an expression for the reservation wage when combined with the value of $S_{63}$ in expression (15):

$$w_{63} = \left[ 1 + \beta_{63} + \beta_{64} A_{65} \frac{1}{D} \left( 1 + \frac{D-1}{D} \right) \right]^{-1} \left[ S_{63} - \beta_{64} A_{65} \left( \hat{w} \left( \frac{D-1}{D} \right)^2 - l \right) \right]$$

As one may expect, good prospects of future job-offers (summarized in a high value of searching $S_{63}$) may lead workers to turn down a low wage offer currently available. Of course, the empirical importance of this possibility at such advanced ages seems rather small.

4.2 Simulation Experiments

The theoretical analysis in the previous sections leads to a number of valuable insights into the basic trade-offs controlling optimal individual behavior. In this section we go one step further by exploring the basic predictions of the complete model (without any simplification) with the help of illustrative examples. We proceed as follows. We first (section 4.3) introduce our baseline case and discuss the implications of the different values assigned to the parameters of the model. We then, discuss the details of the optimal retirement and search behavior by individual state in our baseline case (section 4.4.1). Next, section 4.4.2 is devoted to compare the aggregate predictions of the model with the broad empirical trends discussed in sections 2.1 and 2.2. Finally, in section 4.4.3 we undertake some extra aggregate simulations. On the one hand, we explore the relative contribution of (perverse) institutional incentives vs. bad labor market conditions to the low reemployment rates observed at advanced ages. On the other hand, we explore a couple of institutional reforms aimed at facilitating the return into employment by reducing the incidence of non-participation.

4.3 Calibration

As the complete model can not be solved analytically, the quantitative properties of the solutions must be established via simulations. A brief description of the numerical solution method is provided in Appendix B. In this section we discuss the calibration process. We proceed in two steps: we choose a baseline economy by applying an informal calibration approach, and then check the robustness of the findings to changes in the parameter values through a complete set of sensitivity simulations\(^{16}\).

\(^{16}\)To keep the length of the manuscript within reasonable length, we only report the main findings of this second step.
The general idea of calibrating a model implies choosing properties of the data, first, and then finding the parameters of the model that best match the data in the chosen dimensions. To implement it, however, we must distinguish the parameters with direct empirical counterparts from those that lack direct observable references. The calibration process only really applies to the latter. In our case, the institutional parameters and the mortality process belong to the former group and so it is relatively easy to assign them concrete numerical values. Preference parameters and the properties of the job offers are unobservable and should be dealt with in a more indirect way. We discuss the functional forms and parameters values chosen in both situations next.

- **Institutions, Survival and dismissal rate**

  The parameters describing the pension and unemployment schemes are set to reproduce their direct empirical counterparts as of 2002. Retirement pensions are, then, first available at the age of 60, with an annual early-retirement penalty of 7.5% of the accrued pension rights. $\hat{w}$ is computed to approximate a moving average of the 15 years immediately before retirement (according to equation (4)). The full pension is granted at the normal retirement age of 65. The value of the minimum and maximum pensions are, respectively 5.7 and 23.8 (thousand of 2002 Euros, per annum).

  Unemployed workers receive 65% of their previous wages as benefits during the first year out of work (and average of the 70% being provided in the first six months, and the 60% provided thereafter, till a maximum of two years). This figure is then reduced to 60% in the second year and just 75% of the minimum contribution (6.2 thousand Euros) in subsequent years. The general proportionality of benefits and wages (for durations of less than two years), then, is broken by the minimum subsidy and a ceiling of two times the minimum contribution (12.4 thousand Euros)\(^\text{17}\).

  Finally, we calibrate the dismissal rate ($\delta$) to 6.5%, the average annual value observed in the entire MCVL sample for workers in the age range 50-65. This is coherent with our treatment of transitions from employment into unemployment as an exogenous process.

