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How sensitive is the provision of public inputs to specifications?

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JEL Classification numbers: H21, H3, H41, H43

Keywords: firm-augmenting public input, factor-augmenting public input, optimal provision

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How sensitive is the provision of public inputs to specifications?*

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Abstract

This paper studies the sensitivity of provision of public inputs to changes in the specification of technology and consumer preferences. We consider a simple model in which the government, with recourse to three different tax settings (a lump-sum tax, a tax on labour and a tax on economic profit), provides firms with certain productive services. We focus on the numerical results coming from the government optimization problem. We look at several specific cases in which the returns to scale in the production function emerges as a critical issue. Our findings also address the impact of changes in output elasticity, in consumer preferences and in the number of households on the levels of public input and utility.

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1 Introduction

The standard theory of public goods, developed (among others) by Samuelson, states that the sum of the marginal benefits coming from one unit of public spending must be equal to the marginal rate of transformation (of the public good with respect to a private good). However, from the very beginning, this rule was partially contested by Pigou (1947), who argued that the total welfare cost of public spending provision must include not only its marginal production cost, but also the deadweight loss generated by distortionary taxation.

The controversy has extended until present dates, specially when the optimal level of provision is discussed, instead of focussing on the optimality rules. Wilson (1991a,b), Chang (2000) and Gaube (2005) are a sample of such a debate. In general, they provide theoretical arguments and numerical examples (and counterexamples) in which the usual result is that using distortionary taxation will open the door to an optimal spending scenario that falls short of its first-best level. However, this result is sensible to the presence of taxed goods which are complements to the public expenditure (Gaube, 2000, 2007).

Much less attention has received the provision of productivity-enhancing public expenditures, despite it usually has positive effects on tax revenues, which could involve a feedback effect stimulating the provision of public spending with distortionary taxation. To the best of our knowledge, with the exception of Feehan and Matsumoto (2000, 2002) -in terms of optimality rules- and Martínez and Sánchez (2010) -in terms of optimal levels of provision- no previous papers have dealt with the optimal provision of public inputs taking as reference the Pigou's criticism. Moreover, the sensitivity of the optimal provision level to consumer preferences, number of taxpayers and returns to scale in the production function has not been sistematically studied (see, for instance, Matsumoto (1998) and Colombier and Pickhardt (2005)). Precisely this issue serves us as a starting point in this paper.

We use a simple model in which a government with recourse to three different tax settings (a lump-sum tax, a tax on labour and a tax on economic profit) draws on public funds to provide firms with certain productive services. We focus on the numerical results coming from the government optimization problem. With this aim, we look at several specific cases in which the nature of the returns to scale in the production function emerges as critical to the conclusions that may be drawn. In fact, only cases involving the so-called factor-augmenting public input suggest that optimal policy must collect taxes from sources other than economic rents. Consequently, the controversy on level comparisons of public inputs with lump-sum *versus*

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distortionary taxation only makes sense when this case is considered.

In this regard, our numerical results show that the first-best level of public input will always be higher than the second-best one. In general, this result accords with those found for public goods, in which the level reversal appears to be unusual; and it holds despite the feedback effect that public inputs exert on tax revenues by lowering the marginal cost of provision and thus potentially encouraging public spending to rise within a second-best framework. Our results also address the impact of changes in output elasticity, consumer preferences and of changes in the number of households on public input provision and utility levels. In particular, we observe that a higher elasticity of production with respect to factor-augmenting public input is welfare-enhancing, while the opposite is true for the case of firm-augmenting public input.

This paper is structured as follows. Section 2 presents our model. Section 3 carries out some numerical simulations. Finally, section 4 concludes.

2 The model

We assume an economy of n identical households whose utility function is expressed as $u(x,l)$, where x is a private good used as numeraire and l the labor supply[1]. Let Y be the total amount of time available, such that $h = Y - l$ denotes the leisure. Economic output derives from labour services and public input g, according to the aggregate production function $F(nl, g)$. This function satisfies the usual assumptions: increasing in its arguments and strictly concave. Output can be costlessly used as x or g.

Because the labour market is perfectly competitive in this case, the wage rate ω equals the marginal productivity of labour:

$$\omega = F_L(nl, g), \tag{1}$$

where $L = nl$ and firms take g as exogenous. Profits (if any), which are entirely taxed away by the government[2], may arise and be defined as:

$$\pi = F(nl, g) - nl\omega, \tag{2}$$

We distinguish two different tax settings. First, we consider a lump-sum

[1] The properties of $u(x,l)$ are the standard ones to ensure a well-behaved function: strictly monotone, quasiconcave and twice differentiable.

[2] Pestieau (1976) analyzed how the optimal rule for the provision of public inputs has to be modified when these rents are not taxed away.

tax T so that the representative household faces the following problem:

$$\begin{aligned} & Max \quad u(x,l) \\ s.t. \quad : \quad & x = \omega l - T, \end{aligned} \tag{3}$$

which yields the labour supply $l(\omega, \omega Y - T)$ and the indirect utility function $V(\omega, \omega Y - T)$. We assume that $l_\omega \geq 0$.

For later use, we describe some comparative statics of $\omega(g, T, n, Y)$ and $\pi(g, T, n, Y)$[3]:

$$\omega_g = \frac{F_{Lg}}{1 - nF_{LL}l_\omega} > 0 \tag{4}$$

$$\omega_T = \frac{nF_{LL}l_T}{1 - nF_{LL}l_\omega} > 0 \tag{5}$$

$$\pi_T = -\frac{n^2 l F_{LL} l_T}{1 - nF_{LL}l_\omega} < 0 \tag{6}$$

The government's optimization problem is expressed as:

$$\begin{aligned} & \underset{R}{Max} \quad V(\omega(g,T), \omega(g,T)Y - T) \\ s.t. \quad : \quad & g = nR = nT + \pi(g,T), \end{aligned} \tag{7}$$

where $R = T + \pi(g,T)/n$ is the renevue per person[4]. An alternative scenario involves a specific tax on labour τ. Under this tax setting, the consumer's optimization problem can be expressed as:

$$\begin{aligned} & Max \quad u(x,l) \\ s.t. \quad : \quad & x = (\omega - \tau) l \end{aligned} \tag{8}$$

obtaining $l(\omega_N, \omega Y)$ and $V(\omega_N, \omega Y)$ where $\omega_N = \omega - \tau$ is the net wage rate.

Again for future reference we derive the following results:

$$\omega_\tau = \frac{-nF_{LL}l_\omega}{1 - nF_{LL}l_\omega} > 0 \tag{9}$$

$$\pi_g = F_g - (nF_{LL}l_\omega + 1)\, nlF_{Lg} \gtreqless 0 \tag{10}$$

$$\pi_\tau = (1 - \omega_\tau)\, n^2 l F_{LL} l_\omega < 0 \tag{11}$$

[3]Note that variables n and Y are exogenously determined. For the sake of simplicity, hereafter they will not be used as arguments in these functions.

[4]Here it is useful to consider that rents accrue to consumers before being taxed away by government.

In the second-best scenario, the government's optimization problem is expressed as:

$$\begin{aligned} &\underset{R}{Max} \quad V\left(\omega_N, \omega(g,\tau)Y\right) \\ s.t. \quad : \quad & g = nR = n\tau l + \pi\left(g,\tau\right), \end{aligned} \tag{12}$$

with $R = \tau l + \pi\left(g,\tau\right)/n$. Under both tax settings and after some manipulations involving the FOCs of both problems and expressions (4)-(6) and (9)-(11), an important condition for the optimal provision of public inputs is obtained:

$$F_g = 1 \tag{13}$$

According to this condition, the production effects of the public input must equal its marginal production cost at the optimal level. This result is consistent with the production-efficiency theorem set forth by Diamond and Mirrlees (1971).

