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Should we raise public expenditure on basic education and reduce expenditure at college?

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Keywords: Basic education, college education, public expenditure in education.

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Abstract

This paper analyzes public intervention in education, taking into account the existence of two educational levels: basic education and college education. The government decides per capita expenditure at each level and the subsidy for college education. We explore the effect of transferring money from one level to the other on equity and efficiency. We find that there is always a policy reform that satisfies both the objectives of equity and efficiency, where efficiency refers to average productivity of college graduates. For developed countries, this policy consists of transferring resources from college education to basic education.

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1 Introduction

In most countries, public expenditure on education accounts for a large proportion of total expenditure on education.[1] Public intervention is present at all educational levels, from pre-primary to tertiary education. However, countries differ dramatically according to how they allocate resources across the different educational levels. Columns 1 and 2 of Table 1 show data on yearly per student expenditure at basic and tertiary education, respectively.[2] In Column 3 we compute the ratio between expenditure per student in tertiary education and in basic education. We observe a large heterogeneity. The ratio ranges from 1.00 in Italy to 3.23 in Mexico, with an average for the OECD of 1.68. Columns 4 and 5 show the change in annual expenditure per student from 1995 to 2004 and the ratio between both indexes is reported in Column 6. Fifteen countries out of twenty one have a ratio lower than one, meaning that in this period they have diverted resources from tertiary to basic education, at least in relative terms.

Since Table 1 refers only to annual expenditure, we present some additional evidence in Table 2. Here we show data on *cumulative* expenditure at basic and tertiary education, taking into account the duration of each educational level. Again we observe large differences across countries. In Column 3 we compute the ratio between cumulative expenditure at both levels and we see that it ranges from 0.36 in Korea and New Zealand to 0.98 in The Netherlands.

Tables 1 and 2 document the existence of large differences in educational policies across countries. One possible explanation is that each country has different objectives.[3] In fact, the role of education is at the heart of current and quite di-

[1]For the OECD countries, an average of 87% of expenditure on all levels of education came from public sources in 2004. See Table B3.1, Education at a Glance 2007, OECD.

[2]Basic education corresponds to primary, secondary, and post-secondary non-tertiary education.

[3]Alternatively, if two countries have exactly the same fixed and marginal costs, but different proportions of college students they will not have the same expenditure per capita.

verse debates, such as poverty reduction or the challenge of new technologies. A crucial question when analyzing public intervention is to establish the objectives of educational policies. Most governments care for efficiency and equity issues in a wide sense. However, sometimes the problem is to give a precise meaning to these general principles. To circumvent this problem we propose that equity concerns imply that the objective of the government should be to facilitate everybody the access to education, irrespective of family background. Regarding efficiency we propose two alternative objectives. The first objective consists of increasing the productivity of college students, while the second one is to increase the average productivity of the whole population.[4] We study how these two objectives relate to each other and then we analyze which policies should implement the government to improve efficiency and equity at the same time. In particular, we want to study whether both objectives are compatible or not and, if they are, which policies makes them compatible. Second, we explore whether all countries, rich and poor, should apply the same policy to satisfy these two objectives or if the policy reform is country-specific.

To study these issues, and in line with the data in Tables 1 and 2, we build a model with two educational stages: basic and college education. Basic education comprises all mandatory levels of education and we assume it is fully financed by the government. In contrast, college education is voluntary and students may have to pay a part of the cost. Another difference is that expenditure on basic education affects the quality of education, but not enrollment, since attendance is mandatory. On the other hand, expenditure on college education affects not only quality, but also enrollment. Individuals who go to college get a skilled job, while the rest remain unskilled. Due to capital markets imperfections, some individuals suffer from borrowing constraints.

Most governments, instead of introducing large reforms, introduce small reforms

[4]See for example Lloyd-Ellis (2000), Su (2004) and Blankeneau et al (2007) who consider similar criteria.

in several steps. Then, we focus on educational reforms, instead of focusing on the design of an optimal educational policy. We start from a given division of the budget between the two levels of education and we study the effect of diverting resources from one educational level to the other.[5]

What we find is that there is always a policy reform that simultaneously satisfies the objectives of equity and efficiency, where efficiency means to improve the average productivity of college graduates. We also find that for rich countries, this policy consists precisely of transferring resources from college education to basic education. The intuition is that this policy reduces the threshold level of income needed to attend college, but at the price of raising the threshold level of ability. Since higher education is heavily subsidized in most of the rich countries, the first effect is smaller in size and attendance reduces. However, due to the increase in the threshold level of ability the productivity of skilled workers rises. In addition, this policy has a positive effect on the productivity and the number of unskilled workers. Therefore, we also find that by transferring resources from college to basic education the average productivity across the population as a whole rises. On the contrary, for low income countries the policy reform that has a positive effect on equity and at the same time improves the productivity of skilled workers consists of transferring resources from basic to college education. However, in general we find that this policy will have a negative effect on the average productivity across the population.

We discuss briefly some previous works related to ours. Lloyd-Ellis (2000) studies the impact of alternative allocations of public resources between basic and higher education on enrollment, income distribution and growth, while Blankenau, Cassou and Ingram (2007) investigate its output and welfare implications. However, none of them consider individual heterogeneity with respect to family background, which is

[5] This approach is similar to that in much of the literature on optimal commodity taxation which focuses on local effects of tax changes. See Feldstein (1975), Guesnerie (1977), and King (1983) among others.

one of our main focuses. Driskill and Horowitz (2002) study optimal investment in human capital in a standard growth model, and they find that developing countries should concentrate on advanced human capital, a result similar to ours. Restuccia and Urrutia (2004) focus on intergenerational mobility and find that an increase in expenditure on early education has more impact than an increase in college subsidies. Su (2004) studies the dynamic effects of allocating public funds between basic and college education. However, she abstracts from private education expenditure which is a crucial factor affecting education outcomes. Finally, Romero (2008) considers that voters decide how to split the budget between basic and college education and he studies how the possibility of opting out from public education affects that decision.

The paper is organized as follows. In Section 2 we describe the economy. In Section 3 we consider the effect of public policies on the different objectives of the government and we illustrate our main result with numerical examples. Finally, Section 4 concludes.

2 Model

2.1 Individuals and Educational Sector

We build a model with two periods. In the first period there is a continuum of families made up by one mother and one child. Families are characterized by the income of the mother $y \in [0, Y]$ and the innate ability of the child $a \in [0, A]$, where $Y, A > 0$. The respective cumulative distribution functions are $F(y)$ and $G(a)$, although to get closed-form solutions we will assume that a is uniformly distributed on its support. We also assume that y and a are independently distributed.[6]

In the first part of the first period all children attend basic education, which is

[6] As we will see below, college attendance is the proportion of individuals with ability and income above some given thresholds. The assumption that a and y are independently distributed allows us to study separately the effect of policy changes on college attendance through the effect on the two thresholds.

compulsory. In the second and last part of the first period, children can either get a job as unskilled workers or, alternatively, they can enrol in higher education to become skilled workers. We call δ the fraction of time of the first period spent at college and, thus, out of the labor force, where $0 < \delta < 1$. Since education takes place only in the first period, we assume that it is financed by means of a proportional tax on all income obtained in the first period.

In the second period, only the children live. Those who attended college in the first period get a skilled job, while those who did not remain in an unskilled job. Children care only for their own consumption in the second period (C_2) which is equal to the value of their lifetime income. Mothers have to decide how much income to spend in family consumption in the first period (C_1) and how much to spend in college education (E). We assume that they have homothetic preferences over (C_1, E), which implies that all of them want to spend the same fraction of income in each item. We call φ the fraction of income that they are willing to spend on E.

We assume a simple structure for the educational sector. The per capita cost of providing basic education is denoted as $c_L > 0$. Since basic education is compulsory, we assume that its cost is paid in full by the government.[7]

Regarding higher education we want to separate public provision from public financing. The level of public provision is captured by $c_H > 0$, which is the per capita cost of providing higher education. This includes wages paid to teachers, the cost of college equipment, laboratories, etc. The level of public financing of higher education is captured by s, which represents the proportion of the total cost that the government subsidizes through grants, zero-interest rate loans, etc. That is, the government pays $c_H s$, while college students pay $c_H(1-s)$, with $0 \leq s \leq 1$. To simplify things, we assume that the subsidy is the same for all individuals. We define

[7] In 2003, only 7.4% of total expenditure in basic education in the OECD (primary, secondary and post-secondary non-tertiary education) was privately financed.

as $\beta = c_H/c_L$ the ratio between both costs.[8] Finally, as we said above, all public expenditure is financed by imposing a proportional tax over period one income.