- **Preferences and Job-wage offers**

  Strictly speaking, all preference and labor market parameters impact (to some extend) the retirement and reemployment decisions of the individuals and their associated accepted wages. It is not possible, then, to (1) make a partition of the parameter space; (2) make a partition of the space of empirical properties and (3) find a one-to-one relationship among the resulting subgroups. A substantial degree of separability, however, is possible. This is intensively exploited in our informal calibration process, as we describe next:

  1. Reemployment behavior (ie, the reemployment hazards by age and duration) depends on the willingness to search and to accept job offers after searching. They, in turn, depend on the labor market conditions and on the cost of searching. It turns out the

\(^{17}\)This value is a compromise between the figure for those with more than two descendants (2.25 times the minimum contribution) and the value for smaller families (1.75).
job-acceptance decisions are essentially controlled by, first, the rate of arrival of job offers ($\lambda_1$) and, second, by the cost of searching and the relative value of leisure.\footnote{Note that, in all cases, the number of empirical moments that we try to replicate is much larger than the number of parameters that we can handle.} We add a “histeresis” parameter that reduce the job offer arrival rate depending on the duration in unemployment $\phi$. To quantitatively replicate the stylized facts S1 and S2 as best as possible we set the constant annual arrival rate to 45%, the duration dependence value to 60% (meaning that $\lambda_2 = 0.6 \lambda_1$ and $\lambda_3 = 0.6^2 \lambda_1$), the search costs $c$ to 5% and a leisure value $l$ of 5%. The results are discussed in detail in section 4.4.2.

2. The wages accepted by workers that re-enter the labor market are most dependent on the properties of the process generating the size of wage offers. As is standard in the literature, we assume them to be lognormally distributed with annual mean $\mu$ and standard deviation $\sigma$. To quantitatively approximate S4 we set $\mu$ to 8 thousand euros per annum, with a pretty large standard deviation $\sigma$ of 4.5 thousand euros per annum.

3. We finally target retirement behavior, paying special attention to the hazards by age and by duration in unemployment. The discount factor $\beta$ and the relative risk aversion $\eta$ emerged as the crucial parameters controlling this dimension of behavior. The values we select for the benchmark case are a 10% pure time discount and a relative risk aversion value of 4. These values are within the ranges usually encounter in the literature and permit a reasonable reproduction of the empirical regularities (with the limitations discussed in section 4.4.2).

4.4 Simulation Findings

4.4.1 Optimal policy functions

In this section we summarize the basic properties of the optimal “policy functions” of the model, ie. the optimal retirement, non-participation and job-acceptance rules of unemployed workers conditional on their age, previous wage, pension rights and duration in unemployment.

Table 1 summarizes the optimal decisions of our baseline workers in some selected ages. Each matrix is computed for one particular age (58, 62 and 64) and duration in unemployment ($h=1$ and $h=3$). The layout is similar for all of them: each shows whether it is optimal to search (value 1 in the table), to retire (zero) or to stay inactive (letter N) for some illustrative combinations of previous wage $\pi$ -increasing by row- and pension rights $\hat{w}$ -increasing by column-.

It is more illustrative to discuss the results starting by the intermediate matrix, corresponding to age 62. For the unemployed with less than one year on the rolls ($h=1$) we see that retirement tends to prevail in the top right corner of the matrix, ie. for workers with high pension rights and low previous wages. This is not surprising: the value of the retirement option is larger the higher the pension (which is proportional to $\hat{w}$), while higher $\pi$ leads to a larger value of all the unemployment options (through both higher current and future income) and, consequently, to...
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<td>5 8 10 13 16 19</td>
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<td>1 1 1 1 N N</td>
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Table 1: **Optimal individual decisions** at selected ages in the benchmark simulation. Notation: 1= Search 0=Retire N= Non participation
a lower incidence of retirement. Non-participation, in turn, is common for the combination of simultaneously high pension rights and previous wages. With homogeneous search cost and value of leisure, the key element for the decision to stay inactive is the expected value of searching. This is lower the higher the reservation wages, which, in turn, crucially depend on out-of-the-job income. Therefore, $\bar{w}$ is higher the higher the pension income (i.e. the higher $\hat{w}$) or the higher the Unemployment Benefits (i.e. the higher $\pi$). This explains why searching is the first best choice for only relatively poor unemployed (those with low previous wages and accumulated pension rights). For the long-term unemployed (intermediate right matrix of Table 1) retirement becomes the highest valued option. The reason lies in the drop in both current unemployment income and future pension rights experienced by these workers. In our simulations, this drop is strong enough to dominate the potential gains from reducing the early retirement penalties by staying in the rolls for one additional period.