3 A comparison of public input provision levels

The literature cites a number of approaches mostly used to compare optimal public goods levels under different tax settings. Based on optimality rules and the relationships of complementarity or substitutability between different private goods, and between private and public goods, Gaube (2000) and Chang (2000) suggested several criteria for comparing the first and second-best levels of public goods. Under an alternative framework, Gronberg and Liu (2001) relate the sign of the marginal excess burden to level comparisons of public good provision. But, unfortunately, all these approaches present a critical drawback in our case: public input does not directly enter utility function as an argument and thus there is no scope for the translation of these methologies to the discussion of public input provision.

Alternatively, a standard way of evaluating what happens with the provision of public inputs under different scenarios is to consider specific situations that can be numerically solved. To this end, and in an attempt to obtain results as general as possible and which can be usefully related to previous studies focusing on public goods, we consider three different utility functions: the quasi-linear (Gronberg and Liu, 2001), Cobb-Douglas utility (Atkinson and Stern, 1974; Wilson, 1991a), and CES utility functions (Wilson, 1991b; Gaube, 2000). Specifically,

$$U(x,h) = x + 2h^{\frac{1}{2}} \tag{14}$$

$$U(x,h) = a \log x + (1-a) \log h \quad (15)$$

$$U(x,h) = (x^{\rho} + h^{\rho})^{\frac{1}{\rho}}, \quad (16)$$

where $a \in (0,1)$ and $\rho = 0.5$.

The specification of the production function plays a critical role here, given that private and public factors may enter into this function in a number of different ways. Of particular importance to the debate is whether the function exhibits constant returns to scale in public and private inputs (firm-augmenting public input), or only constant returns to the private factors (factor-augmenting public input). For the sake of simplicity, a Cobb-Douglas production function will be used in both scenarios.

For each of the above optimization problems, we choose the numerical method that best suited to the problem's particular features[5]. Therefore, we use the well-known Newton-Raphson algorithm to solve the case of firm-augmenting public input, but choose the standard Nelder-Mead algorithm and the Rational Iterative Multisection (RIM) methods for the factor-augmenting public input case, since it involves problems with non-convexities[6].

INSERT TABLE 1 ABOUT HERE

Table 1 summarizes the optimal values achieved at the benchmark scenarios (described below). Panels A and B refer to the cases of the firm and factor-augmenting, respectively.

Firm-augmenting public input

We assume a production function given by $F(nl, g) = (nl)^{\alpha} g^{1-\alpha}$, where $\alpha \in (0,1)$. This specification creates firm-specific rents. As Pestieau (1976) proved, if these rents are also an argument in the consumer's indirect utility function, the first-best spending condition will not be the optimal one; however, since our model assumes that all economic rents will be taxed away by the government, the observation is irrelevant here.

[5] In an attempt to obtain comparable solutions, the same level of precision, 10^{-4}, was demanded of all of them.

[6] See Kelley (1999) and Mathews and Fink (2004) for further details on Nelder-Mead algorithm. The RIM method consists of a selective iterative subdivision of the initial decision variables set, in which the objective function is then evaluated. For a detailed explanation of this method, see Sánchez and Martínez (2008). Matlab routines implementing the different methods are available upon request.

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With regard to parameters, we assume that $a \in \{0.1, \mathit{0.5}, 0.9\}$, that $\alpha \in \{0.6, 0.64, 0.67, \mathit{0.7}, 0.74, 0.77, 0.8\}$ for the production function and that $n \in \{10, 40, 70, \mathit{100}, 400, 700, 1000\}$ for population, where the benchmark values have been emphasized.

INSERT FIGURES 1-4 ABOUT HERE

Figures 1-4 show the main results for several scenarios and each utility function. The debate over optimal levels of public inputs is irrelevant in situations where the rents generated by firm-augmenting public input are completely taxed away by the government. Both the analytical solution of our model and its numerical resolution give the intuitive result that the optimal level of productive public spending must be exclusively financed with the economic rents. As a result, there is no distinction between distortionary and lump-sum taxes.

This situation can be compared to that studied by Feehan and Batina (2007), in which a (semi)public input is treated as a common property resource. The appropriate policy instrument in this case is a Lindahl pricing system that charges firms for their use of the public input/resource, in accordance with the value of its marginal contribution to each firm's profits (Sandmo, 1972). All in all, the complete taxation of rents implies to solve the common problem arising when public input provision is involved.

Beyond the contribution to the optimal levels controversy, these figures present a number of interesting results**.** First, for the quasi-linear utility function the level of public input provision in the production function is non-monotonically related to its output share $(1 - \alpha)$ while for the CES and Cobb-Douglas utility functions the optimal level is increasing in $(1-\alpha)$. The latter result is easily explained. It can be proven that F_g is decreasing in α. Since a Cobb-Douglas utility implies a fixed labor supply, the only way to hold $F_g = 1$ is to provide less public input.

Second, all of the utility specifications show a direct link between the number of households and the level of public inputs. More specifically, public input and the population increase at the same rate. Since the production function exhibits a constant return to scale (i.e., it is homogeneous of degree 1), it may be claimed that function $\pi(.)$ is also homogeneous of degree 1. Accordingly, a rise in the number of households is always followed by a proportionate rise in profits (and, consequently, in the provision of public inputs).

Third, the output share of public inputs $(1 - \alpha)$ and the level of utility are inversely related for all the utility functions. The higher the productivity of public input, the smaller the utility of the representative household. This situation can be attributed to the decrease in the output share of labour resulting from constant returns to scale in public and private inputs.

Finally, only for the Cobb-Douglas case, the level of public input rises as preferences for the private good increase (parameter a). The greater the preference for the private good, the lower will be the preference for leisure and consequently the more time will be devoted to work. Under the assumptions of our model, this situation leads to increase the production and to decrease the wages, which together serve to increase economic rents[7].

Factor-augmenting public input

The main difference between the above scenario and one involving factor-augmenting public input lies with the assumptions regarding the returns to scale in the production function. More specifically, we assume a function that yields increasing returns on all of the inputs (constant returns on labor): $F(nl, g) = nlg^{\beta}$, where $\beta \in (0, 1)$. In this context, the debate on the level of public spending in alternative tax settings again becomes relevant. Indeed, the use of lump-sum or distorting taxes is necessary here as long as rents are null. Our parameters in this case are $a \in \{0.1, \mathit{0.5}, 0.9\}$, $\beta \in \{0.1, 0.14, 0.17, \mathit{0.2}, 0.24, 0.27, 0.3\}$ and $n \in \{10, 40, 70, \mathit{100}, 400, 700, 1000\}$, again with an emphasis on benchmark values.

INSERT FIGURES 5-9 ABOUT HERE

Figures 5-9 report the levels of utility, public input and labor supply for each tax setting and each of the three utility functions. Our results accord with the generalized finding in the literature that a level reversal (a second-best level of public input over the first-best one) is unusual.

Other interesting conclusions may also be drawn. First, the level of provision in the second-best scenario is always increasing in the output share of public inputs β for the three utility functions. This contrasts with the firm-augmenting case, for which this occured in only two of the three utility functions. Second, there exists a direct relationship (not necessarily linear) between the number of households and the level of public inputs. Third, the level of utility at optimum is increasing in the output share of public inputs β. In other words, the higher the elasticity of output with respect to

[7] Given our Cobb-Douglas utility function, it can be written that $l(\omega_N) = Ya$. Since $\frac{\partial \pi}{\partial a} = -nYa\frac{\partial \omega}{\partial a}$ and n, Y and a are positive, then $sign(\frac{\partial \pi}{\partial a}) \neq sign(\frac{\partial \omega}{\partial a})$. As we know that $\frac{\partial \omega}{\partial a} = \alpha(1-\alpha)\left(\frac{g}{n}\right)^{1-\alpha}(Ya)^{\alpha}\frac{-1}{Ya^2} < 0$, therefore $\frac{\partial \pi}{\partial a}$ has to be positive.

