Working papers series

WP ECON 09.04

Tracking can be more equitable than mixing

Marisa Hidalgo Hidalgo (U. Pablo de Olavide)

JEL Classification numbers: D63, I28, J24

Keywords: Peer Effects, Tracking, Mixing, Equality of Opportunity

Department of Economics

Tracking can be more Equitable than Mixing *

Marisa Hidalgo-Hidalgo†

Universidad Pablo de Olavide, Seville

July 31, 2012

Abstract

Parents and policy makers often wonder whether and how the choice of a tracked or a mixed educational system affects opportunity equalization. We answer this question by analyzing the impact of peers on future educational results. We define an equal-opportunity policy as one that maximizes the average lifetime income of the worst-off type, poor students. We find that, provided that tracking maximizes average education at the compulsory level, it will also maximize average lifetime income if the opportunity cost of college attendance is sufficiently high.

Keywords: Peer Effects, Tracking, Mixing, Equality of Opportunity

JEL Classification: D63, I28, J24.

***Address for correspondence:** Marisa Hidalgo Hidalgo, Departamento de Economía (Area de Análisis Económico), Universidad Pablo de Olavide, Ctra. Utrera, Km.1, E-41013, Sevilla, Spain. E-mail: mhidalgo@upo.es

†I am most grateful to Iñigo Iturbe-Ormaetxe for his invaluable help and ongoing support. I thank the comments on earlier versions of the paper to Gianni de Fraja, Elena del Rey, Moshe Justman, Francois Maniquet, Juan D. Moreno-Ternero, Ignacio Ortuño-Ortín and John Roemer. The comments of two anonymous referees are gratefully acknowledged. I thank seminar participants at Aarhus, Alicante, Málaga, Seville, XIX Annual ESPE Conference, 11th International Public Economic Theory Conference, Louis-André Gérard-Varet 2009 Conference and 10th International Meeting of the Society for Social Choice and Welfare. Finally, I would like to thank CORE and the University of Leicester, where part of this research was carried out, for their hospitality. Financial support from the Spanish Ministry of Education and Science (SEJ 2007-67734), Junta de Andalucía (SEJ-2905, SEJ-426) and Instituto Valenciano de Investigaciones Económicas is gratefully acknowledged.

1 Introduction

Education is one of the most important means by which governments attempt to equalize opportunities for economic success among citizens. Parents and policy makers often wonder whether and how the choice of a tracked or a mixed educational system affects opportunity equalization. Under a tracking system, schools are hierarchically organized to accommodate a range of student performance levels, and students are placed in the school that best suits their ability level. By contrast, mixing works by grouping students of differing ability levels within the same school. When comparing these systems, it is critically important to recognize the existence of peer interactions and account for their impacts on students' outcomes.[1]

It is commonly accepted that equality of opportunity is best achieved in a mixing educational system (see Hanushek and Woessmann (2006) and Brunello and Checchi (2007)).[2] However, we argue here that this is not always the case. We analyze the conditions under which tracking best guarantees opportunity equalization for lifetime income. To do so, we study the distributions of lifetime income across socioeconomic types produced by these two educational systems and evaluate them according to the opportunity-egalitarian criterion. This implies to recommend the system that maximizes the average lifetime income of the worst-off type of individual in the society.

To address these issues, a model with two educational stages is introduced: compulsory and college education. Students differ in parental backgrounds as well as in school achievement levels. Some positive dependence between these two defining variables is assumed. After attending compulsory education, each individual chooses either attending college or entering the labour market. The peer group characteristics at compulsory education level indirectly affect an individual's lifetime income.

Several findings result from our analysis. The first one is that the opportunity egalitarian policy maker implements tracking (mixing) if he is concerned only with students from poor families with high (low) achievement levels upon entering high school. Furthermore, by proceeding in this way, the selected policy coincides with the one that maximizes college attendance among poor students. Second, we define an equal-opportunity policy as one that maximizes the

[1] There is a large empirical literature on peer effects, see Sacerdote (2011).

[2] Another recent example is Pekkarinen et al (2009) where, albeit with a different approach, it is found that mixing enhances intergenerational earnings mobility.

average lifetime income of the worst-off type, poor students. We find that tracking better guarantees equality of opportunity in most cases. In particular, we find that, provided that tracking maximizes average education at the compulsory level (for instance, because peer effects and individual achievement are close complements), it will also maximize average lifetime income if the opportunity cost of college attendance is sufficiently high.[3] In this case, tracking not only provides students with better preparation on average, but also guarantees that the number of students who attend college (i.e., those with high school achievement levels) is maximized. In this context most low school achievers are excluded from college under either system. Thus, intervention should target those with high achievement levels as they are the only ones who will potentially attend college. Choosing tracking is the right way to do so as the peer effect is stronger for these students in a tracking system than in a mixing one. In addition, these students experience an increase in lifetime income after college attendance that is larger than the increase that would have taken place under mixing. To summarize, tracking guarantees equality of opportunity better than mixing in countries with a high opportunity cost of college attendance. This is the case, for example, of countries with good labour market conditions for young workers.

This paper is based on the literature that compares the performance of tracking versus mixing education systems from a theoretical perspective (see Betts (2011)). The most closely related works are Takii and Tanaka (2009) and Hidalgo-Hidalgo (2011). The former examines the effect of these two alternative grouping systems on aggregate output. They find that whether ability tracking increases aggregate output depends on the technology of human capital accumulation. Similar results are found by Hidalgo-Hidalgo (2011). In particular, she finds that the higher the complementarity between peer effects and individual school achievement, the larger the difference between average education in the two systems. Several empirical papers cited above explore equality of opportunity for tracked versus mixed education systems and find that mixing better guarantees it.[4] However, these studies suffer from two main limitations, which may explain the difference between their conclusions and our own. They either look

[3] If peer effects matter more for high-ability students than for low-ability students, then average education at the compulsory level will be maximized in a tracking system as it is the educational system where high-ability students enjoy a stronger peer effect.

[4] Hanushek and Woessmann (2006) find that based on international comparisons of early outcomes early tracking increases educational inequality. Brunello and Checchi (2007), who focus on later outcomes such as employability and earnings, find that tracking reinforces family background effects on labour market outcomes.

just at compulsory-level outcomes, which impede them from observing the long-term effects of either system or, analyze later outcomes, but fail to consider the distributional impact of family background on educational outcomes and instead focus solely on mean impacts. Our study goes a step further than previous work by focusing not only on mean impacts but also on the distributional outcomes of different grouping policies and by determining how those policies may alter such outcomes by hindering or enhancing the college attendance possibilities for specific groups of students.

This paper complements the existing theoretical literature on tracking by explicitly discussing the notion of equality of opportunity. By proceeding in this way, we find unexpected arguments for a tracking educational policy. In addition, this paper complements the literature by considering the effects of tracking versus mixing (each of which entails different peer interactions) on college choice and lifetime income acquisition. This effect has been neglected in the literature, although some empirical studies have shown that the quality of students' peers at school can influence their college attendance and performance rates (see Betts and Morell (1999) and Hahn et al. (2008)).