As we approach the normal retirement age (i.e., as we move downwards in the table) the previous findings are modified in two ways. First, the retirement option becomes more valuable, as the early retirement penalties vanishes. This manifests in our simulations in two ways: by increasing outright transitions into retirement, and by substantially increasing the reservation wages of unemployed of shorter durations. This leads to the second clear pattern: active search becomes much less relevant (it actually disappears with the parameter values in our benchmark case). The drop in the relative value of searching induced by higher reservation wages can be clearly appreciated in expression (16). In contrast, search is more valuable if we consider pre-retirement ages (i.e., before the age when pensions are first available). This can be appreciated in the matrices on the top row of Table 1. Non-participation is still the predominant option for most long-term unemployed, due to the rapid deterioration of the job arrival rate with the duration of the unemployment spell. For the rest, it is mainly confined with workers with very high pension rights.

4.4.2 Aggregate predictions: Base case

The basic idea in this section is simple: we compute the labor transitions predicted by the model in a population whose observable characteristics are distributed as in the Spanish economy in 1998/2004 (as reflected by the MCVL05 sample). The model predictions are subsequently compared with the empirical transitions recorded for that very same population (discussed in section 2). A more detailed description of the design of the simulations is confined to Appendix B.1.

The basic aggregate predictions of our baseline model are shown, along with their empirical counterparts, in Figure 6. It is apparent that the model does an acceptable job in reproducing the broad empirical patterns of retirement, reemployment and accepted wages by age. Under closer examination, however, some relevant discrepancies do appear. This is better appreciated in Figures 7 and 8. Of course, this is not surprising, given the rather stylized character of the implemented model.

\[19\text{Note that while maximum unemployment benefits pose a clear limit to the strict proportionality between }\pi \text{ and current benefits, the constraint on future payments via higher contributions is only binding at rather high wages.}\]
Consider first the predictions of the model regarding re-employment. The predicted hazard by age is monotone decreasing (top-left panel of Figure 6) which conforms nicely with stylized fact S1 in the pre-retirement ages. However, the model has difficulties in generating enough reemployment after the ERA. Given the patterns of retirement at that age (discussed below) this may be pointing to composition effects: if early retirement is higher for the group of unemployed with weaker re-employment prospects, the unemployed remaining after 60 may have better average re-employment hazards than those predicted by a model with a homogenous (in terms of unobservable characteristics) population. Regarding duration (fact S2) we see (bottom panels of Figure 7) that, again, the model performs well before the ERA and fall short after 60. We learn that the main difficulties lie within the group of long-term unemployed: in the model, the option of searching is dominated by the alternatives of retirement or non-participation, while in the data there exists a significant incidence of re-employment. This hints at similar composition effects to those mentioned above. The predicted behavior conditional on the size of pension rights (bottom panels of Figure 8) results, once more, in the same pattern: much better
Figure 7: Comparison of the data (-) and the theoretical predictions (- -): re-employment hazard and retirement by age and duration in unemployment.

Figure 8: Comparison of the data (-) and the theoretical predictions (- -): re-employment hazard and retirement by age and pension rights.
agreement before 60, and difficulties after the ERA. The model reproduces the lower reentry levels of the unemployed with higher pensions rights, but goes too far away in the process (predicting no reentry at all for workers above the 2/3 percentage of the distribution by $\hat{w}$).