We want to distinguish between public provision and public financing as each one of them can be used by the government to achieve different objectives. The parameter c_H, as well as c_L in the case of basic education, captures the quality of education. Increasing c_H could be seen as a way of improving the quality of college education which, in turn, may have a positive effect on the human capital of college graduates. However, for a fixed level of c_H, an increase in s can be seen as a way of easing access to college for individuals from low-income families.

2.2 College Attendance

Since children care only for their consumption in the second period C_2, their only concern will be to maximize lifetime income. Next we define lifetime income for the two types of workers, unskilled and skilled.

An individual who only attends basic education will be an unskilled worker for all her remaining lifetime. We assume that her productivity and, thus, her wage w_U will be determined exclusively by per capita expenditure at basic education c_L. In particular we set $w_U = c_L$. Since they work a fraction δ of the first period, their lifetime income is $(1-t)\delta c_L + c_L$, where t is the constant marginal tax rate.[9]

If an individual goes to college she becomes a skilled worker. Her productivity and thus, her wage, rises to $w_S(a) = c_L + c_H a$.[10] We assume that c_L represents a positive effect of education that applies irrespective of ability, while c_H interacts with ability.

[8] As in Blankeneau et al (2007) and Lloyd-Ellis (2000) we do not consider the existence of fixed costs. One reason is of tractability. Another reason is that we are interested only in marginal changes in per capita costs.

[9] To simplify the analysis we assume that individuals do not discount future payoffs.

[10] We have tried with a more general version of the model in which public expenditure has decreasing returns. In particular, we have used $w_L = (c_L)^\alpha$, and $w_H = w_L + (c_H)^\alpha a$, where $0 < \alpha \leq 1$. As we will comment below, assuming decreasing returns to expenditure reinforces our main results, although the expressions become much more complicated.

Lifetime income of a skilled worker will be $w_S(a) - c_H(1-s) = c_L + c_H a - c_H(1-s)$. An individual will choose higher education if:

$$c_H a - c_H(1-s) \geq (1-t)\delta c_L. \tag{1}$$

We assume that this condition always holds for those individuals with the highest ability A since, otherwise, nobody will choose a college education. In particular we require:

$$c_H A - c_H(1-s) > (1-t)\delta c_L. \tag{2}$$

Condition (1) will hold for those individuals with ability a above a threshold $\widehat{a}$, with $0 < \widehat{a} < A$, which is defined as:

$$\widehat{a}(c_L, c_H, s) = 1 - s + (1-t)\delta\frac{c_L}{c_H} = 1 - s + (1-t)\frac{\delta}{\beta}. \tag{3}$$

The threshold value $\widehat{a}$ increases with c_L and δ, and it decreases with c_H, t, or s.

However, individuals must also be able to afford the tuition cost $c_H(1-s)$ to attend college. As we saw above all mothers spend the same fraction of after-tax income in college education. That is, a mother with pre-tax income y is willing to spend on college education at most $\varphi(1-t)y$. This amount can be interpreted as the value of the transfer children get from their mothers to spend on college education. On top of this amount, we assume that children have also access to a loan from a bank. However, and due to capital market imperfections we assume that they can borrow only up to an amount $\gamma c_H(1-s)$, where $0 \leq \gamma \leq 1$. The parameter γ captures the "quality" of capital markets. This borrowing constraint, an exogenous feature of the model, is assumed to be the same across individuals.[11] The two polar cases are $\gamma = 0$, which means complete impossibility of borrowing, and $\gamma = 1$ which means

[11] Lochner and Monge-Naranjo (2002) build a model with endogenous borrowing constraints. Individuals of heterogeneous abilities or those making different schooling choices face different borrowing constraints. We implicitly assume that banks cannot condition loans on ability, as they cannot observe it.

that capital markets are perfect. So, the higher is γ the better is the quality of capital markets.[12] To attend college, therefore, individuals must have pre-tax income satisfying:

$$y \geq \widehat{y}(c_H, s) = \frac{(1-\gamma)c_H(1-s)}{\varphi(1-t)}. \tag{4}$$

Those who get a transfer above $c_H(1-s)$ do not need to ask for a loan. Those with a transfer below $(1-\gamma)c_H(1-s)$ cannot afford college. Finally, those with a transfer between $(1-\gamma)c_H(1-s)$ and $c_H(1-s)$ need a loan to attend college. The proportion of individuals who can afford college is the proportion of individuals with pre-tax income above $\widehat{y}$, namely, $1-F(\widehat{y})$. When $\gamma = 1$ or $s = 1$, we get $\widehat{y} = 0$ and the constraint is not binding for any individual. To simplify notation we call $p = 1 - F(\widehat{y})$, and in the sequel we assume that $p > 0$. The proportion of individuals attending college π is the proportion of individuals who satisfy at the same time Conditions (1) and (4). Since y and a are independently distributed, we can write π as:

$$\pi(c_L, c_H, s) = \frac{p}{A} \times (A - \widehat{a}). \tag{5}$$

It is immediate to check that π is decreasing with c_L and increasing with γ and s. However, we cannot sign easily the partial derivatives of π with respect to c_H and t. For example, while an increase in c_H rises the tuition cost reducing p, it also affects positively the term $(A - \widehat{a})$ since now college students get higher wages. In the Appendix we study in more detail the sign of the partial derivative $\frac{\partial \pi}{\partial c_H}$. We see that, except for the cases when s is so low that nobody wants to attend college or when it takes value 1, the sign of $\frac{\partial \pi}{\partial c_H}$ will be negative. To be more precise, if we define as $\varepsilon^p_{c_H} = \frac{\partial p}{\partial c_H}\frac{c_H}{p}$ the elasticity of p with respect to c_H, we can show that $\frac{\partial \pi}{\partial c_H} < 0$ if and only if $\varepsilon^p_{c_H} < \frac{-(1-t)\delta c_L}{c_H A - c_H(1-s) - (1-t)\delta c_L}$. This term is the ratio between earnings foregone in the first period by college students and the boost in lifetime income obtained from

[12]The parameter γ could be alternatively interpreted as a policy variable. Many countries are offering students' loans to overcome this constraint. Then $\gamma = 1$ means that there is such a policy in place, while $\gamma = 0$ means a complete absence of it.

attending college by the highest ability individual. In principle we should expect this ratio to be very low. However, as we said above, when s takes either a very low value or it is 1, the condition will not hold because then $\varepsilon^p_{c_H} = 0$.[13]

2.3 The Government Budget Constraint

Here we study how the three instruments of the government (c_L, c_H, s) are related through the budget constraint. We define total expenditure in education E as:

$$E \equiv c_L + s c_H \pi. \tag{7}$$

Total government revenue R is the sum of taxes collected from mothers plus taxes collected from children who do not attend college in the first period:

$$R \equiv t\overline{y} + t\delta c_L(1-\pi), \tag{8}$$

where $\overline{y}$ represents mean income of mothers. We assume that the government cannot run a deficit. Then the budget constraint is:

$$E \equiv c_L + s c_H \pi \leq t\overline{y} + t\delta c_L(1-\pi) \equiv R. \tag{9}$$

For fixed values of c_L and c_H, we call $\widehat{s}$ the value of the subsidy for which the constraint is satisfied with equality. If $E < R$ for all values of the subsidy, then we set $\widehat{s} = 1$. In the next proposition we provide conditions that guarantee that $\widehat{s}$ exists and that it is unique.

Proposition 1 *Consider any combination (c_L, c_H) and assume that the tax rate t satisfies that $t \geq t_0 = \frac{c_L}{\overline{y}+\delta c_L}$. Then, there is a unique value $\widehat{s}(c_L, c_H) > s_0 = 1 - A + (1-t)\frac{\delta c_L}{c_H}$ that satisfies the budget constraint. Moreover, there is a second threshold $t_1 > t_0$ such that if $t < t_1$ then $\widehat{s}(c_L, c_H) < 1$, while if $t \geq t_1$, then $\widehat{s}(c_L, c_H) = 1$.*

[13] The elasticity can be written as:

$$\varepsilon^p_{c_H} = -\frac{f(\widehat{y})(1-\gamma)(1-s)}{\varphi(1-t)}\frac{c_H}{p}, \tag{6}$$

where f is the density function of y.