The rest of the paper is organized as follows. Section 2 describes the model. Section 3 derives the distributions of lifetime income produced by the two educational systems at the compulsory level. Section 4 evaluates both systems according to the opportunity-egalitarian criterion, and Section 5 concludes.

2 Model

2.1 Individuals

We consider an economy in which individuals live for two periods, the educational and the labour one, which last one unit of time each. Individuals differ in two aspects: their family background and their school achievement, x, where $x \in [0, 1]$. We interpret x here as student's achievement upon entering high school.[5] To make the model tractable, we assume that family background takes only two values; that is, individuals can have either poor or rich parents with

[5] In the related literature x denotes "innate ability". As the focus here is on grouping policies, we think it is more appropriate to interpret x as the educational accomplishment at the age of first selection into different groups, which is around 15 years-old in most OECD (see Education at a Glance 2008).

probabilities $1-\lambda$ and λ, respectively.[6] Population size is 1. We denote by $G_b(x)$ the cumulative distribution function (CDF) of x, conditional on having family background b, where $b = p, r$ for poor and rich parents, respectively. To capture the possibility that some level of positive dependence exists between parental background and school achievement, we assume that the CDF of x for rich parents dominates that for poor ones.[7] Finally, we assume that the support of x for both poor and rich individuals is full. Thus, the aggregate CDF of school achievement, denoted by $G(x)$, with a domain $x \in [0, 1]$, can be expressed as:

$$G(x) = (1 - \lambda)G_p(x) + \lambda G_r(x). \tag{1}$$

In the first period of their lives, the educational one, individuals acquire compulsory education during a fraction $1 - \alpha$ of this period. During the remaining fraction α, some individuals attend college, and some others work as unskilled workers. This parameter α also represents the opportunity cost of investment in human capital, that is, the fraction of earnings that would have been received in the absence of the investment. During the second period of their lives, the labour one, all individuals have one unit of time and all of them work. Those who attended college are now skilled workers, whereas those who did not attend college remain as unskilled workers. Figure 1 below illustrates the timing of the model. Individuals care for their lifetime income, which, as we shall see below, depends on their education acquired.

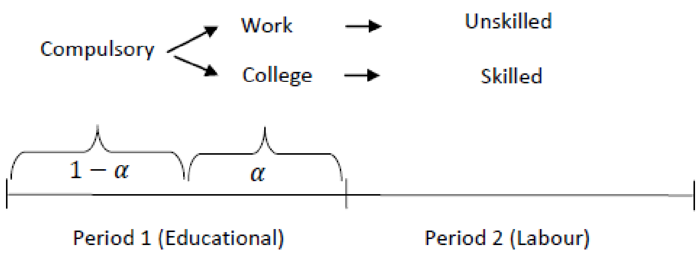

Figure 1: The timing of the model

We assume that the cost of education is paid in full by the government. However, in this paper we do not focus on financial reallocations of educational resources. Instead we focus on a different set of policy instruments, the grouping criteria, and investigate how the redistribution of non-monetary inputs like peer group quality can reduce the magnitude of financial

[6] Alternatively, we could interpret the two parent types as white or black, natives or immigrants, etc.
[7] That is, $G_r(x) \leq G_p(x)$ for any $x \in [0, 1]$, and $G_r(x) < G_p(x)$ for some $x \in [0, 1]$.

redistribution needed at later stages. In Section 5 we discuss the possibility of introducing fees.

2.2 Education Production

At the compulsory high school level, students are separated into different groups, according to their school achievement levels. For the sake of simplicity, we consider only two groups here. The production of education at this level depends on two factors: the individual's school achievement, x, and a "peer group" effect that depends on the characteristics of the group in which the student is placed. These characteristics are summarized by the mean achievement of the group j, or the "peer" effect, denoted by $\overline{x}^j$, which depends on the grouping policy selected, as we will see next.[8] An individual with school achievement x will have an education e upon graduation from compulsory high school:

$$e = \Phi(x, \overline{x}^j), \tag{2}$$

where $\Phi(x, \overline{x}^j)$ is a twice differentiable, monotonically increasing and concave function in each argument, and thus it has an inverse that we denote by $\phi = \Phi^{-1}(e, \overline{x}^j)$. Finally, note that $e \in [\Phi(0, \overline{x}^j), \Phi(1, \overline{x}^j)]$, where we assume that $\Phi(0, \overline{x}^j) > 0$.

Consider now the college decision of the individual. If she attends college, and thus becomes a skilled worker, her lifetime income y will be equal to $\Psi_c(e)$, where Ψ_c is a monotonically increasing and concave function in education acquired at compulsory school. If an individual does not attend college, she works as unskilled worker during a fraction α of the first period and throughout the second period. To simplify, we assume that the unskilled wage is linear with e and thus, her lifetime income y is $\Psi_u(e, \alpha) = e(1 + \alpha)$. The main results in the paper will not qualitatively change had we assume that $\Psi_u(e, \alpha)$ is concave with e. We assume that the marginal returns to compulsory education are at least as high for college graduates or skilled workers than for unskilled ones, i.e., $\partial\Psi_c(e)/\partial e \geq \partial\Psi_u(e, \alpha)/\partial e$. This assumption captures the well-known fact that early education fosters further learning which in turn yields higher earnings.[9] Therefore, an individual will choose to attend college if $\Psi_c(e) \geq \Psi_u(e, \alpha)$. An equilibrium

[8]There is an intense debate on the influence of peers on individual educational attainment. However, this assumption is commonly accepted in the literature. See, among others, Bishop (2006) and Epple, Newlon and Romano (2002), who also assume that peers affect an individual through the mean of their characteristics.

[9]See, for example, Heckman (2000) where this idea is extensively developed. Moreover, it follows directly from Lucas (1988) that existing human capital improves learning efficiency in subsequent education.

is characterized by a threshold level of education $\widehat{e}(\alpha)$ that satisfies the previous equation with equality, that is, the minimum level of education that students must acquire by the end of their compulsory education if they are to attend college. That is, we are implicitly assuming that α is such that $\widehat{e}$ is interior. This implies that at least some individuals attend college but not all of them, which seems reasonable.[10] From the properties of $\Psi_c(e)$ and $\Psi_u(e,\alpha)$ we have that a rise in α increases $\widehat{e}$, meaning that a smaller proportion of students will attend college.[11]. Finally, we denote by $\widehat{y}$ the lifetime income acquired by the individual who is indifferent between college attendance or not, that is, $\widehat{y} = \Psi_c(\widehat{e}) = \Psi_u(\widehat{e},\alpha)$. Thus, from Equation (2), individual's lifetime income is monotonically increasing with school achievement and peer group characteristics. In particular, the peer group characteristics at compulsory level condition her lifetime income in two ways. First, directly as one's compulsory school peers affect the amount of education that one acquires at that level. And second, indirectly because such education also determines the individual lifetime income and, thus, can influence the student's decision regarding college attendance.[12]

3 The distributions of lifetime income under the two educational systems

This section describes the two contrasting educational systems, mixing and tracking, and analyzes the distributions of lifetime income produced by these two systems.