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public input, the greater the utility of the representative agent. This result has a clear policy implication: governments must be aware of the potential impact on productivity of factor-augmenting public input, since such input is welfare-enhancing.

4 Concluding remarks

The focus of this paper -the sensivity of provision of public inputs to changes in specifications under different tax settings- remains largely unexplored in the literature. Previous analyses have focused on the case of public goods or on the optimal rules for productive public spending. Yet the welfare implications of taxation and the growth-enhancing potential of public inputs make this topic a highly relevant one for policy-makers.

In our simple model, public inputs provide productive services to firms and government has recourse to three alternative tax settings: a lump-sum tax, a tax on labour, and taxes on economic profits. In principle, the feedback effect, which arises as public inputs encourage the tax bases, might reduce welfare losses in a distortionary tax setting and, consequently, it may (i) increase the likelihood of having a second-best level higher than the first-best one (it would be an exception in the mainstream of literature on public good levels comparisons), and (ii) make less evident which the reaction to changes in the specification is.

To shed some light on these issues, we have performed a numerical simulation solving the optimization problem of government. With the aim of obtaining the most general result as possible, we have worked with three standard utility functions and two types of public inputs (firm-augmenting and factor-augmenting), which involve different assumptions on returns to scale and the role played by rents.

Our first group of results refers to the case of firm-augmenting public inputs. In this case, rents are taxed away by the government precisely because they suffice to finance the optimal public spending. Under such circumstances, we found that the debate on level comparisons under different tax settings is irrelevant. We also obtained a level of public inputs that is linearly proportionate to the number of households. The relationship between the output share of public inputs and their levels of provision is positive in the case of Cobb-Douglas and CES utility functions, but non-monotonic for quasi-linear preferences.

A second group of results refers to the case involving factor-augmenting public inputs for which we used appropriate numerical methods to address non-convex optimization problems. The controversy over optimal public spending levels matters in this case, since it describes a situation in which government financing comes not from rents but from labor or lump-sum taxes. Our numerical results are clear and in line with previous studies regarding public goods: the level of public input in the first-best scenario always exceeds that of the second-best one. Moreover, we find that the optimal level of public inputs is increasing in the output share of public inputs and the number of households. In terms of utility levels, the results for firm and factor-augmenting public inputs differ from one another. In this sense, the higher elasticity of production with respect to factor-augmenting public input, the higher the household's utility, while the relationship is the inverse for the case of firm-augmenting public input.

Finally, some policy implications can be derived. Shortly, the paper highlights the critical importance of identifying the returns to scale in production functions involving public inputs. On the one hand, only in the case of the so-called factor-augmenting public input, the optimal policy require taxation drawn from sources other than economic rents. By contrast, when firm-augmenting public inputs are considered, comprehensive taxation of economic rents offers the best solution to the optimization problem in a second-best scenario. On the other hand, policymakers must be aware of the fact that public inputs have a tremendous impact on welfare (with opposite findings for firm and factor-augmenting public inputs). Therefore, these public expenditures affect not only the production side of the economy, but also play a significant role in household utility levels.

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References

[1] Atkinson, A. B. and Stern, N. H. (1974) Pigou, taxation and public goods, *Review of Economic Studies*, 41, pp. 119-128.

[2] Browning, E. K., Gronberg, T. and Liu, L. (2000) Alternative measures of the marginal cost of funds, *Economic Inquiry*, 38 (4), pp. 591-599.

[3] Chang, M. (2000) Rules and levels in the provision of public goods: the role of complementarities between public good and taxed commodities, *International Tax and Public Finance*, 7, pp. 83-91.

[4] Colombier, C. and Pickhardt, M. (2005) A note on public input specifications, *International Advances in Economic Research*,11, pp. 13–18.

[5] Diamond, P. A. and Mirrlees, J. A. (1971) Optimal taxation and public production I-II, *American Economic Review*, 61, pp. 8-27 and 261-278.

[6] Feehan, J. P. and Batina. R. G. (2007) Labor and capital taxation with public inputs as common property, *Public Finance Review*, 35, 5, pp. 626-642.

[7] Feehan, J. P. and Matsumoto, M. (2000), Productivity-enhancing public investment and benefit taxation: the case of factor-augmenting public inputs, *Canadian Journal of Economics*, 33 (1), pp. 114-121.

[8] Feehan, J. P. and Matsumoto, M. (2002), Distortionary taxation and optimal public spending on productive activities, *Economic Inquiry,* 40 (1), pp. 60-68.

[9] Gaube, T. (2000) When do distortionary taxes reduce the optimal supply of public goods?, *Journal of Public Economics*, 76, pp. 151-180.

[10] Gaube, T. (2005) Financing public goods with income taxation: provision rules vs. provision level, *International Tax and Public Finance*, 12, pp. 319-334.

[11] Gaube, T. (2007) A note on the link between public expenditures and distortionary taxation, *Economics Bulletin* 8 (9), pp. 1-10.

[12] Gronberg, T. and Liu, L. (2001) The second-best level of a public good: an approach based on the marginal excess burden, *Journal of Public Economic Theory*, 3 (4), pp. 431-451.

[13] Kay, J. A. (1980) The deadweight loss from a tax system, *Journal of Public Economics*, 13, pp. 111- 119.

[14] Kelley, C. T. (1999) Detection and remediation of stagnation in the Nelder-Mead algorithm using a sufficient decrease condition, *SIAM Journal on Optimization*, 10, pp. 43–55.

[15] Konishi, H. (1993) A note on public good provision and commodity taxes, *The Economic Studies Quarterly*, 44 (2), pp. 178-184.

[16] Martínez, D. and Sánchez, A. J. (2010) A note on the optimal level of public inputs, *Social Choice and Welfare*, forthcoming.

[17] Mathews, J. H. and Fink, K. D. (2004) *Numerical methods using Matlab*, Fourth edition, Prentice-Hall, New Jersey, USA.

[18] Matsumoto, M. (1998) A note on tax competition and public input provision, *Regional Science and Urban Economics*, 28, pp. 465–473.

[19] Pazner, E. A. and Sadka, E. (1980) Excess burden and economic surplus as consistent welfare indicators, *Public Finances/Finances Publiques*, 35, pp. 439-449.

[20] Pestieau, P. (1976) Public intermediate goods and distortionary taxation, *European Economic Review*, 7, pp. 351-357.

[21] Pigou, A. C. (1947) *A study in Public Finance*, Third edition, Macmillan: London.

[22] Samuelson, P. A. (1954) The pure theory of public expenditure, *Review of Economics and Statistics*, 36, pp. 387-389.

[23] Sánchez, A. J. and Martínez, D. (2008) Optimization in non-standard problems. An application to the provision of public inputs, WP ECON 08.07, University Pablo de Olavide, Department of Economics.

[24] Sandmo, A. (1972) Optimality rules for the provision of collective factors of production, *Journal of Public Economics*, 1, pp. 149-157.

[25] Triest, R. K. (1990) The relationship between the marginal cost of public funds and the marginal excess burden, *American Economic Review*, 80, pp. 557-566.

[26] Wildasin, D. E. (1984) On public good provision with distortionary taxation, *Economic Inquiry*, 22, pp. 227-243.