Proof. First we study the function E. When $s \leq s_0 = 1 - A + (1-t)\frac{\delta c_L}{c_H}$, nobody attends college and expenditure E is constant at the level $E = c_L$. When $s > s_0$, E is strictly increasing in s. Second, we study the function R. When $s \leq s_0$, since $\pi = 0$ revenue R is constant at the level $R = t\overline{y} + t\delta c_L$. When $s > s_0$, R is strictly decreasing in s. Then, to have an equilibrium a necessary condition is that the function R must be above the function E when $s \leq s_0$. This will be the case if $t\overline{y} + t\delta c_L > c_L$ or if $t > \frac{c_L}{\overline{y} + \delta c_L}$. Note now that since E increases and R decreases to the right of s_0, we have two possibilities. Either E is always below R, in which case $\widehat{s}(c_L, c_H) = 1$, or they cross at a value of the subsidy $\widehat{s}(c_L, c_H)$ strictly below 1. To see which case prevails we need to compare the values of both E and R when $s = 1$. When $s = 1$, $\pi = 1 - \frac{1}{A}(1-t)\delta\frac{c_L}{c_H}$ and from Equations (7) and (8):

$$E(s = 1) = c_L + c_H - \frac{1}{A}(1-t)\delta c_L, \tag{10}$$

$$R(s = 1) = t\overline{y} + \frac{t(1-t)\delta^2 c_L^2}{c_H A}. \tag{11}$$

If $E(s = 1) \leq R(s = 1)$ we have $\widehat{s}(c_L, c_H) = 1$, while if $E(s = 1) > R(s = 1)$ we have $\widehat{s}(c_L, c_H) < 1$. There is a unique value of t for which $E(s = 1) = R(s = 1)$ and $R(s = 1)$ cuts $E(s = 1)$ from below. This is the value we call t_1. To prove that $t_1 > t_0$, it suffices to prove that if we set $t = t_0$, then $E(s = 1) > R(s = 1)$. But we have defined above t_0 as the value of the tax for which $E(s) = R(s)$ for all $s \leq s_0$. Since $E(s)$ is increasing with s for $s > s_0$ while $R(s)$ is decreasing with s for $s > s_0$, we have that $E(s = 1) > R(s = 1)$ for $t = t_0$. ■

The condition that t must be above t_0 is required in order to have at least some individuals attending college. For given values of γ, δ, A, t and $\overline{y}$, the fact that the government has to satisfy the budget constraint implies that it has only two free policy instruments. We choose c_L and c_H as the two free parameters and we assume that the subsidy always adjusts to satisfy the constraint. Since we are interested in policy changes, we want to know what is the effect of changes in c_L and c_H on $\widehat{s}(c_L, c_H)$. To do this we assume that the equilibrium is interior. In particular we define $\widehat{s}(c_L, c_H)$ as the value that satisfies:

$$E(c_L, c_H, \widehat{s}) - R(c_L, c_H, \widehat{s}) = 0. \tag{12}$$

Computing the corresponding derivatives:

$$\begin{aligned}
\frac{\partial \widehat{s}}{\partial c_L} &= -\frac{\frac{\partial E}{\partial c_L} - \frac{\partial R}{\partial c_L}}{\frac{\partial E}{\partial s} - \frac{\partial R}{\partial s}} = -\frac{[1 - t\delta(1-\pi)] + [t\delta c_L + \widehat{s}c_H]\frac{\partial \pi}{\partial c_L}}{\frac{\partial E}{\partial s} - \frac{\partial R}{\partial s}}, \\
\frac{\partial \widehat{s}}{\partial c_H} &= -\frac{\frac{\partial E}{\partial c_H} - \frac{\partial R}{\partial c_H}}{\frac{\partial E}{\partial s} - \frac{\partial R}{\partial s}} = -\frac{\widehat{s}\pi + [t\delta c_L + \widehat{s}c_H]\frac{\partial \pi}{\partial c_H}}{\frac{\partial E}{\partial s} - \frac{\partial R}{\partial s}}.
\end{aligned} \tag{13}$$

Since $\frac{\partial E}{\partial s} - \frac{\partial R}{\partial s} \geq 0$, the sign of $\frac{\partial \widehat{s}}{\partial c_L}$ and $\frac{\partial \widehat{s}}{\partial c_H}$ will be negative if the terms in the numerator are positive. Assume first that college attendance π is not affected by either c_L or c_H. Then, it is clear that both derivatives are negative. That is, rising either c_L or c_H reduces the resources that can be used to subsidize higher education. However, college attendance can also be affected negatively by the increase in c_L or c_H, reducing the absolute value in the numerators. Intuitively, the negative effect on the subsidy is attenuated, since now fewer individuals receive it. What we do is to assume that the indirect effect through π is not that large so as to offset the initial negative effect.[14]

Assumption 1 (A.1): The following conditions hold: (i) $\frac{\partial \widehat{s}}{\partial c_H} < 0$ and (ii) $\frac{\partial \widehat{s}}{\partial c_L} < 0$.

We define an *"iso-subsidy"* curve as the set of all combinations (c_L, c_H) giving rise to the same value of the subsidy $\widehat{s}$. From Equations (12) and (13), the slope of an iso-subsidy is:

$$\frac{\partial c_H}{\partial c_L}\Big|_{s=\bar{\widehat{s}}} = -\frac{\frac{\partial \widehat{s}}{\partial c_L}}{\frac{\partial \widehat{s}}{\partial c_H}} = -\frac{[1 - t\delta(1-\pi)] + [t\delta c_L + \widehat{s}c_H]\frac{\partial \pi}{\partial c_L}}{\widehat{s}\pi + [t\delta c_L + \widehat{s}c_H]\frac{\partial \pi}{\partial c_H}}. \tag{14}$$

[14] In the Appendix we discuss which are the conditions that need for this to be true. Basically we need to assume that the elasticities of π with respect to both c_L and c_H are small in size.

By Assumption 1, this slope is negative, implying that there is always a trade-off between expenditure on basic education and expenditure on college education. Holding the subsidy fixed, if we increase one of them we have to reduce the other.

From Proposition 1 we also see that a fixed combination (c_L, c_H) corresponds to different values of the subsidy in two countries that have different values of mean income $\overline{y}$. Using the equilibrium condition above, for a fixed (c_L, c_H), the equilibrium value of the subsidy in a rich country will be higher than in a less rich country. The equilibrium value of college attendance will be also higher in the rich country than in the less rich country.

In Figure 1 we illustrate our policy space. We represent in black (respectively, red) a rich (respectively, poor) country. The closer to the origin, the higher is the value of the subsidy. The curved lines in the bottom part of the figure represent the restriction that the subsidy s must be above a threshold s_0. The interpretation of this condition is that, for a fixed value of the subsidy $\widehat{s}$, only those combinations (c_L, c_H) satisfying $c_H > \frac{(1-t)\delta}{\widehat{s}-(1-A)} c_L$ are allowed. Point A in the figure corresponds to a higher subsidy in a rich country than in a poor country. In the example of the figure the values of the subsidy are 0.85 and 0.6, respectively.

Finally we also see that for a fixed combination (c_L, c_H), the iso-subsidy curve $\widehat{s}(c_L, c_H)$ is flatter in a rich country rather than in a less rich country, provided that both $\varepsilon^{\pi}_{c_H}$ and $\varepsilon^{\pi}_{c_L}$ are of small size.[15] The intuition is simple. Consider a rich country where both $\widehat{s}$ and π are large. If the government rises c_H this policy will have a large impact on expenditure because a lot of people are getting a large subsidy. If nothing else is done, the subsidy should be reduced in a large amount. To hold the subsidy fixed, a large reduction of c_L is needed. In the less rich country, on the contrary, rising c_H has a smaller effect on expenditure, since few people attend college and the subsidy is low. The reduction needed in c_L to keep the subsidy constant is smaller.

[15] See the Appendix.

Here Figure 1 (The policy space)

3 Policy Reforms

We want to analyze policy reforms from an initial situation described by a particular combination (c_L, c_H) through their effects on different government objectives. In particular, we assume that the government will make policy changes that do not harm the objective of equity. That is, only those policy reforms that do not have a negative effect on the income threshold are considered. The implication of this restriction is that the government cannot raise at the same time both c_L and c_H, since this would rise the income threshold. Given this restriction, we start by discussing different objectives that the government might consider in order to enhance efficiency.