3.1 Mixing

In a mixing system, the school achievement distribution is the same in both classrooms. The average school achievement within each classroom, denoted by $\overline{x}^m$, coincides with the average school achievement in the population. Individual lifetime income, y will lie in the support $[y_m, y^m]$, where y_m and y^m denote the lifetime income y acquired in a mixing system by the "worst" (unskilled and lowest school achiever) and the "best" (skilled and highest

[10] In addition we assume that $\Psi_u(\Phi(0,\overline{x}^j),\alpha) > \Psi_c(\Phi(0,\overline{x}^j)) > 0$.

[11] A rise in α can be interpreted either as the result of an increase in the difficulty of college studies or as an increase in the length of time spent at college.

[12] Betts and Morell (1999) find a direct link between high-school peer group quality and college grade point average. Hahn et al. (2008) also find that peers in high school affect the student's decision regarding college attendance.

school achiever) individuals in the population, respectively, $y_m = \Psi_u(\Phi(0, \overline{x}^m), \alpha) < \widehat{y}$ and $y^m = \Psi_c(\Phi(1, \overline{x}^m)) > \widehat{y}$. Therefore, the CDF of y in a mixing system, conditional on parental background b, denoted $F_b^M(y)$, and with domain in all $y \in [y_m, y^m]$ is as follows:

$$F_b^M(y) = \begin{cases} F_{b,u}^m(y) & if \quad y_m \le y \le \widehat{y} \\ F_{b,c}^m(y) & if \quad \widehat{y} \le y \le y^m, \end{cases} \tag{3}$$

where $F_{b,w}^m(y) = G_b(\phi(\Psi_w^{-1}(y), \overline{x}^m))$ for $b = p, r$ and Ψ_w^{-1} denotes the inverse of the lifetime income function for unskilled workers (if $w = u$, where $\Psi_u^{-1}(y)$ stands for $\Psi_u^{-1}(y, \alpha)$ henceforth) and skilled or college graduated workers (if $w = c$).

3.2 Tracking

Tracking students implies grouping them on the basis of school achievement. To simplify, we permit only two tracks.[13] We denote by δ and $(1-\delta)$ the proportions of students in the low and high tracks, respectively.[14] In addition, we denote by t the threshold level of school achievement used for grouping students into one track or the other. Thus, a student is assigned to the high (low) track when her school achievement x is above (below) t. From (1), the threshold level t is implicitly defined as follows:

$$(1-\lambda)G_p(t) + \lambda G_r(t) = \delta. \tag{4}$$

We use $\overline{x}^l(\lambda, \delta)$ and $\overline{x}^h(\lambda, \delta)$ to denote the average school achievement in the low and high tracks, respectively. In a tracking system, and for those students who joined the low track, y will lie in the support $[y_l, y^l]$, where y_l denotes the lifetime income y acquired in a tracking system by the "worst" (unskilled and lowest school achiever) individual in the low track, that is, $y_l = \Psi_u(\Phi(0, \overline{x}^l), \alpha)$. We denote by y^l the lifetime income y acquired in a tracking system by the "best" (highest school achiever) individual in the low track, that is, the student with school achievement $x = t$. Note that her final lifetime income depends on whether or not she

[13] See Epple, Newlon and Romano (2002) who also allow for two tracks.

[14] We assume that $\delta \in (0, 1)$. That is, there is some grouping policy in place that separates students according to their previous achievement. If $\delta = 0$ or $\delta = 1$, then a tracking system is equivalent to a mixing one, and the comparison between the two educational systems is meaningless. Thus, the government's choice between tracking and mixing can alternatively be interpreted as the choice between any $\delta \in (0, 1)$ on one hand and $\delta = 0$ or $\delta = 1$ on the other hand.

attends college, that is, on whether her education level acquired at compulsory level is above or below the minimum required to attend college, $\widehat{e}$. Let $e^l = \Phi(t, \overline{x}^l)$ denote the education acquired at the compulsory level by the best student in the low track. Thus, if this student does not attend college (i.e., $e^l < \widehat{e}$), then $y^l = \Psi_u(e^l, \alpha) < \widehat{y}$, whereas if she attends college (i.e., $e^l > \widehat{e}$), then $y^l = \Psi_c(e^l) > \widehat{y}$. Likewise, for those individuals who joined the high track, y lies in the support $[y_h, y^h]$, where y^h denotes the lifetime income y acquired in a tracking system by the "best" (skilled and highest school achiever) individual in the high track, that is, $y^h = \Psi_c(\Phi(1, \overline{x}^h))$. We denote by y_h the lifetime income y acquired in a tracking system by the "worst" (lowest school achiever) individual in the high track, that is, the student with school achievement $x = t$. Again, her final lifetime income depends on whether she attends college or not. We denote by $e_h = \Phi(t, \overline{x}^h)$ the education acquired at the compulsory level by the worst student in the high track. Thus, if she does not attend college (i.e., $e_h < \widehat{e}$), then $y_h = \Psi_u(e_h, \alpha) < \widehat{y}$, whereas if she attends college (i.e., $e_h > \widehat{e}$), then $y_h = \Psi_c(e_h) > \widehat{y}$. Finally, let $\underline{y} = \min\{y^l, \widehat{y}\} < \overline{y} = \max\{y_h, \widehat{y}\}$. The CDF of y in a tracking system, conditional on parental background b, denoted by $F_b^T(y)$ and with domain in all $y \in [y_l, y^h]$ is:

$$F_b^T(y) = \begin{cases} F_{b,u}^l(y) & if \quad y_l \leq y \leq \underline{y} \\ F_{b,c}^l(y) & if \quad \underline{y} \leq y \leq y^l \\ \delta & if \quad y^l \leq y \leq y_h \\ F_{b,u}^h(y) & if \quad y_h \leq y \leq \overline{y} \\ F_{b,c}^h(y) & if \quad \overline{y} \leq y \leq y^h, \end{cases} \tag{5}$$

where $F_{b,w}^j(y) = G_b(\phi(\Psi_w^{-1}(y), \overline{x}^j))$ for $b = p, r$, $j = l, h$ and $w = u, c$.

4 On the design of an equal-opportunity educational system

According to Roemer's theory of equality of opportunity (Roemer (1998)), an individual's acquisition of her objective is influenced by three factors: circumstances beyond her control, the effort she expends and the policy environment. In our model, the objective is lifetime income, y.