[27] Wilson, J. D. (1991a) Optimal public good provision with limited lump-sum taxation, *American Economic Review*, 81 (1), pp. 153-166.

[28] Wilson, J. D. (1991b) Optimal public good provision in the Ramsey tax model, *Economics Letters*, 35, pp. 57-61.

Tables and figures

Table 1: Firm-augmenting public input scenarios

Panel A: Firm-augmenting public input

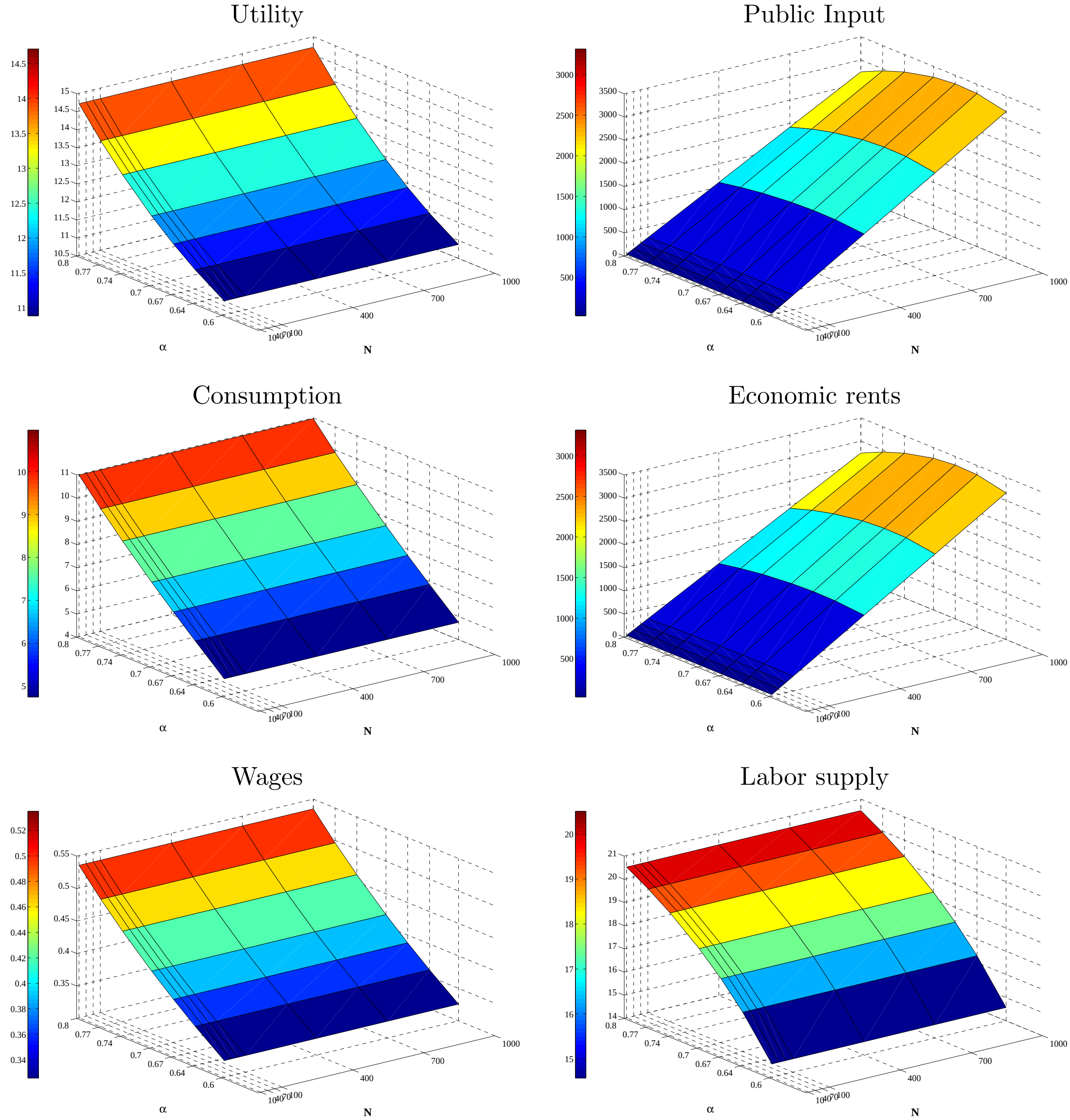
	Quasi-linear	Cobb-Douglas	CES ($\rho = 0.5$)
	$n = 100, \alpha = 0.7$	$n = 100, \alpha = 0.7, a = 0.5$	$n = 100, \alpha = 0.7$
Utility	12.42137	2.04858	34.02809
Public Input	327.20646	214.8877	126.65501
Labour Supply	18.27223	12	7.07281
Profits	327.20646	214.8877	126.65501

Panel B: Factor-augmenting public input

		Quasi-linear	Cobb-Douglas	CES ($\rho = 0.5$)
		$n = 100, \beta = 0.2$	$n = 100, \beta = 0.2, a = 0.5$	$n = 100, \beta = 0.2$
Utility	LS	90.0830	3.0654	109.4471
	D	90.0697	3.0584	108.4200
Public	LS	2241.3885	1077.6083	1830.5908
Input	D	2238.5800	944.6400	1643.5273
Labour	LS	23.9543	13.3333	20.3724
supply	D	23.9286	12	18.6873

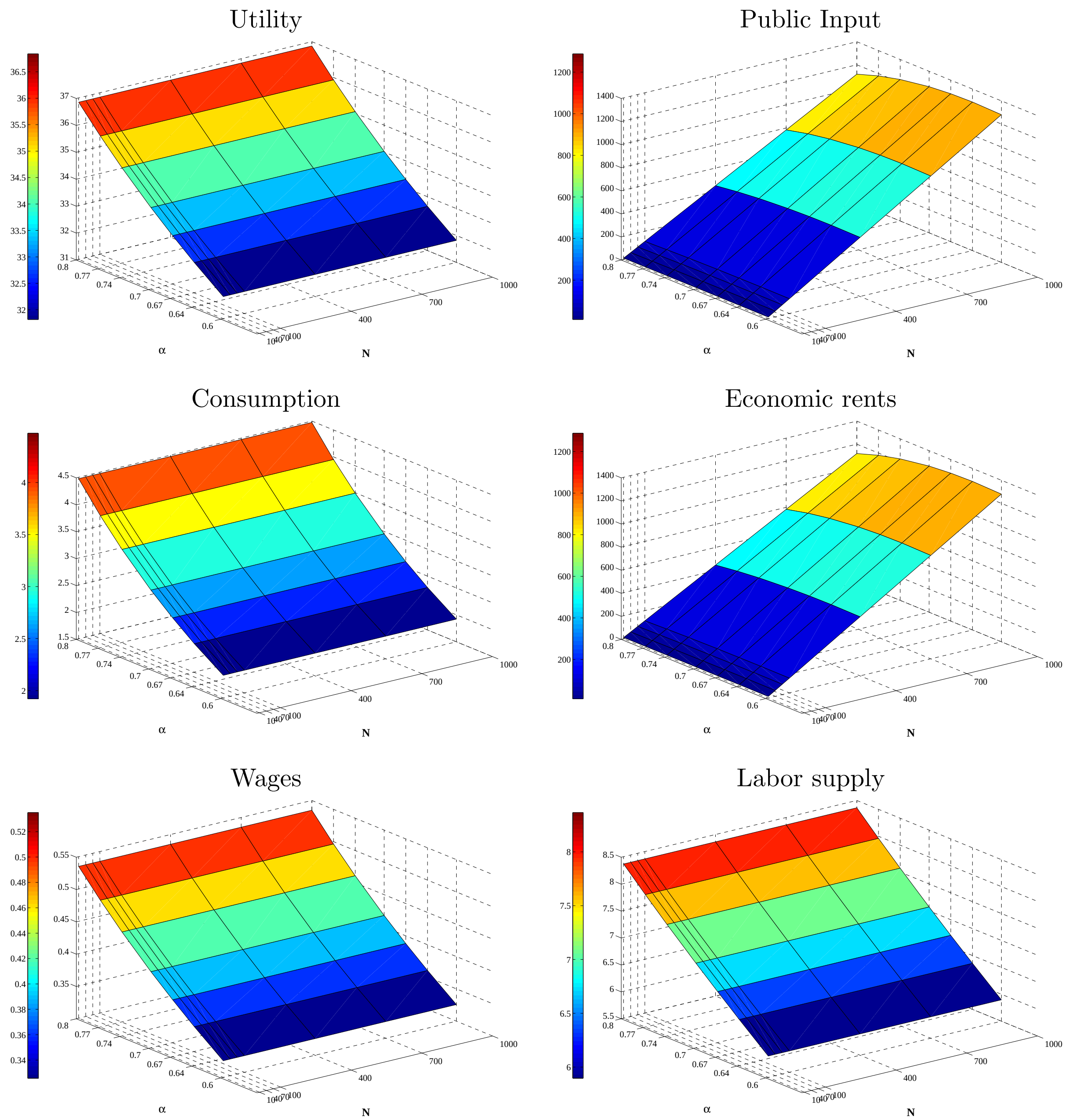
Note: (1) LS=Lump-sum, D=Distorting.