Finally, and back to Figure 6, we see that the model does a good job in reproducing the average level of re-employment wages. The overall picture is that the empirical regularities seem to fit well with the economic incentives included in the model in the age range 55/60. After 60 the model has difficulties due to the low reentry rate of the long-term unemployed and of the workers entitled with high pension rights. Extending the model to include some unobservable heterogeneity (eg. in the labor market opportunities) will undoubtedly help to bridge the gap between the model and the data after the early retirement age.

Predictions in terms of retirement seem to fit the data better that predictions about reemployment. Again, there are differences when looking into the details, but the overall view is very satisfactory: empirical behavior seems to be very responsive to the economic incentives included in the model. First, the model generate clear spikes of retirement at the Normal and at the Early retirement ages, in accordance with R1 (top right panel of Figure 6). The model does not generate as much early retirement as in the data, both at the age of 60 and at the immediately following ages. This may reflect unobserved heterogeneity in the discount factor (interacting with borrowing constraints) and/or the existence of health shocks. By conditioning on unemployment duration (top row of Figure 7) we see that the mismatch tends to concentrate on unemployed with less than 2 years of duration. But this discrepancy seems small in comparison with the overall success of the model in this dimension. Both in the data and in the model, shifting from the generous contributive unemployment benefit into the unemployment subsidy after two years on the rolls goes hand in hand with a huge increase in retirement hazard (stylized fact R2).

Finally, the model also performs remarkably well in predicting retirement conditional on the level of the pension rights (top row of Figure 8). As in the data (R3), the unemployed with lower pension rights tend to leave in larger numbers at the ERA, while the hazard become monotone increasing in $\hat{w}$ after that age. For workers with high pension rights, however, the model seems to overestimate the extend of non-participation, resulting in lower retirement hazards than in the data.

4.4.3 Aggregate predictions: Experiments

The bottom-right panel of Figure 6 plots the proportion of the sample population (by age) for whom staying in the labor force without searching is the optimal decision. The quantitative importance of this behavior is quite astonishing. The average value in the age range 55/65 is 56.8% (first row of Table 2), although the value undergoes significant changes by age: it grows systematically till reaching a peak at a value close to 70% at the age of 58. After that it remains pretty constant until approaching the Normal Retirement Age. The incidence also changes with duration: before 60 it mainly affects the long-term unemployed, while after 60 it is predominant among the unemployed with less than two years of duration. These numbers suggest that the moral hazard problem associated with unemployment insurance is specially acute at advanced ages. The reason, as discussed in detail in section 4.1.1, lies in the interaction of a generous unemployment benefit (for durations of up to 2 years) and the quite strong early
Experiment Simulation Average %

<table>
<thead>
<tr>
<th></th>
<th>Non participation</th>
<th>Re-employment</th>
<th>Re-employment 60/65</th>
<th>Retirement 60/65</th>
</tr>
</thead>
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<tr>
<td>BASE</td>
<td>56.8</td>
<td>4.96</td>
<td>1.61</td>
<td>25.36</td>
</tr>
<tr>
<td>$\lambda=90%$</td>
<td>34.9</td>
<td>13.23</td>
<td>5.03</td>
<td>25.36</td>
</tr>
<tr>
<td>b rep 50%</td>
<td>53.8</td>
<td>4.68</td>
<td>1.17</td>
<td>38.43</td>
</tr>
<tr>
<td>Separate $\mu$</td>
<td>27.8</td>
<td>7.10</td>
<td>5.09</td>
<td>53.61</td>
</tr>
</tbody>
</table>

Table 2: Simulation results: Incidence of Non-participation and average re-entry and retirement hazards in the age range 55/65 (60/65 when stated) in 4 economic environments: Baseline case (BASE); High offer arrival rate ($\lambda=80\%$); Low unemployment benefit (b rep 50%); Early retirement penalties of the unemployed fixed at the age of departure from the labor force (Separate $\mu$).