The circumstance is the parental background, b, and thus, individuals are partitioned into two types: poor and rich. If two students of the same type are exposed to the same grouping policy but their lifetime incomes differ, we attribute this to differential "effort", which in our model is captured by school achievement, x.[15] Recall from Section 3 above that the grouping system s determines the average school achievement of the group j, $\overline{x}^j$. That is, if $s = M$ (Mixing), then $\overline{x}^j = \overline{x}^m$ within each classroom, whereas if $s = T$ (Tracking), then $\overline{x}^j = \overline{x}^l$ within the low track and $\overline{x}^j = \overline{x}^h$ within the high track. Thus, from Equation (2), we can rewrite the lifetime income acquired by an individual of type b who is exposed to the grouping policy s and has school achievement x as $y^b(x, s)$.[16]

According to this theory, to equalize opportunity is to choose a grouping policy such that the chance that an individual acquires a given amount of lifetime income is a function only of her school achievement and not of her parental background.[17] As school achievement x depends on students' type, we view the distribution of achievement within a type as a circumstance itself and thus, if we wish to equalize opportunity, we must compensate them for this circumstance. To do so, we define a student's effort level by his rank π in the school achievement distribution of his type, $\pi = G_b(x)$. And thus, we compare her to others with her same circumstances. Let $H_b = G_b^{-1}$ be the inverse CDF of school achievement. As suggested by Roemer's theory (1998), we may now define the "indirect advantage function" $v^b(\pi, s)$ as the level of lifetime income obtained by an individual of type b who expends the π^{th} degree of effort while facing the grouping policy s, that is, $v^b(\pi, s) = y^b(H_b(\pi), s)$. Thus, from a moral viewpoint, we declare that two students coming from different parental backgrounds have equal school achievement if they lie at the same rank of the school achievement distributions of their types (rich or poor), and our grouping criteria are proposed as a policy instrument for rendering such students as close

[15] As it is well known in the literature on compensation and responsibility, it is extremely difficult to make the distinction between circumstances and effort. The use of two types allows for a relatively intuitive discussion of the optimal grouping policy. In addition, in reality, whereas the distribution of post-school scores presents a large variance, the distribution of school achievement is fairly homogeneous (see, among others, Heckman (2006)). Thus, any partitioning of students based on their school achievement levels would lead to very similar types. In Section 4.1 below we comment on the implications of this assumption.

[16] The effect of parental background on an individual's lifetime income has two sources. First, it has an indirect effect on education acquisition through the positive dependence on an individual's school achievement. Second, in a tracking educational system, parental background influences the peer group effect that she enjoys there as the probability of being in the high track, conditional on having poor parents, is lower than conditional on having rich parents.

[17] We are not interested here on the issue of income inequality or differences between classes, although we agree that this could be other potential objective of public intervention. In addition, the government could use other instruments to achieve this goal as the fiscal policy, which is not the focus in this paper.

as possible in lifetime income. In general it is not feasible to achieve this goal and, therefore, we also wish to constrain the minimum value of the lifetime income.

First, let us fix a particular centile π of school achievement. Suppose we were only concerned with equalizing lifetime incomes across types at that π. For this case, we propose choosing the policy that maximizes the minimum level of advantage of these individuals. Propositions 1 and 2 below provide some results related to the equal-opportunity policy if we fix π. Formally, the equal-opportunity policy is:

$$s_\pi^{EOp} = \arg\max_{\{s\}} \ \min \{v^p(\pi, s), v^r(\pi, s)\}. \tag{6}$$

Recall that $F_b^s(y)$ denotes the distribution function of lifetime income y in type b at grouping policy $s = M, T$. Then, it follows that those at the π^{th} centile of the school achievement distribution are exactly those at the π^{th} centile of the lifetime income distribution by the monotonicity of lifetime income in school achievement. We can therefore write $F_b^s(v^b(\pi, s)) = \pi$. Now, assuming that the distribution function is strictly increasing, it has an inverse, and we can write $v^b(\pi, s) = F_b^{s-1}(\pi)$. It now follows that the equal-opportunity program in (6) can be written as:

$$s_\pi^{EOp} = \arg\max_{\{s\}} \ \min \left\{F_p^{s-1}(\pi), F_r^{s-1}(\pi)\right\}. \tag{7}$$

Hence, we can compute the equal-opportunity policy knowing just the distributions of lifetime income for the two types as a function of the grouping policy, that is, Equations (3) and (5). If s_π^{EOp} were the same policy for all π, that would be, unambiguously, the equal-opportunity policy. As Proposition 1 below shows, this is not the case in our model.

Proposition 1 *Let $\pi < (>)t$. Then $s_\pi^{EOp} = M(= T)$.*

Proof. First, from Equation (3), we have that $F_p^M(y) > F_r^M(y)$ as $F_{p,w}^m(y) > F_{r,w}^m(y)$ for $y \in (y_m, y^m)$ and $w = u, c$. Similarly, from Equation (5), we have that $F_p^T(y) > F_r^T(y)$ for any $y \in (y_l, y^h)$. As a result, $\min \left\{F_p^{s-1}(\pi), F_r^{s-1}(\pi)\right\} = F_p^{s-1}(\pi)$ and thus $s_\pi^{EOp} = \arg\max F_p^{s-1}(\pi)$. Now, if $s = M$, then $\overline{x}^j = \overline{x}^m$ regardless of π. However, if $s = T$, then $\overline{x}^j = \overline{x}^l$ if $\pi < t$, whereas $\overline{x}^j = \overline{x}^h$ if $\pi > t$. The rest is immediate from Equations (3) and (5), and $\overline{x}^l < \overline{x}^m < \overline{x}^h$. ■

Proposition 1 tells us which is the equal-opportunity policy s_π^{EOp} if the government is concerned only with the π-slice of the population. This implies to compare individuals with the same π from poor and rich backgrounds, and select the policy that maximizes the minimum lifetime income among them.[18] Proposition 1 shows that if the government is concerned only with low school achievers (i.e., students with $\pi < t$), then it should select a mixing grouping policy as it is the system where these students would enjoy a stronger peer effect. In contrast, if, it is concerned only with high school achievers (i.e., students with $\pi > t$), then tracking is the appropriate system as it is the one where these individuals would enjoy a stronger peer effect. Thus, contrary to the general belief that equality of opportunity is best achieved under mixing, we find that tracking may be a more effective means of achieving that goal in some cases. This result does not depend on the proportion of students in the low track, δ. In other words, as long as δ is such that there is some grouping policy in place that separates students according to their previous achievement, then precisely that grouping policy, tracking, will be selected if we focus only on those poor students who would be placed in the high track.