Figure 1: Firm-augmenting (Lump-sum). Quasi-linear utility function.



Notes: (1) Newton-Raphson is used to solve these scenarios. (2) Results corresponding to the distorting tax setting equal those shown here. For the sake of brevity, we skip their presentation.

Figure 2: Firm-augmenting (Lump-sum). CES ($\rho = 0.5$) utility function.

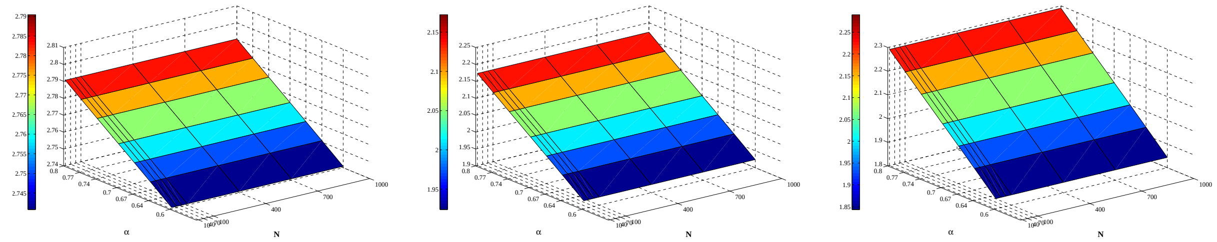


Notes: (1) Newton-Raphson is used to solve these scenarios. (2) Results corresponding to the distorting tax setting equal those shown here. For the sake of brevity, we skip their presentation.

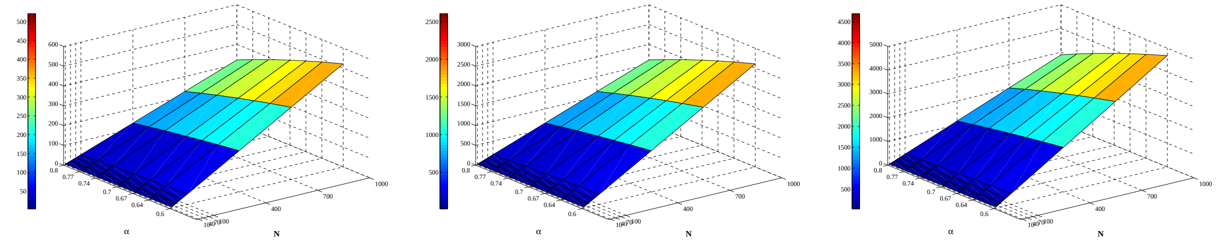
Figure 3: Firm-augmenting (Lump-sum). Cobb-Douglas utility function.