3.1 On the Different Government's Objectives

First, we consider that the objective of the government consists of increasing the average productivity or the average human capital of college graduates. This would be the case if the government is particularly concerned in improving the productivity of skilled workers. Since an individual with ability a who attends college has productivity $c_L + c_H a$, the productivity of graduates, denoted by Q_S is:

$$Q_S = c_L + c_H \left(\frac{\widehat{a} + A}{2} \right) = \frac{1}{2} \left[(2 + \delta(1-t))c_L + (1 - \widehat{s} + A)\, c_H \right]. \tag{15}$$

In the space (c_L, c_H) we can define an *"iso-productivity"* curve as the set of combinations (c_L, c_H) giving rise to the same level of Q_S. From (15) and (3) the slope of an iso-productivity curve can be written as:

$$\left. \frac{\partial c_H}{\partial c_L} \right|_{Q_S = \overline{Q}_S} = \frac{c_H \frac{\partial \widehat{s}}{\partial c_L} - (2 + (1-t)\delta)}{A + 1 - \widehat{s} - c_H \frac{\partial \widehat{s}}{\partial c_H}}. \tag{16}$$

By Assumption 1 this slope is negative. The government cannot raise at the same time c_L and c_H since this harms equity. Then, there are only two ways of improving the productivity of skilled workers. Either to raise c_L and reduce c_H transferring resources from college to basic education, or the opposite policy reducing c_L and rising c_H. Our main result below proves that the first policy reform is the only one that does not harm equity in high income countries, while in low income countries it is the second one.

A second efficiency objective consists of rising the average level of human capital of the entire cohort of individuals, and not only that of college graduates. Recall that a proportion $1-\pi$ of the cohort has human capital c_L, while those attending college have human capital $c_L + c_H a$. If we call Q_T the average level of human capital, we have:

$$Q_T = c_L + c_H \pi \left(\frac{\widehat{a} + A}{2} \right) = \frac{1}{2} \left[(2 + \delta(1-t)\pi) c_L + \pi \left(1 - \widehat{s} + A\right) c_H \right]. \tag{17}$$

Using the definition of Q_S above, we can also write Q_T as:

$$Q_T = (1-\pi) c_L + \pi Q_S, \tag{18}$$

where the first term captures the aggregate level of human capital of unskilled workers and the second term takes into account both the quantity and the quality of college graduates. The main difference with Q_S above is that now we also care for college attendance. In particular, consider a policy change that rises Q_S by transferring resources from college education to basic education. If π does not change, it is obvious that Q_T will rise as well. However, in general we should expect a change in college attendance. If college attendance diminishes the term on the left $((1-\pi)c_L)$ gets higher, while the term on the right (πQ_S) can either rise or diminish. In the next Proposition we show that in general this second effect is of a smaller size meaning that any policy that transfers resources from college to basic education that has a positive effect on Q will also have a positive effect on Q_T.

Proposition 2 *Suppose that* $\frac{\partial \pi}{\partial c_H} \leq 0$ *and that* $p \leq \frac{2}{2+\delta(1-t)}$. *Then, the slope of* Q_T *is steeper than the slope of* Q_S *at any combination* (c_L, c_H).

Proof. From (18) the slope of Q_T can be written:

$$\frac{\partial c_H}{\partial c_L}\,|_{Q_T=\overline{Q}_T} = -\frac{(1-\pi)+\pi\frac{\partial Q_S}{\partial c_L}+(Q_S-c_L)\frac{\partial \pi}{\partial c_L}}{\pi\frac{\partial Q_S}{\partial c_H}+(Q_S-c_L)\frac{\partial \pi}{\partial c_H}}. \tag{19}$$

If both $\frac{\partial \pi}{\partial c_L}$ and $\frac{\partial \pi}{\partial c_H}$ are zero, we are done. Then, we consider the case in which $\frac{\partial \pi}{\partial c_L}$ and $\frac{\partial \pi}{\partial c_H}$ are not zero. The condition that guarantees that Q_T is steeper than Q_S is:

$$\frac{(1-\pi)+(Q_S-c_L)\frac{\partial \pi}{\partial c_L}}{(Q_S-c_L)\frac{\partial \pi}{\partial c_H}} < \frac{\frac{\partial Q_S}{\partial c_L}}{\frac{\partial Q_S}{\partial c_H}}. \tag{20}$$

Since the term on the right is positive and the denominator on the left is negative, a sufficient condition for this inequality to hold is that the numerator is positive. Substituting the values of $\frac{\partial \pi}{\partial c_L}$ and Q_S and rearranging terms, the numerator is positive if:

$$(1-\pi) \geq \frac{p(1-t)\delta}{A}\left(\frac{\widehat{a}+A}{2}\right). \tag{21}$$

Since $\pi = p(1-\widehat{a}/A)$, this can be written as:

$$2(1-p)A + 2p\widehat{a} \geq p(1-t)\delta A + p(1-t)\delta\widehat{a}. \tag{22}$$

Since $2 > (1-t)\delta$, a sufficient condition to ensure the previous inequality is that $2(1-p) \geq p(1-t)\delta$. This is the case if $p \leq \frac{2}{2+\delta(1-t)}$. ■

In the Appendix we discuss the assumption that $\frac{\partial \pi}{\partial c_H} \leq 0$. Regarding the second assumption about p, it is very weak since the threshold is close to 1 for reasonable values of the parameters. The worst case corresponds to $t = 1$. If, for example, we fix $\delta = 0,1$ the condition is $p \leq 0.95$.

In the next section we focus on our narrower definition of efficiency, namely Q_S, taking into account that the government will not implement a policy that harms the objective of equity. We focus on Q_S for two reasons. First, the analysis is simpler than with Q_T. Second, given the current trend in most Western countries towards cutting expenditure in higher education, we are interested in studying when it is the case that the policy that makes equity and efficiency compatible consists of rising c_L and reducing c_H. Thus, given the relationship between the indifference curves of Q_S and the other two concepts, if that policy has a positive effect on equity and Q_S, it will also have a positive effect on Q_T.

3.2 Equity and College Productivity

Since we assume that the government cares about equity and also about the quality of college graduates, there are only two candidates for a policy change: either we raise c_L and we reduce c_H, or we raise c_H and we reduce c_L. Having these two objectives in mind imposes, therefore, a trade-off between basic education and college education. However, which one of the two alternative policies needs to be implemented depends on the particular values of the parameters of the model. In particular, in the next proposition we show that the particular policy to pursue depends crucially on the values of the elasticities of $\widehat{s}(c_L, c_H)$ with respect to c_L and c_H.

Proposition 3 *There is always a policy change that rises the productivity of college graduates Q_S without rising the income threshold $\widehat{y}$. The way of achieving these two objectives at the same time depends on the size of the elasticity of $\widehat{s}(c_L, c_H)$ with respect to c_L. If this elasticity is small in absolute terms, the government should rise c_L and reduce c_H. If the elasticity is large in absolute terms, the government should rise c_H and reduce c_L.*

Proof. To check whether or not these two policies are compatible, we have to compare the slopes of Q_S and $\widehat{y}$ in the space (c_L, c_H). The slope of Q_S is described in Equation (16). Regarding the slope of $\widehat{y}$, we see that they have a slope:

$$\left.\frac{\partial c_H}{\partial c_L}\right|_{\widehat{y}=\overline{\widehat{y}}} = \frac{\frac{\partial \widehat{s}}{\partial c_L}}{\frac{(1-\widehat{s})}{c_H} - \frac{\partial \widehat{s}}{\partial c_H}}, \tag{23}$$

which is clearly negative. There are two possibilities: either the slope in Equation (16) is smaller than the slope in Equation (23) or it is the other way round. In the first case, the only possibility of achieving both objectives is by increasing c_L while reducing c_H. This is the situation of point A in Figure 2a below, where we represent in green all policies satisfying both objectives. In the second case, the way to achieve both objectives is by increasing c_H while reducing c_L. This is point B in the figure. We then see that which one of these two cases prevails depends on the value of the elasticity of $\widehat{s}(c_L, c_H)$ with respect to c_L. From Equations (16) and (23) we check that the first case will arise as long as:

$$\frac{\frac{\partial \widehat{s}}{\partial c_L} - \frac{(2+(1-t)\delta)}{c_H}}{\frac{A+1-\widehat{s}}{c_H} - \frac{\partial \widehat{s}}{\partial c_H}} < \frac{\frac{\partial \widehat{s}}{\partial c_L}}{\frac{(1-\widehat{s})}{c_H} - \frac{\partial \widehat{s}}{\partial c_H}}. \tag{24}$$

This will be the case provided that:

$$(2+(1-t)\delta)c_H \frac{\partial \widehat{s}}{\partial c_H} < A c_H \frac{\partial \widehat{s}}{\partial c_L} + (2+(1-t)\delta)(1-\widehat{s}). \tag{25}$$

Defining the elasticities of $\widehat{s}(c_L, c_H)$ with respect to c_L and c_H as $\varepsilon^s_{c_L} = \frac{\partial \widehat{s}}{\partial c_L}\frac{c_L}{\widehat{s}}$ and $\varepsilon^s_{c_H} = \frac{\partial \widehat{s}}{\partial c_H}\frac{c_H}{\widehat{s}}$, respectively, the expression above can be simplified into:

$$\varepsilon^s_{c_L} > \left(\frac{2+(1-t)\delta}{A\beta}\right)\left(\varepsilon^s_{c_H} - \frac{(1-\widehat{s})}{\widehat{s}}\right). \tag{26}$$

■

Here Figure 2 (Iso-productivity and iso-equity lines)

This proposition shows that the objectives of equity and productivity are always compatible and provides a characterization of the direction of the policy reform. In particular, if Condition (26) holds, the only policy reform that rises the productivity of college graduates without hurting the objective of equity consists of transferring resources from college education to basic education. Alternatively, every time that the government implements the opposite policy and moves resources towards higher education, either equity or college productivity or both are hurt.