Now, an immediate question arises: is there any particular government aim that might justify the government's caring for only one of these two sets of individuals when choosing its grouping policy? The answer is yes. As long as the government is interested in maximizing college attendance among poor students, it should care more for one particular set of students depending on the prevailing opportunity cost of college attendance. Moreover, by proceeding in this way, the selected policy coincides with the equal-opportunity policy as defined above. Although college attendance is not the critical outcome in our model, one could think of college attendance as a crucial means to improve lifetime income.[19] Proposition 2 shows that the opportunity cost of college attendance determines which system better guarantees maximum college attendance among poor students. Let $\widetilde{\alpha}$ denote the opportunity cost such that $\widehat{e}(\widetilde{\alpha}) = \Phi(t, \overline{x}^m)$. This value $\widetilde{\alpha}$ implies that when the opportunity cost of college attendance is equal to $\widetilde{\alpha}$, no student in the low track attends college, whereas all high-track students attend college.

Proposition 2 *Let $\alpha \geq (\leq)\widetilde{\alpha}$. Then Tracking (Mixing) maximizes college attendance among*

[18] The minimum lifetime income is that of poor individuals here. The reasons for this are, first, that we assume that the student's circumstances are just parental background and thus individuals are partitioned into two types, poor and rich and, second, that we assume that the CDF of x for rich parents dominates that for poor ones. Thus, henceforth, social welfare is reduced to the welfare of students with poor parental background. The analysis can be easily extended to consider a wider set of circumstances and types.

[19] We could think of societies where there is underinvestment in tertiary education. That is, the optimal investment is lower than the equilibrium outcome, for example, because of positive externalities generated for the entire society by more highly educated people. In this respect, Moretti (2004) finds empirical evidence suggesting that an increase in the supply of college graduates increases not only high school graduates' wages but also high school dropouts' wages. Thus, it is crucial to promote college attendance.

poor students.

Proof. See the Appendix. ∎

The intuition is as follows. If α is very low, nearly all students will attend college, in particular, most high school achievers. Thus, intervention must be targeted toward those who performed poorly in school. As the peer effect is stronger for these students in a mixing system than it is under tracking, college attendance is maximized by the former. When the opportunity cost of college attendance is very high, the college student body will be relatively small, and the opposite result is obtained. In this context, most low school achievers are excluded from college under either system. Intervention should target those who performed well during school as they are the only ones who will potentially attend college. Choosing tracking is the best way to meet this goal as the peer effect is stronger for these students in a tracking system than it is in a mixing one.[20] In other words, if the opportunity cost of tertiary education is so high such that a small portion of the population attends college, and the educational system is mixing, then attendance would increase if the policy changes to tracking. This result is consistent with observed stylized facts. Some European countries where a mixing system prevails (in particular Spain) experienced a decrease in college entry rates, especially among students coming from poor backgrounds, during the first decade of the 2000s (see Education at a Glance 2008). That period was characterized by low unemployment rates and high wages among unskilled workers which rose up the opportunity cost of college attendance.

From Propositions 1 and 2 above we know that the selected policy to achieve maximum college attendance coincides with the equal-opportunity policy as defined above, s_{π}^{EOp}. Nevertheless, s_{π}^{EOp} is not the same for every π, and, of course, we wish to equalize lifetime income across types for every π. In what follows we consider every π and thus, we need some compromise solution. As suggested by Roemer's theory (1998, 2002), we propose to form a social objective function consisting of the average of the objective functions of each school achievement slice of the population. Formally, from (7):

$$s^{EOp} = \arg\max_{\{s\}} \int_{0}^{1} \min\left\{F_p^{s-1}(\pi), F_r^{s-1}(\pi)\right\} d\pi. \tag{8}$$

[20] This result may explain the empirical evidence found by Hahn et al. (2008) based on Korean data regarding high school graduates. They conclude that the number of high school graduates who entered top universities (i.e., those universities for which $\widehat{e}$ is high) was higher in a tracking system than it was in a mixing system.

The problem in (8) implies to choose the grouping policy that maximizes the minimum average value of lifetime income across types. As $F_p^s(y) > F_r^s(y)$ for $s = M, T$, this criterion implies the following:

$$s^{EOp} = \arg\max_{\{s\}} Y_{p,s}, \tag{9}$$

where $Y_{p,s} = \int_0^1 F_p^{s-1}(\pi)d\pi$ denotes the average lifetime income for poor students in an education system s. Let $\overline{Y}$ denote the difference in average lifetime income for poor students between both systems, $\overline{Y} = Y_{p,T} - Y_{p,M}$. Now, we compute $\overline{Y}$ based on the primitives of the model, x. In order to do so, we define $\widehat{x}^s$ for $s = M, T$ as the minimum school achievement such that it is optimal to go on to college in the education system s. That is, $\widehat{x}^s = \phi(\widehat{e}, \overline{x}^j)$ where $\overline{x}^j = \overline{x}^m$ if $s = M$ and $\overline{x}^j = \overline{x}^l$ if $s = T$ and $\widehat{e}$ lies on the low track (i.e., $\widehat{e} \leq e^l$) and thus $\widehat{x}^T < t$ and $\overline{x}^j = \overline{x}^l$ if $s = T$ and $\widehat{e}$ lies on the high track (i.e., $\widehat{e} \geq e_h$) and thus $\widehat{x}^T > t$. Let $\underline{x} = \min\{t, \widehat{x}^T\}$, $\overline{x} = \max\{t, \widehat{x}^T\}$ and then $\overline{Y}$ is as follows:

$$\overline{Y} = \left(\int_0^{\underline{x}} \Psi_u^l g_p dx + \int_{\underline{x}}^{\overline{x}} \Psi g_p dx + \int_{\overline{x}}^1 \Psi_c^h g_p dx \right) - \left(\int_0^{\widehat{x}^M} \Psi_u^m g_p dx + \int_{\widehat{x}^M}^1 \Psi_c^m g_p dx \right), \tag{10}$$

where $g_p = g_p(x)$ denotes the probability density function (p.d.f.) of school achievement x for poor students, $\Psi_u^j = \Psi_u(\Phi(x, \overline{x}^j), \alpha)$, $\Psi_c^j = \Psi_c(\Phi(x, \overline{x}^j))$ for $j = l, m, h$ and $\Psi = \Psi_u^h$ if $\underline{x} = t$ or $\Psi = \Psi_c^l$ otherwise. The first and second terms in (10) reflect the average lifetime income for poor students in tracking and mixing, respectively. We see now that the difference in average lifetime income between both systems depends on the difference in average education between them. To do so, and similar to $\overline{Y}$, let denote by $\overline{E}$ the difference in average compulsory education for poor students between both systems, which is:

$$\overline{E} = \left(\int_0^t \Phi(x, \overline{x}^l) g_p dx + \int_t^1 \Phi(x, \overline{x}^h) g_p dx \right) - \int_0^1 \Phi(x, \overline{x}^m) g_p dx. \tag{11}$$

The first and second terms reflect the average compulsory education for poor students in tracking and mixing, respectively. Previous literature (see, among others, Arnott and Rowse (1987)) finds that the difference in average education between tracking and mixing is determined by the properties of the education production function $\Phi(x, \overline{x}^j)$. Recent empirical evidence suggest that the degree of complementarity between individuals' school achievements x and peer effects