$a = 0.1$ $a = 0.5$ $a = 0.9$

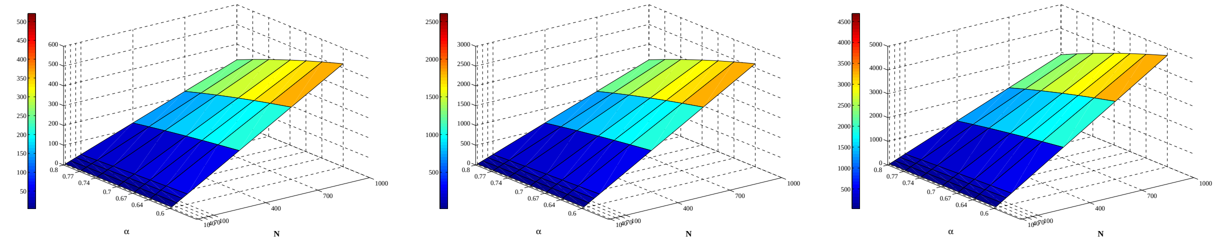
Utility



Public Input

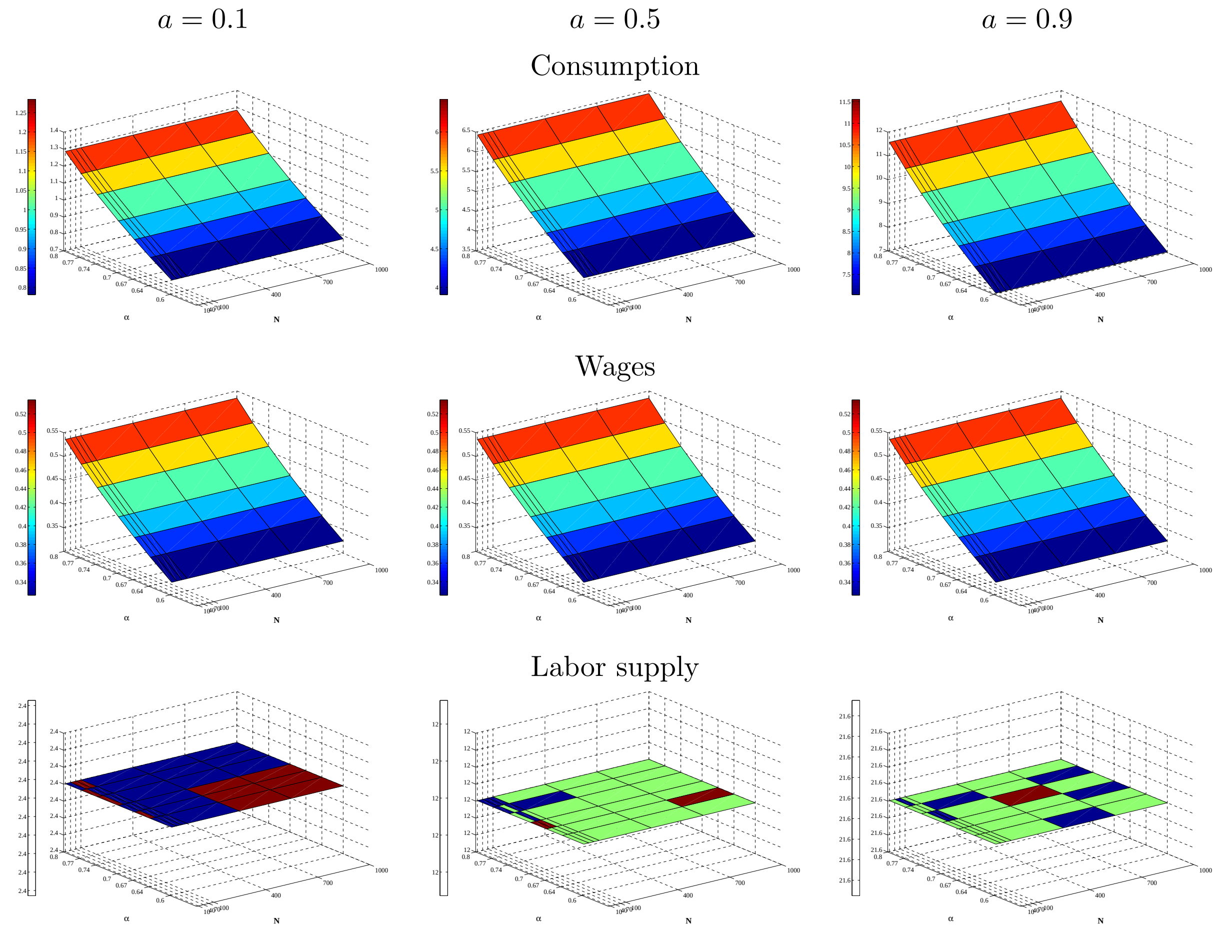


Economic rents



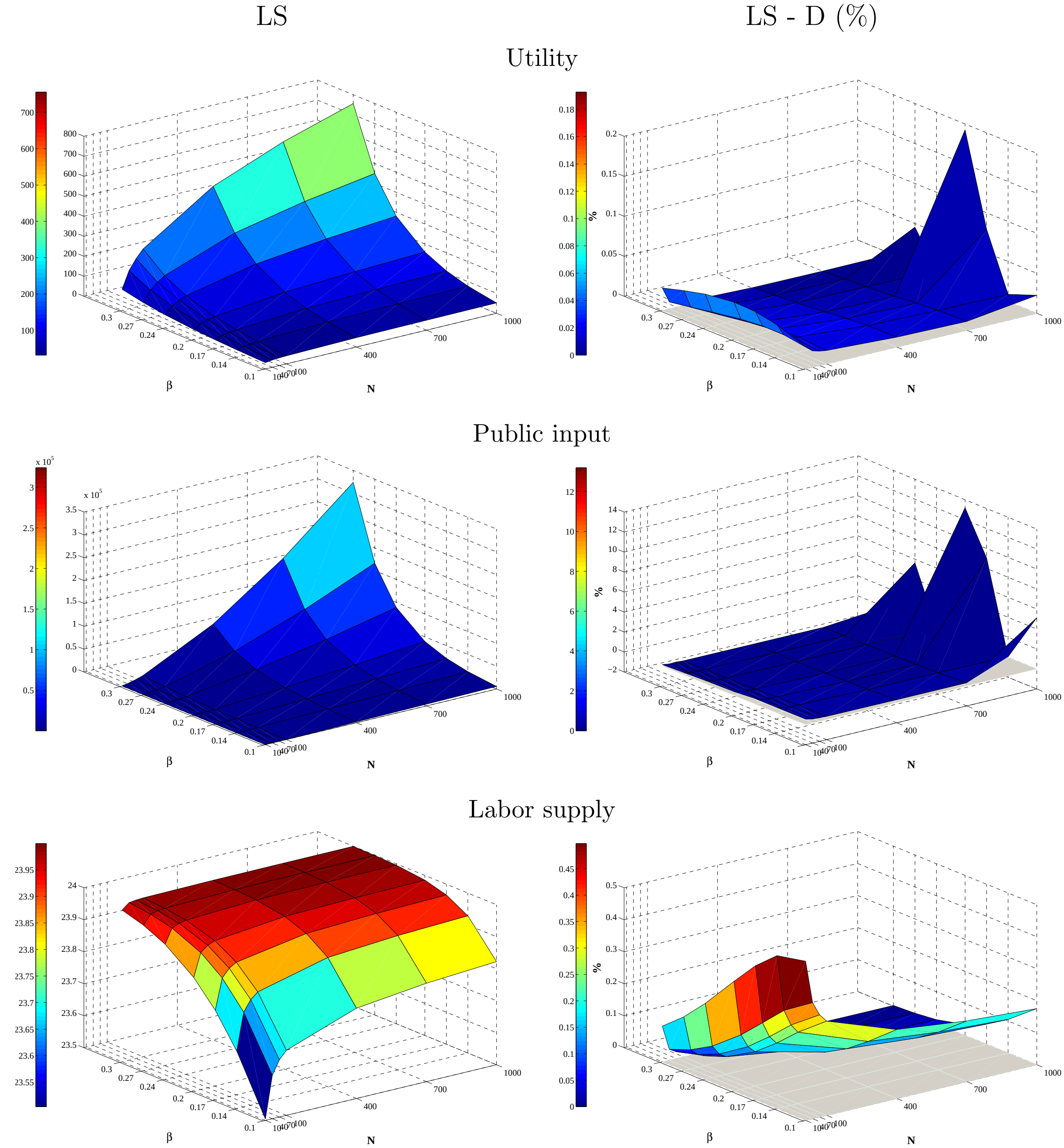
Notes: (1) Newton-Raphson is used to solve these scenarios. (2) Results corresponding to the distorting tax setting equal those shown here. For the sake of brevity, we skip their presentation.

Figure 4: (Cont.) Firm-augmenting (Lump-sum). Cobb-Douglas utility function.



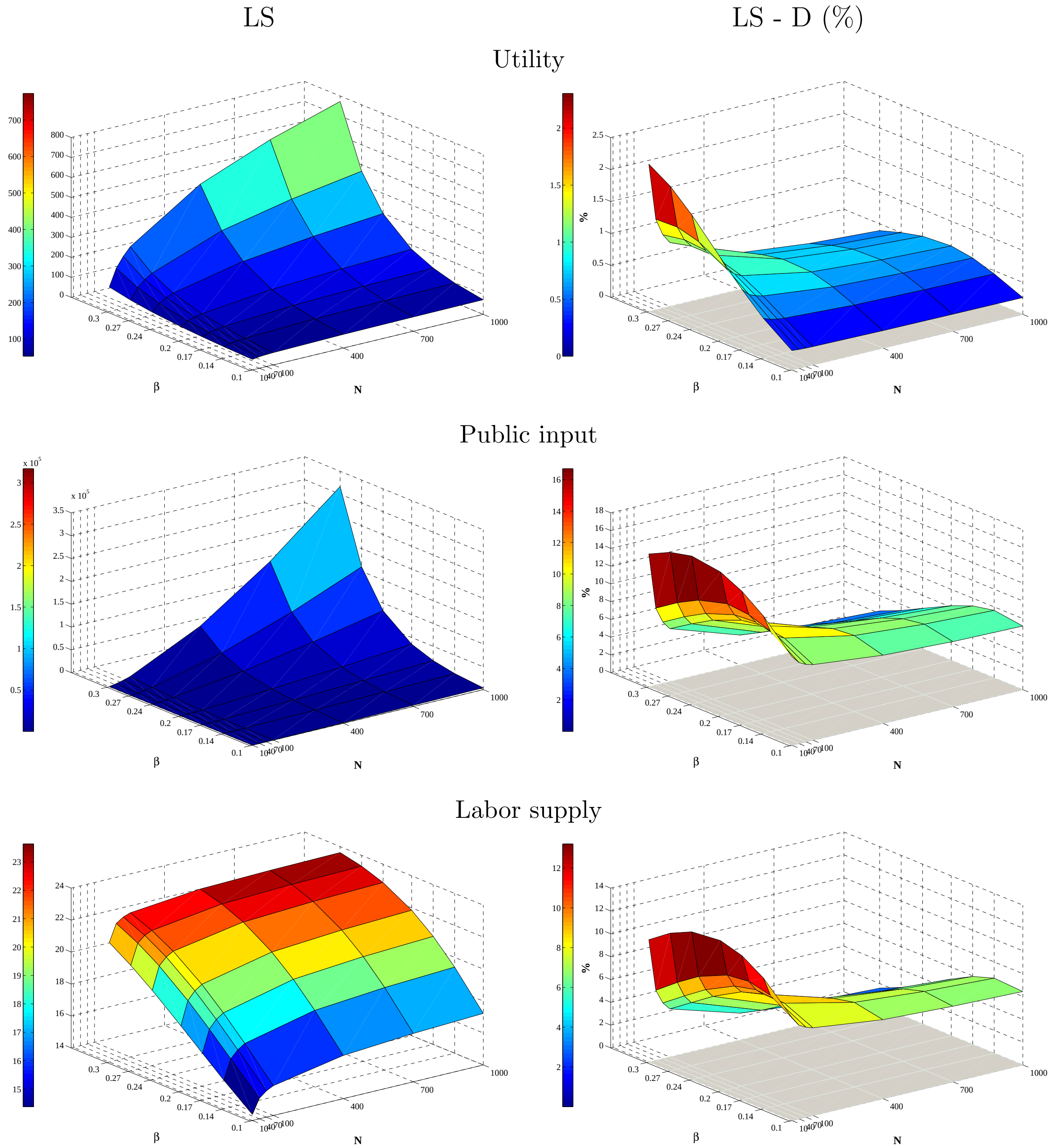
Notes: (1) Newton-Raphson is used to solve these scenarios. (2) Results corresponding to the distorting tax setting equal those shown here. For the sake of brevity, we skip their presentation.