Condition (26) is more complicated than it seems, as it depends on $\widehat{s}$ and the elasticities $\varepsilon^s_{c_H}$ and $\varepsilon^s_{c_L}$ which, in turn, depend on the specific values of c_L and c_H. In the next section we present some examples to give some intuition for the result and to highlight the conditions under which Condition (26) may hold.

As both elasticities are negative, Condition (26) will be true whenever $\varepsilon^s_{c_L}$ is close to zero and $\varepsilon^s_{c_H}$ is far from zero. This is always the case when the subsidy is close to one, as long as A is not large. When $s = 1$, the condition is true as long as $A < 2 + 2(1 - t)\delta$. That is, when the subsidy is equal to one, the iso-productivity lines are always steeper than the iso-equity lines (as represented in point A of Figure 2a). By continuity, this will also be the case as long as the subsidy is close to 1, which seems to be the case in most developed countries. As we move into higher values of both c_L and c_H, the equilibrium level of the subsidy gets lower. At the same time, we find that the iso-equity lines become steeper relatively to the iso-productivity lines. This means that, as we move farther away from the origin, eventually Condition (26) will fail. This is represented as point B in Figure 2a.

In addition, Proposition 3 shows that which is the policy reform for a given starting point (c_L, c_H) depends on whether the country is rich or poor. Figure 2b represents the policy reforms for two countries: one rich and one poor. In both cases, the dotted lines represent the iso-productivity curves. The rich country (in black) has mean income $\overline{y}$. The poor country (in red) has mean income $\overline{y}' < \overline{y}$. Thus, for a fixed combination (c_L, c_H), the subsidy in the rich country will be higher than in the poor country and the iso-productivity lines will be steeper than the iso-equity lines. The policy reform will consist of increasing c_L. In the poor country, if the subsidy is low the policy reform will be the opposite one.

Finally, from Proposition 2 we know that any policy that transfers resources from college education to basic education and that respects both equity and college productivity will have a positive effect on Q_T as well. What about when Condition (26) does not hold? We saw that a policy that transfers resources towards higher education improves equity and college productivity. However, this is done at the price of reducing the human capital of those who do not attend college, and this may have at the end a negative effect on Q_T. In fact, the only policy that both respects equity and has a positive effect on Q_T is the one that transfers resources towards basic education.

3.3 An Example: High Income vs Low Income Countries

Here we present a numerical example to illustrate Proposition 3. To do it we look for reasonable values of our parameters. This exercise should not be taken as a full-fledged calibration exercise, since the model is too abstract to be calibrated properly.

We need values for φ, δ, γ, A, Y, together with $F(y)$. Once we have this, for every combination (c_L, c_H) we can compute the equilibrium levels of the subsidy $\widehat{s}(c_L, c_H)$, college attendance and productivity. In Table 3 we present our choice of parameter values. Below we describe briefly our choices.

Table 3: Parameter values

φ	0.005
δ	0.1
γ	0.75
A	1
$\overline{y}$ (high income)	$\$2,418,992$
$\overline{y}$ (low income)	$\$596,009$

We choose $\varphi = 0.005$ because average private expenditure in college education for the OECD represents approximately 0.5% of total family lifetime income. The value that we choose for δ reflects the fact that the working life of an unskilled worker is a 10% longer than that of a skilled worker. Since borrowing constraints are not very important for most OECD countries we think that a high value of γ is appropriate. In particular, we fix $\gamma = 0.75$.

Next we focus on A. The value of A determines the value of the college premium. In particular, the college premium is an increasing function of A. We define the college premium for an individual with ability a as the ratio between net lifetime income attending and not attending college. Since a follows a Uniform distribution on $[0, A]$, the average college premium can be written:

$$\frac{2 + (1-t)\delta + \beta(A - (1-s))}{2(1 + (1-t)\delta)}. \tag{27}$$

To obtain a value for β, s and t we use OECD data from Table 2. Columns 1 and 2 correspond roughly to what we call in the paper c_L and c_H, respectively. In Column 3 (T/B) we compute the ratio β. Taking the mean values of β, s and t for the OECD ($\beta = 0.61, s = 0.782, t = 0.053$), if we choose $A = 1$ we obtain an average college premium of 1.17, which seems reasonable. Moreover, choosing $A = 1$ simplifies our calculations to a great extent.[16]

Regarding income distribution $F(y)$ we find a lot of heterogeneity among the OECD countries. For simplicity we assume that y follows a uniform distribution on $[0, Y]$. We then divide all countries into two groups, high and low income countries, and we present values for the parameters within each group.

Using again the data in Table 2 for c_L, c_H, subsidies for higher education (Column Subs.) and tax rate (Tax) we compute a value of $\widehat{a}$ for each country. Using these values of $\widehat{a}$, given the college attendance levels (C. Att.) we can obtain a value of p. Since $\widehat{y} = \dfrac{(1-\gamma)c_H(1-s)}{\varphi(1-t)}$ and $p = 1 - F(\widehat{y}) = 1 - \frac{\widehat{y}}{Y}$, once we have a value for $\widehat{y}$ and p we can also compute Y. Countries are classified as high income or low income according to their value of Y. In particular, we consider as high income countries

[16] The value of t is chosen such that $t \in (t_0, t_1)$ and $\widehat{s} \in (0, 1)$. This implies that the values of both t and $\widehat{s}$ might not coincide with the mean values for the OECD since that would imply that their budgets are in equilibrium which is not generally the case. For example, for the high income countries we choose $t = 0.09$.

those with $Y > \$1,124,758.81$ which is the median value of Y. Thus, the median values of Y (shown in Table 3) c_L and c_H will be $Y = \$2,418,992.52, c_L = \$86,295$ and $c_H = \$58,496$ for high income countries and $Y = \$596,009$, $c_L = \$67,567$ and $c_H = \$32,770$ for low income countries. Given that y is uniform on $[0, Y]$ we obtain the values for mean income shown in Table 3.[17]

Once we have chosen appropriate values for all our parameters, we can compute for the two groups of countries the equilibrium values of $\widehat{s}$, p and π corresponding to any given combination (c_L, c_H). Next, we plug these equilibrium values into Equation (26) to see for which values of (c_L, c_H) that condition holds. What we observe is that the policy space is split into two regions as represented in Figure 3. For a given value of c_L, there is a threshold value of c_H called $\theta_1(c_L)$, such that Condition (26) holds for those values of c_H *below* the threshold $\theta_1(c_L)$. This means that for those combinations (c_L, c_H) below $\theta_1(c_L)$, the policy reform to implement consists of rising c_L and reducing c_H. Above the line $\theta_1(c_L)$, the policy reform is just the opposite. This result is in line with the interpretation we gave to Condition (26) immediately after Proposition 2. In fact, we could think of the line $\theta_1(c_L)$ as a way of separating those combinations (c_L, c_H) where the subsidy is large enough and the condition holds (those below the line) from those where the subsidy is low enough and the condition does not hold (those above the line).

We also find that the threshold $\theta_1(c_L)$ depends on how rich a country is. In particular, as we show in Figure 3, the threshold for high income countries (in black), is above the threshold for low income countries (in red).[18] Moreover, if we focus on the group of high income countries, we find that the region where Condition (26)

[17] The values that we use for mean income per capita can be seen as very high. However, recall that they correspond to *lifetime* income, and not to annual income.