$\overline{x}^j$ is high enough (see Ding and Lehrer (2007)). This, in turn, means that average education in tracking is higher than in mixing, as it is the system where high ability students enjoy a stronger effect (see Takii and Tanaka (2009) and Hidalgo-Hidalgo (2011)). Thus, assuming that $\overline{E} > 0$, as in Proposition 3 below, just introduces some restrictions on the primitives of the model that enable a result in line with this empirical evidence. Now, using Equation (11) we can rewrite $\overline{Y}$ as follows:

$$\overline{Y} = (1+\alpha)\overline{E} + \Delta^l + \Delta^h - \Delta^m, \tag{12}$$

where Δ^l, Δ^h and Δ^m denote the increment in lifetime income after college attendance for low-track, high-track and mixing students, respectively. That is, $\Delta^l = \int_{\underline{x}}^{t} (\Psi_c^l - \Psi_u^l) g_p dx$, $\Delta^h = \int_{\overline{x}}^{1} (\Psi_c^h - \Psi_u^h) g_p dx$ and $\Delta^m = \int_{\widehat{x}^M}^{1} (\Psi_c^m - \Psi_u^m) g_p dx$. Proposition 3 shows that the opportunity cost of college attendance and the difference between the increment in lifetime income after college attendance in the mixing and tracking systems, $\Delta^m - (\Delta^l + \Delta^h)$, determine which system better guarantees equality of opportunity.

Proposition 3 *Let $\Phi(x, \overline{x}^j)$ be such that $\overline{E} > 0$. The following statements hold:*
(i) If $\alpha \leq \widetilde{\alpha}$ then $s^{EOp} = T$ if $(1+\alpha)\overline{E} > \Delta^m - (\Delta^l + \Delta^h)$.
(ii) If $\alpha \geq \widetilde{\alpha}$ then $s^{EOp} = T$.

Proof. See the Appendix. ■

From Propositions 2 and 3 we can conclude that, if tracking maximizes both average education at compulsory level, and college attendance rates among poor individuals, then it is the recommended system under the equality of opportunity approach. The intuition is as follows. Suppose that the opportunity cost of college attendance is very high; in this case, not only does tracking provide better preparation to students on average, but it also guarantees that the number of students who attend college (i.e., those with high school achievement levels) is maximized. In addition, these students experience an increment in lifetime income after college attendance that is larger than the one they would have experienced under mixing. Suppose now that the opportunity cost of college attendance is low and, thus, mixing maximizes college attendance among poor students. However, even in this case, tracking could maximize overall lifetime income. Note first that tracking is clearly detrimental for those low-track students who would attend college only in a mixing system (i.e., those with $x \in (\widehat{x}^M, \widehat{x}^T)$). In addition, the increment in lifetime income after college attendance is larger in mixing than it is in tracking for low school achievers (i.e., those with $x \in (\widehat{x}^T, t)$). However, the reverse occurs for high school achievers (i.e., those with $x \geq t$). Therefore, if the gain of the latter compensates for the losses of the former two, then tracking will maximize average lifetime income among poor individuals.

4.1 A brief discussion of alternative welfare criteria

We believe that it is useful to contrast the equal-opportunity policy with two other well-known policies, the Rawlsian (RW) and the Utilitarian (U). The Rawlsian policy maximizes the minimum level of advantage across all individuals, regardless of type. That is:

$$s^{RW} = \arg\max_{\{s\}} \min_{\{b,\pi\}} F_b^{s-1}(\pi). \tag{13}$$

As $F_p^s(y) > F_r^s(y)$ for $s = M, T$, the solution of this program is obtained by solving the problem:

$$s^{RW} = \arg\max_{\{s\}} \underset{s.t.\ 0<\pi<t}{F_p^{s-1}(\pi)}. \tag{14}$$

The Rawlsian approach implies selection of the education system that maximizes the utility of the worst-off individuals in the society, where we take as the worst-off those students with school achievement levels below the threshold level t and with poor parents. The result is quite immediate. Mixing is the optimal educational system, i.e., $s^{RW} = M$.[21] This finding is derived directly from the properties of the education production function. Observe that maximizing the lifetime income of these individuals will imply maximization of their education level, which is higher in a mixing system than in a tracking system as the peer effect they enjoy in the former system is stronger than it is in the latter.

In contrast, the Utilitarian policy maximizes the average level of lifetime income in the population as a whole. It solves the following problem:[22]

$$s^U = \arg\max_{\{s\}} \lambda \int_0^1 F_r^{s-1}(\pi)d\pi + (1-\lambda)\int_0^1 F_p^{s-1}(\pi)d\pi. \tag{15}$$

[21]Note that this applies to all individuals with $\pi < t$, except for the individual with $\pi = 0$.
[22]Hidalgo-Hidalgo (2010) analyzes this criterion and finds similar results to those in Proposition 3.

Thus, following the Rawlsian approach implies treating all factors as morally arbitrary, i.e., placing no behaviour in the jurisdiction of personal responsibility and simply maximizing advantage over the whole population. Utilitarianism, in contrast, maximizes the average advantage over the whole population. In the equality of opportunity analysis above, we defined only two types, poor and rich (see Footnote 15). However, this assumption might be viewed as a conservative one in the sense that it attributes more to effort than perhaps we should. Thus, in this sense, the equal-opportunity policy we find might be seen as less compensatory to disadvantaged types than it would be if we delineated all of the relevant circumstances. As the set of types becomes larger, the equal-opportunity policy approaches the Rawlsian policy (see Roemer (1998)). Now suppose that society decides that there is only one type, that is, individuals are to be held fully responsible for their lifetime income. Then, the equal-opportunity policy reduces to the Utilitarian policy. Roemer (1998) shows that indeed, the equal-opportunity approach takes a middle position between the Rawlsian and the Utilitarian. Here we find a similar result. To the extent that differences in lifetime income are due to circumstance, it approaches Rawlsianism, but to the extent that they are due to effort (here, school outcomes), it approaches Utilitarianism. In our view, this equality of opportunity approach is the most sensible one: individuals should be compensated for certain kinds of bad luck but should be held responsible for much of what they do.

Finally, an important aspect of the analysis above is the properties of the education production function. In particular, we took here a relatively conservative approach regarding the channels through which peers influence individual educational attainment. Namely, we assumed that peers affect an individual only through the mean of their characteristics, thus assuming that tracking is clearly detrimental for those students placed in the low track. On the one hand, this assumption allows an intuitive discussion of the egalitarian policy and on the other hand, it is the most common one in the related literature (see Footnote 8). Nevertheless, removing the specification proposed in this paper will only reinforce our main results without adding further insights.[23]

[23] See Hidalgo-Hidalgo (2011) for a detailed discussion on alternative specifications of peer effects which highlights the conservatism of the approach taken here by modelling the education production function in a way that leads us to consider the worst case for tracking.