Figure 5: Factor-augmenting. Quasi-linear utility function.



Notes: (1) LS = lump-sum, D = Distorting. (2) Nelder-Mead (RIM) is used to solve lump-sum (distorting) scenarios. (3) Differences are expressed in percent terms. (4) Grey surface helps to distinguish between positive and negative values.

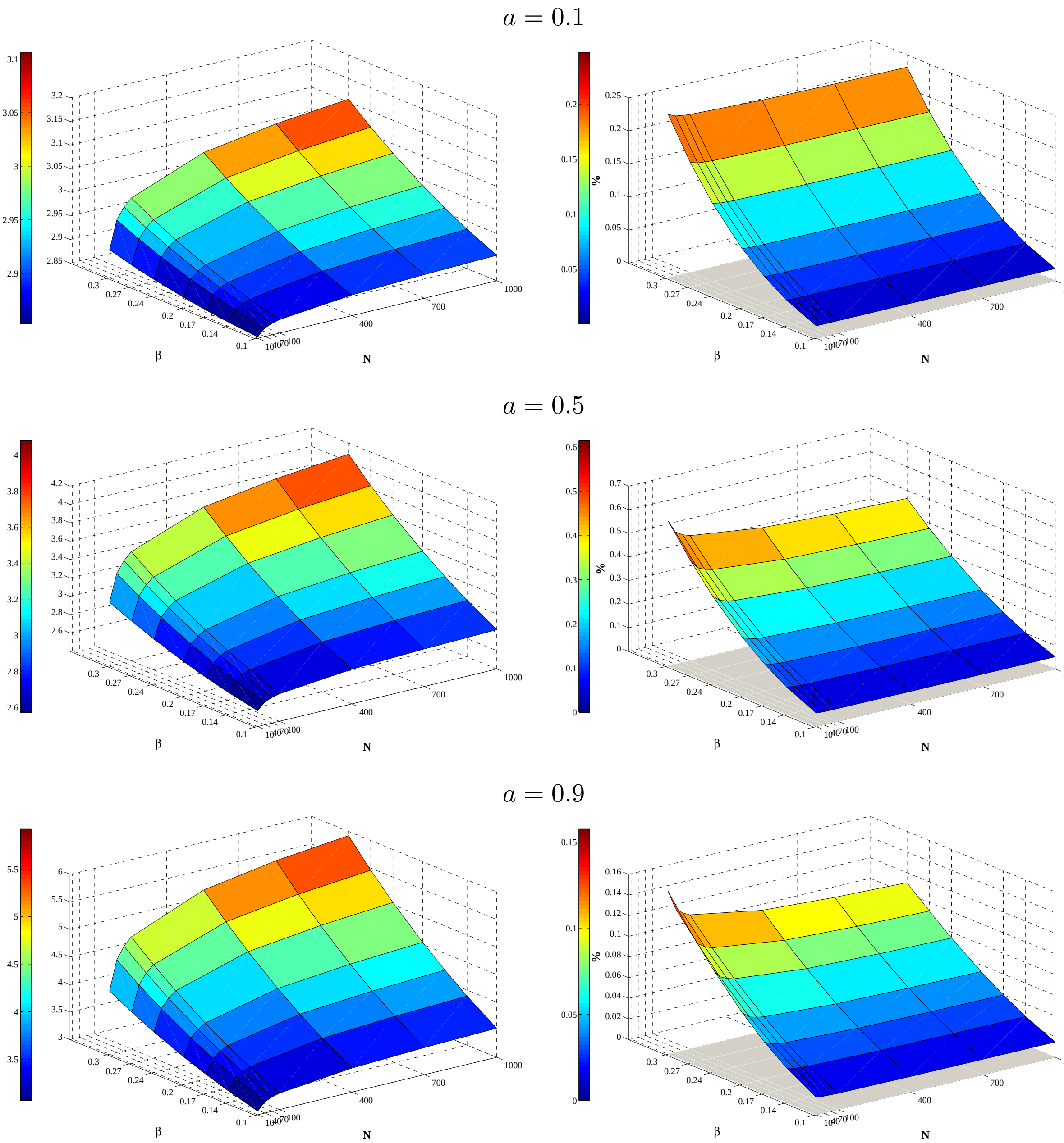
Figure 6: Factor-augmenting. CES ($\rho = 0.5$) utility function.



Notes: (1) LS = lump-sum, D = Distorting. (2) Nelder-Mead (RIM) is used to solve lump-sum (distorting) scenarios. (3) Differences are expressed in percent terms. (4) Grey surface helps to distinguish between positive and negative values.