[18] For both groups, we also obtain that the larger is γ, the larger is the region in the space (c_L, c_H) where Condition (26) holds. We find also a similar effect if we allow for decreasing returns to public expenditure in education (see footnote 9). The lower is α, the larger is the region in the space (c_L, c_H) where Condition (26) holds.

fails (those combinations above $\theta_1(c_L)$) corresponds to extremely low values of college attendance. This allows us to conclude that the empirically relevant region for high income countries corresponds to the situation where Condition (26) holds. However, this is not the case for low income countries, which confirms the result illustrated in Figure 2b. That is, as we move farther away from the origin, Condition (26) will eventually fail. Once we reach low enough values of the subsidy, the iso-equity lines become steeper relatively to the iso-productivity lines. Thus, the policy reform for low income countries will eventually consist of increasing c_H. This policy has the effect of increasing the college subsidy so that a higher proportion of poor individuals can afford college, which in turn implies an increase in college attendance. Although this reform reduces the quality of basic education and the ability threshold for college students, due to the increase in c_H, the final quality of college students increases.

Figure 3: Illustration of Proposition 3.

To illustrate further our results in Propositions 2 and 3, we present in Table 4 an example of the effect of two different policy changes on the different objectives of the government. Since we focus on the case of high income countries, we use the numbers from Table 2 to fix an initial situation with $c_L = \$86,000$ and $c_H = \$59,000$, respectively. The corresponding values of $Q_S(c_L, c_H)$, $\widehat{y}(c_L, c_H)$, $\pi(c_L, c_H)$ and Q_T are also computed, together with the values of $\widehat{s}$ and p.

Table 4: Budget division and public intervention[1]

β	$.68 = \frac{59}{86}$	$.59 = \frac{53}{90}$	$.79 = \frac{66}{83}$
$\widehat{s}$	.801	.783	.807
p	.734	.739	.710
$\widehat{y}$	642,482	631,232	699,335
π	.4914	.4646	.4924
Q_S	125,260	126,339	126,140
Q_T	105,294	106,885	104,244

[1] c_L and c_H are in thousand dollars.

We consider two alternative policies. First we transfer resources from higher education to basic education. Second, we transfer resources from basic to higher education.

In the second column we consider a 10.1% reduction in c_H and a 4.6% increase in c_L. New values are $c_L = \$90,000$ and $c_H = \$53,000$. Once all variables reach a new equilibrium, the subsidy becomes lower. However, we observe that the proportion of individuals who can afford college increases. This policy has also a positive effect on equity and on both measures of productivity, at the cost of a negative effect on college attendance.

In the third column we consider a 3.4% reduction in c_L and a 11.8% increase in c_H. New values are $c_L = \$83,000$ and $c_H = \$66,000$. This policy has a positive impact on the subsidy. Regarding the different objectives of the government, we find a negative effect on equity, since the threshold for income rises, which reduces p. There is also a positive effect on the productivity of college students and on total college attendance. However, the effect on the average level of productivity across the population Q_T is negative. As predicted by Proposition 3, moving resources towards higher education will have a negative effect either on equity or on productivity. On top of this, as seen in the reduction on Q_T, the increase in Q_S (the average productivity of college graduates) is not enough to compensate the reduction in the productivity of unskilled workers (c_L).

4 Conclusion and Discussion

The main result of this paper is that there is always a policy reform that increases the productivity of college graduates without excluding the talented poor from college. For most rich countries, this policy consists of transferring resources from college to basic education. In addition we find that this policy has always a positive effect on the average level of human capital across the population.

Throughout the paper we have assumed that capital markets are imperfect ($\gamma < 1$). Here we want to comment briefly on the effect of removing this assumption. If $\gamma = 1$, then equity is no longer a concern for the government since in that case all individuals can attend college, irrespective of their family income. The trade-off between equity and efficiency disappears. One interesting implication is that, in this case, the government could fix a large value of both c_L and c_H, such that $s = 0$ in order to achieve efficiency. That is, if capital markets work perfectly college education should be privately financed.

There are many possible extensions of this work. One possibility would be to study other objectives to represent efficiency. For example, we could assume that the government tries to increase average lifetime income within a cohort which implies rising average consumption of the cohort in period 2.[19] Thus, the main difference with average human capital above is that now we are subtracting the monetary cost of higher education paid by students. We have some partial results regarding the effects of policy reforms on this objective. In particular, we find that as long as the indirect effect of changes in both c_L and c_H through s and π is not very large, a policy consisting of transferring resources from college to basic education that has a positive effect on Q_S will also have a positive effect on the average lifetime income within a cohort.

Another extension would be to relax the assumption that the two characteristics that define individuals are independent. It is generally assumed that there is correlation between parents' ability and the ability of the child when, for example, IQ is taken as a measure of ability. As parents' income and parents' ability are also correlated, the two characteristics in our model will be positively correlated. However, if the two variables are correlated, the model becomes more complicated as the two

[19] This objective has been analyzed by Lloyd-Ellis (2000) and Su (2004). Blankeneau et al (2002) analyze the lifetime income (or welfare levels) of each group of workers in a separate way.

terms that define college attendance now cannot be separated. The outcome is that the ability threshold will be lower for rich individuals than for poor individuals. One possibility could be to rely on numerical simulations to see whether the results of the paper change.

We believe that our results are relevant for several recent debates in the literature on the economics of education. There is increasing evidence that shows the early emergence and persistence of gaps in cognitive and non-cognitive skills (see among others, Carneiro and Heckman (2003)). This issue is of special concern as, according to recent evidence, family environments have deteriorated (Heckman and Masterov (2004)).[20] Studies that highlight the importance of increasing expenditure in early childhood care in achieving both equity and efficiency provide an interesting illustration since, obviously, at the current level of resources, the rise of expenditure at that level should be done at the expense of reducing expenditure in later educational levels (see Heckman (2006)).

[20] In the US, the percentage of children born into, or living in, "nontraditional" families has increased greatly in the last 30 years (about 25% of children are now born into single parent homes now). "Nontraditional" families include not only single-parent families but also families where the parents are not married. The evidence found by Heckman and Masterov (2004) suggests that children raised in these types of families fare worse in many aspects of social and economic life.

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5 Appendix

Sign of $\frac{\partial \pi}{\partial c_H}$:

We expand $\frac{\partial \pi}{\partial c_H}$ to get:

$$\begin{aligned} \frac{\partial \pi}{\partial c_H} &= \frac{1}{A}\left\{(A-\widehat{a})\frac{\partial p}{\partial c_H}+\frac{p(1-t)\delta c_L}{(c_H)^2}\right\} = \\ &= \frac{1}{A}\left\{-(A-\widehat{a})f(\widehat{y})\frac{(1-\gamma)(1-s)}{\varphi(1-t)}+\frac{p(1-t)\delta c_L}{(c_H)^2}\right\}. \end{aligned} \tag{28}$$

If $s=1$ or $s=s_0$, the negative term is zero and $\frac{\partial \pi}{\partial c_H}>0$. By continuity, this will also be the case when s is close to either s_0 or 1. Apart from these extreme cases, in general $\frac{\partial \pi}{\partial c_H}$ will be negative. The reason is that the positive term is of a much smaller absolute size since it is divided by the large term $(c_H)^2$, while the negative term is divided by φ which is small.

Discussion on Assumption 1:

Sign of $\frac{\partial \widehat{s}}{\partial c_L}$:

We have to study the sign of the term in the numerator of $\frac{\partial \widehat{s}}{\partial c_L}$. Substituting the value of $\frac{\partial \pi}{\partial c_L}$ we get:

$$\text{Sign}\left[\frac{\partial \widehat{s}}{\partial c_L}\right] = \text{Sign}\left[[1-t\delta(1-\pi)]-[t\delta c_L+\widehat{s}c_H]\frac{p(1-t)\delta}{Ac_H}\right]. \tag{29}$$

This is positive if:

$$A\beta\,[1-t\delta(1-\pi)] \geq [t\delta+\widehat{s}\beta]\,p(1-t)\delta, \tag{30}$$

where $\beta = c_H/c_L$. This can be simplified to:

$$\beta \geq \frac{p(1-t)t\delta^2}{A\,[1-t\delta(1-\pi)]-p(1-t)\delta\widehat{s}}. \tag{31}$$

The term on the right is rather small. As an illustration, take $A=1, t=0.05, \delta=0.1, p=0.75, \pi=0.6, \widehat{s}=0.9$. The condition is $\beta>0.00038$.