5 Final Comments

This paper analyzes public intervention in education when the government, taking into account the existence of peer effects at the compulsory education level and their impact at the college level, has to decide how to group compulsory school students. Two different education systems (tracking and mixing) are examined. Equality of opportunity is assumed to be the central governmental concern. Conventional wisdom suggests that equality of opportunity is best guaranteed under mixing. Our study shows that this is not necessarily the case and that the impact on educational results at later stages (i.e., college attendance outcomes) must be taken into account when weighing the pros and cons of each educational system. We find that tracking may perform better than mixing under the opportunity-egalitarian criterion.

Our proposal regarding equality of opportunity could be interpreted as an application of Roemer's theory of equality of opportunity (Roemer (1998)) to an education policy context. Here, equality of opportunity is achieved through the allocation of students across different groups. That is, students who have a low probability of college attendance only because of their parental background should be compensated by grouping them with more competent peers at school.

The paper abstracts from variation in public schooling expenditure at the compulsory and college levels, which have previously been considered by Arnott and Rowse (1987) and Benabou (1996), among others. However, the analysis of equality of opportunity might not be reversed by considering, for example, the direct cost of providing college education. Higher college attendance rates imply a higher total cost of college education, but this cost will always be lower than its associated benefits (otherwise, it makes no sense to promote college attendance, see Footnote 19 above). In addition, abstracting from this concern enables us to isolate the role of compulsory school peer effects on future educational outcomes, which has not been considered in the prior literature. Indeed, this is a major contribution of this paper.

The paper allows for some extensions. An important one is the introduction of prices, which are omitted in this paper under the assumption of free education at both levels. This would imply modelling parental income explicitly and could enable second-best analysis to be introduced in the comparison between tracking and mixing. The crucial assumption in this analysis would be on the degree of complementarity/substitutability between parental income and peer group effects. In this paper the number of tracks is fixed. Another feasible extension includes the analysis of the optimal number of tracks. Observe that creating more tracks does not necessarily imply increasing welfare. First, the complementarity between the peer group effect and the student's achievement might not offset the concavity in peer effects. In other words, the gains of the high achievers (those who will be in tracks with higher mean achievement than they would enjoy in a tracking system with a lower number of tracks) might not compensate the losses of the low achievers (those who will be in tracks with lower mean achievement than they would enjoy in a tracking system with a lower number of tracks). And second, as the distribution of achievement is right-skewed (see also Brunello and Checchi (2007)), the set of poor students who might benefit from an increase in the number of tracks (high achievers) might not always be high enough. In addition, we might consider the effect of college attendance rate on wages. We think that introducing this assumption would not change qualitatively the main results of the paper. Note that this would make the opportunity cost of college attendance an endogenous variable, $\alpha(\theta_p^s)$, where θ_p^s denotes the college attendance rate among poor students under education system s, that is, $\theta_p^s = 1 - G_p(\phi(\widehat{e}(\alpha(\theta_p^s)), \overline{x}^j))$. If we consider only the conventional supply effect on the skilled wage, it can be shown that θ_p^s exists and it is unique. Finally, it might also be interesting to compare the two educational systems in a dynamic setup.

To conclude, we believe our results on the role of compulsory school peers' impact on future educational outcomes, on the design of equal-opportunity grouping policies are of value and seem relevant to several key issues currently under debate among economists of education. In addition these theoretical results yield two hypotheses to be tested empirically: the impact of grouping policies on the deceleration in college entry rates recently observed in some European countries (see Education at a Glance (2008) and Hahn et al. (2008)) and the distributional impact of these grouping policies on students with different background.

References

[1] Arnott, R.J., and J. Rowse (1987): "Peer group effects and educational attainment" *Journal of Public Economics*, **32**: 237-306.

[2] Benabou, R. (1996): "Efficiency and equity in human capital investment: the local connection" *Review of Economic Studies* **63** (2): 237-264.

[3] Betts J.R. and D. Morell (1999) "The determinants of undergraduate GPA: the relative importance of family background, high school resources and peer group effects" *Journal of Human Resources* **107:** 797-817.

[4] Betts, J.R. (2011) "The Economics of Tracking in Education" Chapter 7 in the Handbook of the Economics of Education, Vol. 3 (E. Hanushek, S. Machin, and L. Woessmann, eds.).

[5] Bishop, J. (2006): "Drinking from the fountain of knowledge: student incentive to study and learn externalities, information problems and peer pressure" Chapter 15 in the Handbook of Economics of Education, Vol. 2 (E. Hanushek and F. Welch, eds.).

[6] Brunello, G. and D. Checchi (2007) "Does school tracking affect equality of opportunity? New international evidence" *Economic Policy* **52**, 781-861.

[7] Ding, W. and S.F. Lehrer (2007): "Do peers affect student achievement in China's secondary schools?" *Review of Economics and Statistics*, 89 (2): 300-312.

[8] Epple, D. Newlon, E. and R.E. Romano (2002) "Ability tracking, school competition, and the distribution of educational benefits" *Journal of Public Economics* **83**: 1-48.

[9] Hahn, S., T. Sung and J. Baek (2008) "Mixing versus sorting: entering top universities" *Economics Letters*, **100**: 43-46.

[10] Hanushek, E.A. and L. Woessmann (2006) "Does educational tracking affect performance and inequality? differences-in-differences evidence across countries", *Economic Journal* **116** (510): C63-C76.

[11] Heckman, J.J. (2000) "Policies to Foster Human Capital," Research in Economics **54**: 3-56

[12] Heckman, J.J. (2006) "Skill formation and the economics of investing in disadvantaged children" *Science* **312** (5782):1900-1902.

[13] Hidalgo-Hidalgo, M. (2010) "Tracking can be more equitable than mixing: peer effects and college attendance" ECINEQ Working Papers 2010-162.

[14] Hidalgo-Hidalgo, M. (2011) "On the optimal allocation of students when peer effects are at work: tracking vs. mixing", *SERIEs: Journal of the Spanish Economic Association* **2**(1): 31-52.

[15] Lucas Jr., R.E. (1988) "On the mechanics of economic development" *Journal of Monetary Economics* **22**(1): 3-42.

[16] Moretti, E. (2004) "Estimating the social return to higher education: evidence from longitudinal and repeated cross-sectional data" *Journal of Econometrics* **121**: 175-212.

[17] OECD (2008) "Education at a Glance", OECD, Paris.

[18] Pekkarinen, T., Pekkala S. and R. Uusitalo (2009) "School tracking and intergenerational income mobility: Evidence from the Finnish Comprehensive School Reform" *Journal of Public Economics* 93: 965-973.

[19] Roemer, J.E. (1998) Equality of Opportunity. Harvard University Press, Cambridge.

[20] Roemer, J.E. (2002) "Equality of Opportunity: a progress report" *Social Choice and Welfare,* **19**: 455-471.