Figure 7: Factor-augmenting. Cobb-Douglas utility function. Variable: Utility

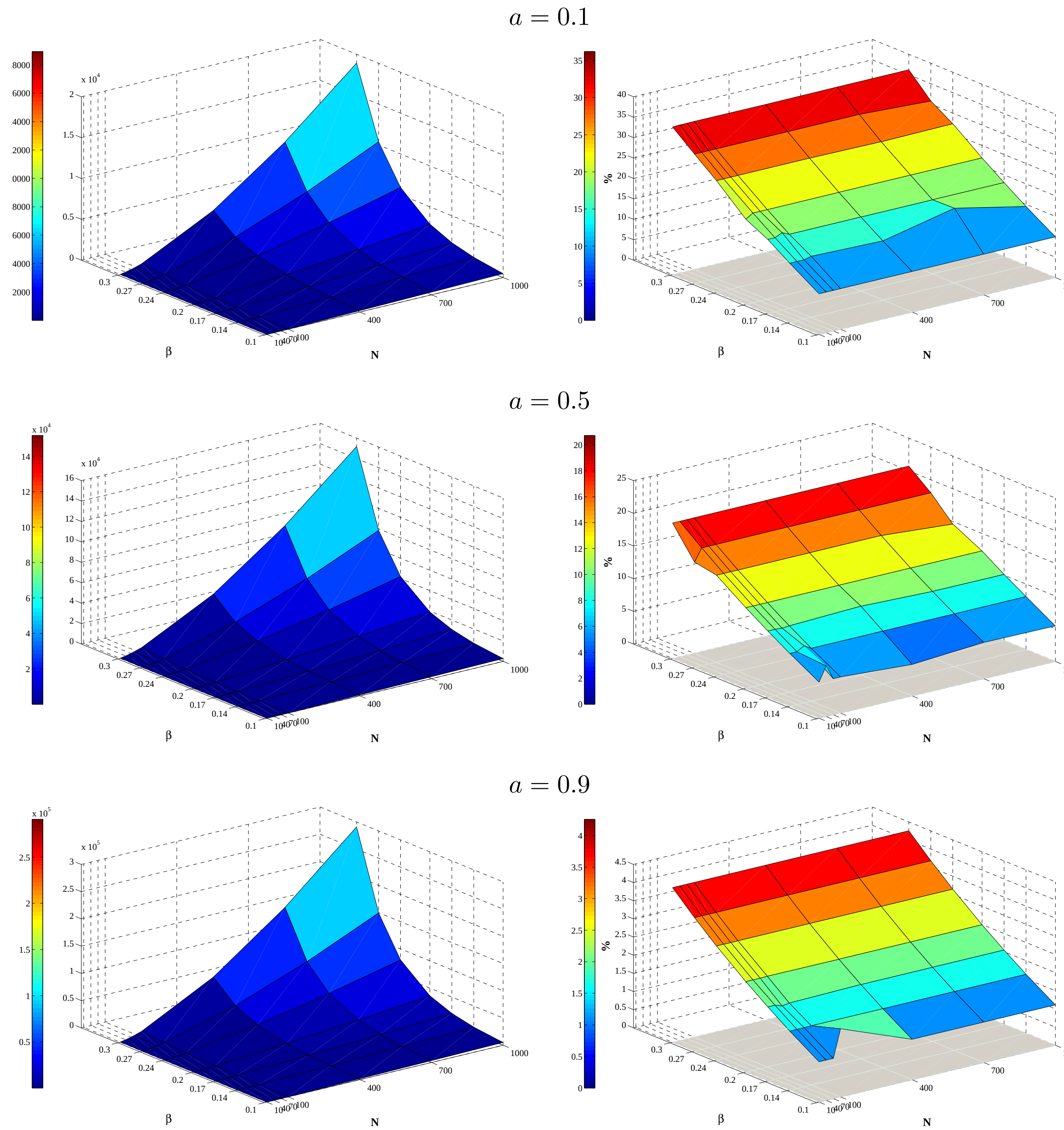
LS LS - D (%)



Notes: (1) LS = lump-sum, D = Distorting. (2) Nelder-Mead is used to solve these scenarios. (3) Differences are expressed in percent terms. (4) Grey surface helps to distinguish between positive and negative values.

Figure 8: Factor-augmenting. Cobb-Douglas utility function. Variable: Public input

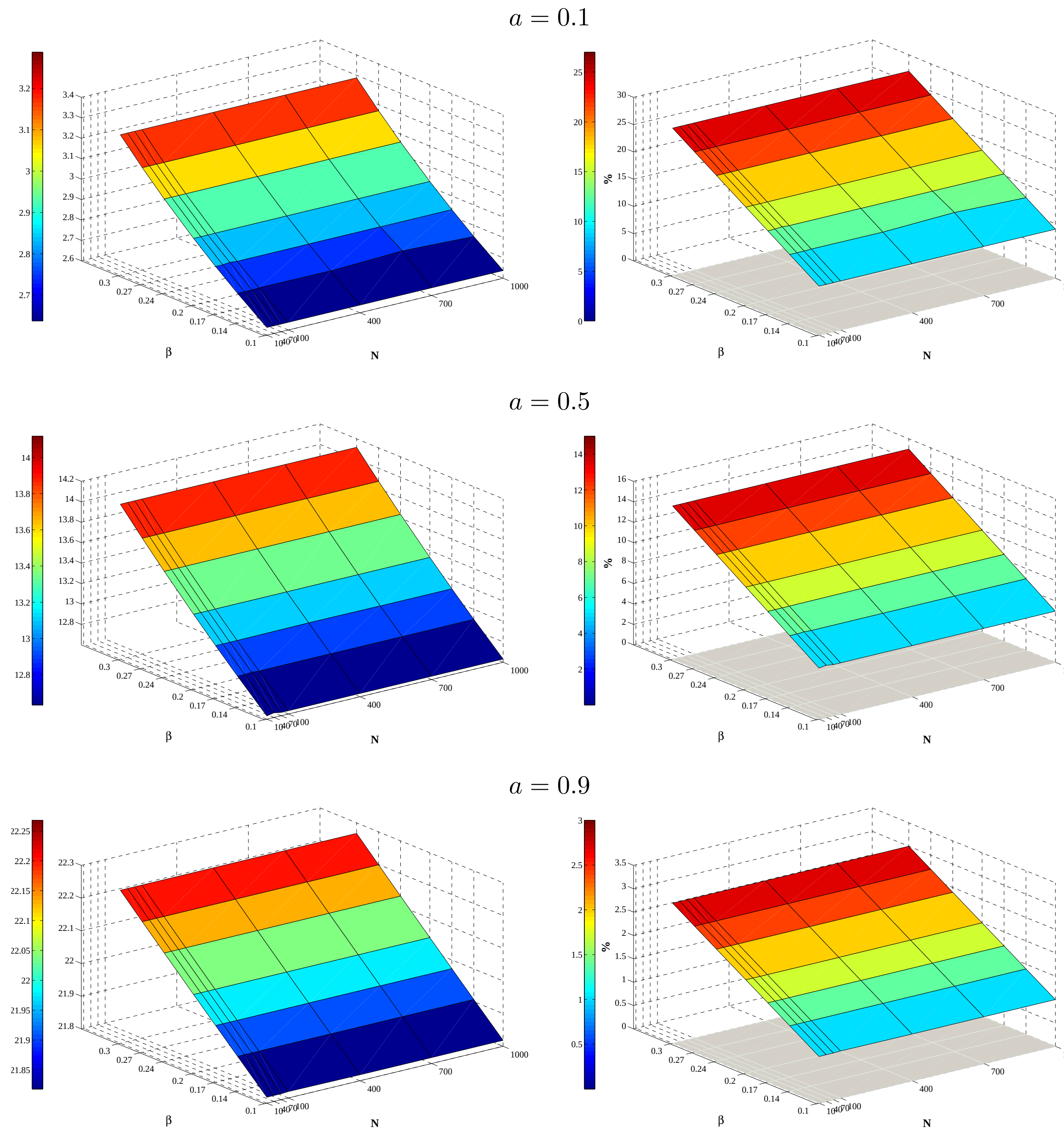
LS LS - D (%)



Notes: (1) LS = lump-sum, D = Distorting. (2) Nelder-Mead is used to solve these scenarios. (3) Differences are expressed in percent terms. (4) Grey surface helps to distinguish between positive and negative values.

Figure 9: Factor-augmenting. Cobb-Douglas utility function. Variable: Labor supply

LS | LS - D (%)



Notes: (1) LS = lump-sum, D = Distorting. (2) Nelder-Mead is used to solve these scenarios. (3) Grey surface helps to distinguish between positive and negative values.