Sign of $\frac{\partial \widehat{s}}{\partial c_H}$:

When $\widehat{s}=1$ or $\widehat{s}$ very low we know that $\frac{\partial\pi}{\partial c_H}$ is positive, and then the term in the numerator of $\frac{\partial\widehat{s}}{\partial c_H}$ is positive as well. We focus on intermediate values of $\widehat{s}$ for which $\frac{\partial\pi}{\partial c_H}$ is negative. We have defined the elasticity of p with respect to c_H as $\varepsilon^p_{c_H}=\frac{\partial p}{\partial c_H}\frac{c_H}{p}$. The numerator of $\frac{\partial\widehat{s}}{\partial c_H}$ is positive if:

$$\varepsilon^p_{c_H} > \frac{-\widehat{s}\beta}{\widehat{s}\beta+t\delta} - \frac{(1-t)\delta}{(A-1+\widehat{s})\beta-(1-t)\delta}. \tag{32}$$

The first term on the right is approximately -1, while the second term is rather small. So this condition says that the elasticity $\varepsilon^p_{c_H}$ has to be above a threshold that is approximately -1.

Slope of the iso-subsidies:

Defining the elasticities of $\pi(c_H,c_L)$ with respect to c_L and c_H as $\varepsilon^\pi_{c_L}=\frac{\partial\pi}{\partial c_L}\frac{c_L}{\pi}$ and $\varepsilon^\pi_{c_H}=\frac{\partial\pi}{\partial c_H}\frac{c_H}{\pi}$, respectively, the slope of the iso-subsidy can be rewritten as:

$$\frac{\partial c_H}{\partial c_L}\mid_{s=\bar{\widehat{s}}}= -\frac{[1-t\delta(1-\pi)]+[t\delta\pi+\widehat{s}\pi\beta]\,\varepsilon^\pi_{c_L}}{\widehat{s}\pi+\frac{1}{\beta}[t\delta\pi+\widehat{s}\pi\beta]\,\varepsilon^\pi_{c_H}}. \tag{33}$$

First, suppose that both $\varepsilon^\pi_{c_H}$ and $\varepsilon^\pi_{c_L}$ are zero. Then the slope is:

$$\frac{\partial c_H}{\partial c_L}\mid_{s=\bar{\widehat{s}}}= -\frac{[1-t\delta(1-\pi)]}{\widehat{s}\pi} = -\frac{(1-t\delta)}{\widehat{s}\pi} - \frac{t\delta}{\widehat{s}}. \tag{34}$$

The lower is $\widehat{s}$ the higher is the absolute value of this expression. That is, the lower is $\widehat{s}$, the steeper are the iso-subsidies. Once we take into account the effect of both $\varepsilon^\pi_{c_L}$ and $\varepsilon^\pi_{c_H}$ the result will hold as long as they are of small size, as we have discussed above.

Table 1. Expenditure on education by level, 2004[a]

	Expenditure per student[b]			Change 1995-2004[c]		
Country	Basic	Tertiary	T/B	Basic	Tertiary	T/B
Australia	6,911	14,036	2.03	138	101	0.73
Austria	8,938	13,959	1.56		122	
Belgium	7,596	11,842	1.62			
Czech Rep.	5,490	6,752	1.68	124	69	0.56
Denmark	8,492	15,225	1.79	121	123	1.02
Finland	6,660	12,505	1.88	122	110	0.90
France	7,262	10,668	1.47			
Germany	6,983	12,255	1.75	105	107	1.02
Greece	4,931	5,593	1.13	192	151	0.79
Hungary	3,833	7,095	1.85	157	73	0.47
Iceland	8,138	8,881	1.09			
Ireland	6,034	10,211	1.69	181	126	0.70
Italy	7,741	7,723	1.00	105	130	1.23
Japan	7,105	12,193	1.72	127	101	0.79
Korea	5,550	7,068	1.27			
Mexico	1,789	5,778	3.23	130	110	0.85
Netherlands	6,914	13,846	2.00	136	101	0.75
New Zealand	5,815	8,866	1.52			
Norway	9,772	14,997	1.53	109	103	0.95
Poland	2,998	4,412	1.47	183	90	0.49
Portugal	5,400	7,741	1.43	154	98	0.64
Slovak Rep.	2,562	6,535	2.55	155	111	0.71
Spain	5,892	9,378	1.59	136	167	1.24
Sweden	7,744	16,218	2.09	117	99	0.84
Switzerland	10,378	21,966	2.12	105	134	1.28
UK	6,656	11,484	1.73	120	93	0.78
USA	9,368	22,476	2.40	130	132	1.01
OECD average	**6,608**	**11,100**	**1.68**	**138**	**109**	**0.79**

[a] Source: Education at a Glance 2007, Tables B1.1a and B1.5 and authors' calculations.

[b] Annual expenditure per student in US dollars, using PPPs.

[c] Index of change in annual expenditure per student, setting expenditure in 1995 at 100.

Table 2: Cumulative public expenditure on education, 2004[a]

	Educational Levels[b]					
Country	Basic	Tertiary	T/B	Subs.[c]	C. Att.[d]	Tax[e]
Austria	106,396	73,983	0.70	93.7	37	0.048
Belgium	86,320	35,406	0.41	90.4	34	0.060
Denmark	109,777	56,332	0.51	96.7	55	0.084
Finland	79,900	60,659	0.76	96.3	73	0.064
France[1]	86,406	42,884	0.50	83.9	39	0.058
Germany	87,659	65,732	0.75	86.4	37	0.046
Greece	58,850	29,361	0.50	97.9	33	0.033
Hungary	47,469	36,353	0.77	79.0	68	0.054
Iceland	113,213	32,770	0.29	90.9	79	0.076
Ireland	82,479	33,083	0.40	82.6	44	0.047
Italy	103,871	55,751	0.54	69.4	55	0.046
Japan	84,930	49,624	0.58	41.2	43	0.036
Korea	67,567	24,242	0.36	21.0	48	0.046
Mexico	22,662	19,761	0.87	68.9	29	0.054
Netherlands	74,339	72,555	0.98	77.6	56	0.052
N. Zealand	74,745	27,042	0.36	60.8	89	0.065
Slovak Rep.	32,856	25,484	0.78	81.3	47	0.042
Spain	69,993	43,699	0.62	75.9	44	0.043
Sweden	92,979	75,901	0.82	88.4	79	0.074
Turkey	15,396	12,474	0.81	90.0	26	0.040
UK	81,732	49,872	0.61	69.6	52	0.053
OECD average	**75,216**	**43,951**	**0.61**	**78.2**	**32**	**0.053**

[a]Source: Education at a Glance 2007, Table B1.3b and authors' calculations.

[b]Cumulative expenditure per student in 2004. In equivalent US dollars converted using PPPS for GDP.

[c]Proportion of public expenditure in tertiary education in 2004. Source: Education at a Glance 2007, Table B3.2b.

[d]Entry rates into tertiary-type A programmes for 2004. Source: Education at a Glance 2006, Table C2.1.

[e]Public expenditure on education as a percentage of GDP. Source: Education at a Glance 2007, Table B4.1.

[1]Year of reference for C. Att.: 2003.