The Impact of unfavorable labor market conditions

A popular explanation for the small number of transitions back into employment observed among the Spanish unemployed of advanced age is the harsh labor market conditions faced by this group. The idea is simply that in a labor market as rigid as that in Spain, the chances of finding an acceptable new job after an unemployment spell at those ages is very low. If this is true, it probably implies that the effort of engaging in costly training to improve one’s re-employment chances is not worthwhile.

Our model gives us a chance to test how much of the high non-participation rates is due to the poor labor market conditions and how much can be attributed to the incentives provided by the institutional design. This can be done with a simple counterfactual experiment: compute the optimal decisions of the unemployed in a world with the same institutional environment but better labor market conditions. Figure 9 and the second row of Table 2 reports the results of such an experiment. We double the rate of arrival of job offers, $\lambda$, independently of the duration in unemployment and keeping all other parameters as in the benchmark case. We find that the reentry hazard rate more than doubles, reaching an average value of 13.2%. Most of the gain is achieved among workers of less than 60 years, specially long term unemployed (which find searching more advantageous under the new conditions); in the age range 60/65 the re-entry hazard is a still very modest 5%. The incidence of retirement is hardly changed and, more importantly, non-participation is still very important, with an average value of 34.9% and peaks close to 60% when approaching the Normal Retirement Age.

Our conclusion from these numbers is that institutional incentives matter a great deal, without denying the crucial role played by the labor market conditions. We conjecture that, even with the harsh re-employment chances in our baseline scenario, alternative institutional frameworks may generate better labor-market performance. Our next experiments test this idea.
Figure 9: Theoretical prediction in the benchmark case (-), ie with $\lambda=40\%$ and under more favorable labor market conditions, ie. with $\lambda=80\%$ (- -)

**Institutional Reforms**

The underlying problem in this section is similar to that addressed by the optimal unemployment literature with imperfect information, ie. to try to avoid the abuse of the insurance system by limiting the protection provided (to the point that claiming the benefit is only profitable for those really suffering the insured condition). The simplest possible way of achieving this is by reducing the size of the unemployment benefit. This should make the option of staying unemployed without searching less attractive. For “good risks”, ie. workers with good chances of re-employment, this institutional change should push them back to active work. For “bad risks” ie. workers whose skills are in low demand, making them very hard to re-employ, this should push them into early retirement (paying the price of a heavily penalized pension).

**Unemployment benefit reform**

Figure 10 and the third row of Table 2 reports the results of reducing the replacement rate of Unemployment Benefits from 65% of the previous wage (60 % for $h=2$) to 50%. We find pretty unsuccessful results. Non-participation does go down (from 56.8 to 53.8), but this happens largely from the self-selection of the “bad risks” into retirement rather than from an increase in the re-entry hazard (which hardly change, despite a pretty substantial drop in accepted wages). Essentially, all behavioral changes limit to some extra retirement after 60 among the unemployed with less than 2 years of duration. In a few cases, this change actually reduces the search effort, due to the higher relative value of retiring. In summary, this policy change achieves very little in terms of extra labor supply for the costs it imposes in the welfare of the unemployed. Alternatively, we explore changes in the pension design next.
Figure 10: **Unemployment benefit reform:** Theoretical prediction in the benchmark case (-) and with unemployment benefits equal to 50% of previous wages (- -).

Figure 11: **Pension benefit reform:** Theoretical prediction in the benchmark case (-), and with early retirement penalties fixed at the effective age of exit from employment (- -).
Reform of the early retirement penalties

In section 4.1.1 we showed that early retirement penalties favor non-participation by making it possible to obtain larger future pension income without searching. Obviously, preventing the early retirement of employees makes perfect sense; That the early retirement penalties will also retain the unemployed in the labor force (largely without searching) is an unintended consequence. This points to the convenience of designing separate pension formulae for the employees and the unemployed.