[21] Sacerdote, B. (2011) "Peer Effects in Education: How Might they Work, How Big are they and How Much do we know thus far?," Chapter 4 in the Handbook of the Economics of Education, Vol. 3 (E. Hanushek, S. Machin, and L. Woessmann, eds.).

[22] Takii, K. and R. Tanaka (2009) "Does the diversity of human capital increase GDP? A comparison of education systems" *Journal of Public Economics*, **93** (7-8): 998-1007.

6 Appendix

Proof. of Proposition 2: Let θ_p^s denote the college attendance rate among poor students under education system s, i.e., $\theta_p^M(\widehat{e}) = 1 - G_p(\phi(\widehat{e}, \overline{x}^m))$ and $\theta_p^T(\widehat{e}) = 1 - G_p(\phi(\widehat{e}, \overline{x}^j))$ where $\overline{x}^j = \overline{x}^l$ if $e \leq e^l$ and $\overline{x}^j = \overline{x}^h$ if $e \geq e_h$ and $\theta_p^T(\widehat{e}) = 1 - G_p(t)$ if $e^l \leq e \leq e_h$. We proof that if $\alpha \leq (\geq)\widetilde{\alpha}$ then $\theta_p^T(\widehat{e}) \leq (\geq)\theta_p^M(\widehat{e})$. To see this, let first $\widetilde{e}$ be some $e \in (e^l, e_h)$ such that $\theta_p^T(\widetilde{e}) = \theta_p^M(\widetilde{e})$. We show first that $\widetilde{e}$ is unique and thus, as $\theta_p^M(e)$ always cuts $\theta_p^T(e)$ from above, a necessary and sufficient condition to ensure that $\theta_p^M(\widehat{e}) < (>)\theta_p^T(\widehat{e})$ is that $\widetilde{e} < (>)\widehat{e}$. Note that as $G_p(\phi(\widehat{e}, \overline{x}^j))$ is decreasing with $\overline{x}^j$ for $j = m, l, h$, we can check that for any $e < e^l$, $\theta_p^T(e) - \theta_p^M(e) = G_p(\phi(\widehat{e}, \overline{x}^m)) - G_p(\phi(\widehat{e}, \overline{x}^l)) > 0$. However, for any $e > e_h$, $\theta_p^T(e) - \theta_p^M(e) = G_p(\phi(\widehat{e}, \overline{x}^m)) - G_p(\phi(\widehat{e}, \overline{x}^h)) < 0$. Thus, there is at least one $e \in (e^l, e_h)$ such that $\theta_p^T(e) = \theta_p^M(e)$. Now suppose there is some $e' \in (e^l, e_h)$ and $e' \neq \widetilde{e}$ such that $\theta_p^T(e') = \theta_p^M(e')$. Then, from the definition of $\theta_p^s(\widehat{e})$, $\theta_p^T(e') = \theta_p^M(e') = 1 - G_p(t)$. However, as $G_p(\phi(\widehat{e}, \overline{x}^m))$ is a monotonically increasing function with e this cannot be true. Now, it can also be checked that $\phi(\widetilde{e}, \overline{x}^m) = t$ or, equivalently, $\widetilde{e} = \Phi(t, \overline{x}^m)$. Now, as $\widehat{e}(\alpha)$ is strictly increasing, then it has an inverse, $\widehat{e}^{-1}$. Thus, from the definition of $\widetilde{e}$ and $\widehat{e}$, $\widetilde{e} < (>)\widehat{e}(\alpha)$ is equivalent to $\widehat{e}^{-1}(\Phi(t, \overline{x}^m)) < (>)\alpha$, or $\widetilde{\alpha} < (>)\alpha$. ■

Proof. of Proposition 3: (i) We first show that if $\alpha \leq \widetilde{\alpha}$ then $\widehat{x}^M < \widehat{x}^T \leq t$ and, thus, $\underline{x} = \widehat{x}^T$ and $\overline{x} = t$. To see this let α_T denote the opportunity cost such that $\widehat{e}(\alpha_T) = e^l$ and α^T denote the opportunity cost such that $\widehat{e}(\alpha^T) = e_h$; thus, it is clear that $\alpha_T < \widetilde{\alpha} < \alpha^T$. (i.1) if $\alpha \leq \alpha_T$, then $\widehat{e} \leq e^l$ which, from Equation (2) and the definitions of $\widehat{e}$ and e^l, is equivalent to $\widehat{x}^T = \phi(\widehat{e}, \overline{x}^l) \leq t$. As $\widehat{x}^M = \phi(\widehat{e}, \overline{x}^m)$, then, from Equation (2), $\widehat{x}^M < \widehat{x}^T \leq t$. (i.2) if $\alpha_T < \alpha < \alpha^T$, $e^l < \widehat{e} < e_h$ and, thus, $\widehat{x}^T = t$. Then, for any α within this interval, if $\widetilde{\alpha} < (>)\alpha$, then $\widehat{e} < (>)\Phi(t, \overline{x}^m)$, which is equivalent to $\widehat{x}^M < (>)t$. Thus, if $\alpha \leq \widetilde{\alpha}$ the result is immediate from Equation (12). (ii) We show that if $\alpha \geq \widetilde{\alpha}$ then $\widehat{x}^M > \widehat{x}^T \geq t$ and, thus, $\underline{x} = t$ and $\overline{x} = \widehat{x}^T$. (ii.1) If $\alpha \geq \alpha^T$, then $\widehat{e} \geq e_h$, which, from Equation (2) and the definitions of $\widehat{e}$ and e_h, is equivalent to $\widehat{x}^T = \phi(\widehat{e}, \overline{x}^h) \geq t$. As $\widehat{x}^M = \phi(\widehat{e}, \overline{x}^m)$, then, from Equation (2), $\widehat{x}^M > \widehat{x}^T \geq t$. (ii.2) If $\alpha_T < \alpha < \alpha^T$ then from (i.2) above we know that if $\widetilde{\alpha} < (>)\alpha$, then $\widehat{x}^M < (>)t$. Thus, if $\alpha \geq \widetilde{\alpha}$ then $\Delta^l = 0$ and from Equation (12) we have that:

$$\overline{Y} = (1 + \alpha)\overline{E} + \Delta^h - \Delta^m, \tag{16}$$

as $\widehat{x}^M > \widehat{x}^T$ we can rewrite it as follows:

$$\overline{Y} = (1 + \alpha)\overline{E} + \int_{\widehat{x}^T}^{\widehat{x}^M} (\Psi_c^h - \Psi_u^h) g_p dx + \int_{\widehat{x}^M}^{1} \left((\Psi_c^h - \Psi_u^h) - (\Psi_c^m - \Psi_u^m)\right) g_p dx, \tag{17}$$

but from the properties of $\Psi_c(e)$ and $\Psi_u(e, \alpha)$ then $(\Psi_c^h - \Psi_u^h) \geq (\Psi_c^m - \Psi_u^m)$. ■