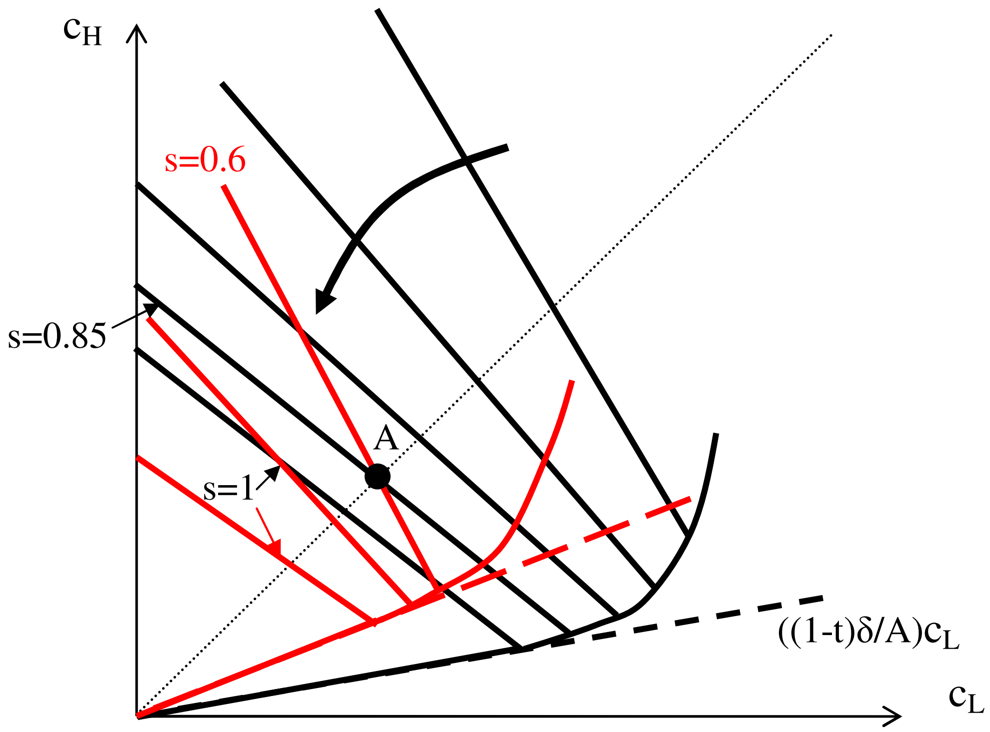

FIGURE 1: THE POLICY SPACE

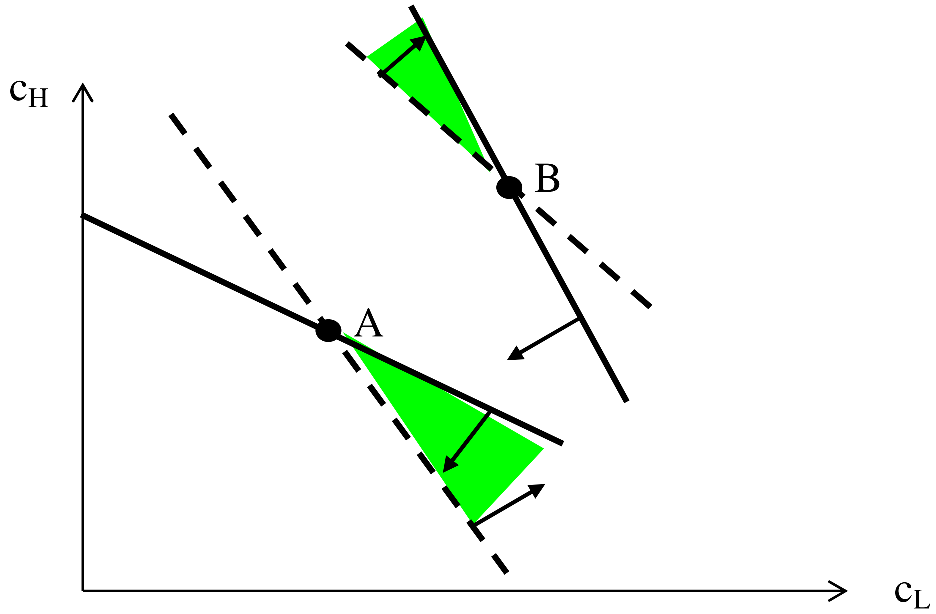

FIGURE 2a: THE OPTIMAL POLICY IN COUNTRY DEPENDS ON THE VALUE OF THE SUBSIDY

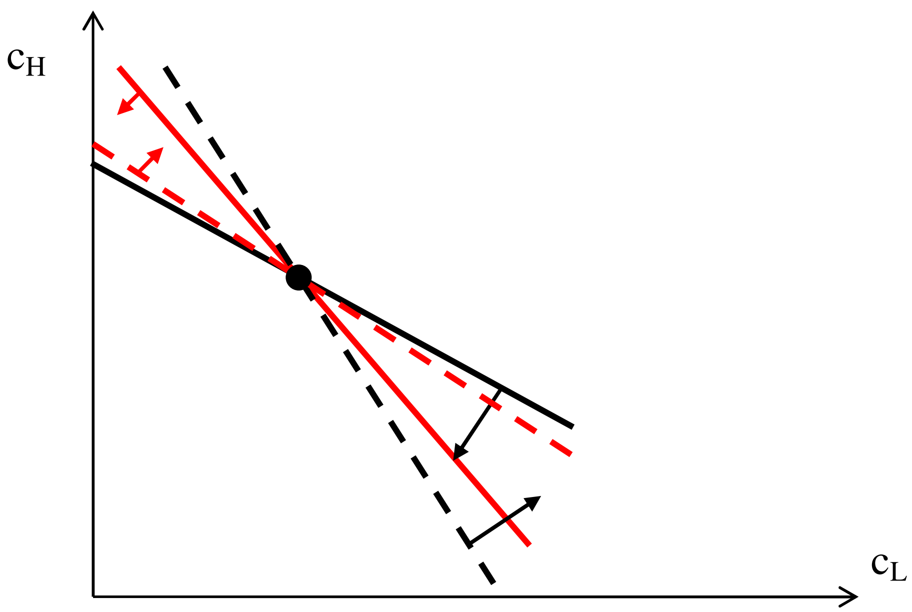

FIGURE 2b: THE OPTIMAL POLICY IN A HIGH AND A LOW INCOME COUNTRY

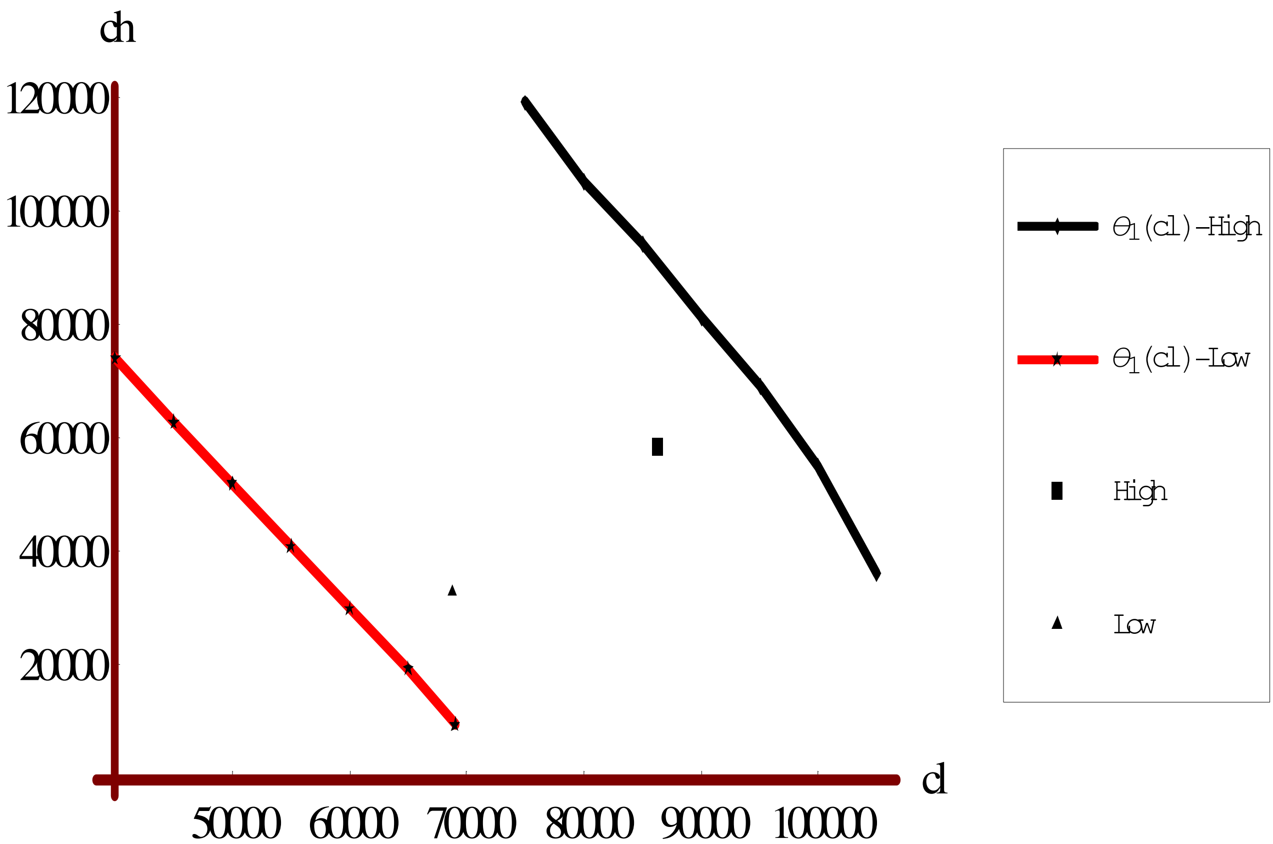

FIGURE 3: ILLUSTRATION OF PROPOSITION 2