Although different alternative schemes can be figured out, the most straightforward idea is simply to stop the automatic increase of the pension while the unemployed stay out of job. A simple way of achieving this is by making the early retirement penalty depend on the age of exit form employment, rather than the age when the individual claim the pension benefit. It would amount to changing the replacement rate $\mu(a)$ in equation (3) to $\mu(a - h)$. So if somebody is made redundant at the end of the year when he/she is 59, his/her “Base reguladora”, $\hat{w}$, will be equally punished if he retires immediately ($a=60, h=0$) or if he/she waits for one year ($a=61, h=1$) or longer. This (rather ruthless) reform eliminate the incentives to stay idle while enjoying the unemployment benefit.\(^{20}\) Figure 11 and the bottom row of Table 2 reports the results of this reform. It is very successful in fostering re-employment after 60: the hazard in that range of ages is almost four times larger than in the base simulation (although this is partly achieved via reductions in the accepted re-entry wages). It also pushes some of the non-participant unemployed of more than 60 years of age into retirement. All these changes are essentially concentrated among the unemployed with shorter durations and average or above-average previous wages. At 60 it affects mostly unemployed with high pension rights, but this changes as we approach the Normal Retirement age. All in all, non participation after 60 is very substantially reduced by the reform.

5 Conclusions

The most basic conclusion we can draw from our analysis so far is that the empirical labor behavior at advanced ages can be rationalized as the response of rational individuals to both the labor market conditions and the institutional incentives. We have shown that a model that combines those two elements goes a long way in reproducing the re-employment and retirement patterns observed in the Spanish data. One important contribution of this paper relates to the role of the institutional incentives: we show via simulation that even in presence of a much larger arrival rate of job offers, the predicted re-employment rate in the current institutional setting would not improve a great deal. The combination of generous Unemployment Benefits (for durations of up to two years), and substantial penalties for early retirement are to be blamed for this. Together, they make staying unemployed without searching an optimal strategy for a very large part (around 50%) of the unemployed in the age range 55/65. Although the Spanish authorities have largely turned a blind eye on this “alternative” exit route into retirement, it

\(^{20}\)The proposed reform treats the unemployed in a harsh way and would undoubtedly generate strong political resistance. We have analyzed some “intermediate” reforms like increasing the pension rights of the unemployed only after a successful return to the labor force. The proposed reforme is, nonetheless, specially interesting as an evaluation of the maximum potential impact of measures of this type.
seems a clear abuse of the original mandate of the unemployment insurance scheme (namely, support workers in their search process).

Our simulations shows that changing this state of affairs would demand substantial reforms of the present institutions. We find that reducing the generosity of Unemployment Benefits would not be particularly successful, as it pushes workers mainly into retirement. A more promising (although rather ruthless) avenue focuses on redesigning the early retirement penalties of the unemployed. If the penalty were fixed according to the age when the individual effectively withdraws from the labor force (rather than when he/she claims the pension for the first time), the incentives to stay idle would be much smaller. Our simulation indicate that such a measure would be very effective in reducing non-participation and increasing labor supply, specially after the early retirement age.

There are some possible extensions of the current work that may improve the quality of our findings. We are specially interested in analyzing the impact of including several dimensions of unobserved heterogeneity. We plan to consider in our future research explicit population distributions for the discount factor and the arrival rate of job offers. This seems to be a promising avenue to tackle the discrepancies observed between the data and the predictions of the model.
References


A The role of labor market conditions in re-employment and retirement

The difference in value between retiring and searching at the age of 63 are:

\[ R_{63} - S_{63} > 0 \iff \mu_{63} \hat{w} + l - b_h \pi + C + \beta [ \mu_{63} \hat{w} + l - E(I_{64}) - E(l_{64})] + \beta^2 A_{65} [B_{63} - E(B_{65}(\hat{w}_{65}))] > 0 \]

with expected future income, leisure and pensions:

\[
E(I_{64}) = (1 - \lambda_{h+1} F_{\pi_m}(\hat{w}_{h+1} \pi)) + \lambda_{h+1} \int_{\hat{w}_m}^{\hat{w}_M} w \, dF_w
\]

\[
E(l_{44}) = (1 - \lambda_{h+1} F_{\pi_m} l)
\]

\[
E(B_{65}(\hat{w}_{65})) = \hat{w} + \frac{\hat{w} - \hat{w} \pi - \hat{w}}{D} (1 - \frac{1}{D}) + (1 - \lambda_{h+1} F_{\pi_m} \frac{\hat{w} - \hat{w} \pi - \hat{w}}{D} + \lambda_{h+1} \int_{\hat{w}_m}^{\hat{w}_M} \frac{w - \hat{w}}{D} \, dF)
\]

The current utility cost of staying in the labor force includes lower income, search cost and foregone leisure. In exchange, waiting not only (typically) increases the future pension by reducing early retirement penalties, but also gives the individual a chance of receiving a good job offer, which would increase future labor income and pensions.

B Numerical solution method

There are no analytical solutions to the functional equations defining the value functions described in the previous sections. Consequently, we employ numerical methods to compute the optimal retirement and search decisions, calculate the value functions and explore the basic properties of the solutions.

Including two continuous states, the value functions are infinite dimensional objects and can only be reproduced in the computer approximately. The use of some numerical approximation method is, then, unavoidable. In particular, we:

- Discretize the continuous state variables when computing the value functions.

Thus, we build an uniform grid in the State Space \( X_N = [\pi_m, \pi_M] \times [\hat{w}_m, \hat{w}_M] \), with \( \pi_m \) and \( \pi_M \) denoting, respectively, the inferior and superior bounds. \( N \) is the number of nodes in the grid. In each iteration \( a = \{55, \ldots, 65\} \) we compute:

\[
U_a(x_j, h) \quad j = \{1, \ldots, N\} \quad h = \{1, 2, 3\}
\]

- We use linear interpolation whenever a value function is evaluated outside the grid. For example, to compute the reservation wage of an unemployed worker in state \((\pi, \hat{w}, h, a)\) we have to evaluate \( W_a(w, \hat{w}) \) for any value of \( w \) (and not just \( w \in \{\pi_m, \ldots, \pi_M\} \)). We also have to repeatedly evaluate the future value of staying unemployed with pension rights that do not exactly match the values in the grid.

We use linear interpolation because, although is more time consuming than other higher order approximation schemes, it guarantees that the shape of the value function is preserved.
B.1 Design of the aggregate simulations

The basic idea is simply to compare the model predictions in term of transition flows with their empirical counterparts, taking the stocks as given.\footnote{Note that the model also generates predictions in terms of stocks, but we abstract from them at this stage.} To implement this simple concept we undertake a sort of Monte-Carlo experiment involving the following steps:

1. Create a large sample of individuals reproducing the empirical distribution of labor states by \( \{a, \pi, \hat{w}, h\} \):

   \[
   \mu(\pi, \hat{w}, a, h) \quad a \in \{50, \ldots, 65\} \quad h \in \{1, \ldots, 3\} \quad \pi \in [\pi_m, \pi_M] \quad \hat{w} \in [\hat{w}_m, \hat{w}_M]
   \]

2. Simulate the arrival of job offers and the size of the wage proposed, according with the parametric functions included in our model.

3. Let the individual in the simulated sample react to the job offers (and to the alternative of retiring from the labor force) in accordance with the model policy functions. Keep records of the job acceptances and retirement decisions.

4. Aggregate the decisions, compute the implicit re-employment and retirement hazards and compare with empirical counterparts.