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On the optimal management of teams under budget constraints*

Dunia López-Pintado[†] **Juan D. Moreno-Ternero**[‡]

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Abstract

We study optimal wage schemes for teams, under the presence of budget constraints, in a model in which agents' effort decisions are mapped into the probability of the team's success. We show that (first-best) efficiency can only be attained with complex contracts that are vulnerable to ex post manipulations and off-equilibrium path violations of the budget constraints. Within the domain of simple (and budget-balanced) contracts, an interesting scheme, which treats equal members of the team unequally, emerges as optimal.

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1 Introduction

The design of wage schemes is central to almost any debate on the optimal functioning of organizations. In this paper, we explore that issue in a basic and common model of team production in which agents' effort decisions are mapped into the probability of the team's success. Our aim is to scrutinize the effect of budget constraints in such context.

We build on a model of team production developed by Winter (2004). More precisely, imagine a project that has to be managed by a team of (risk-neutral) agents. Each agent decides simultaneously (i.e., without observing other agents' decisions) whether to exert effort or not in order to perform their tasks. Exerting effort is a costly action and the higher the skill, the lower the cost. The overall project succeeds with a probability which is an increasing function (known as a *technology*) of the number of agents exerting effort. The success of the project yields proceeds for the (risk-neutral) principal, who aims to maximize expected benefits. The principal, who knows each agent's skill and observes each agent's effort, is committed to limited liability while designing wage schemes. She also faces a budget constraint: total payment cannot exceed the proceeds from the project.

We start showing that the *first best*, i.e., the optimal (expected) benefit for the principal, subject only to individual rationality of the agents, is easily achievable. To do so, the principal only needs to offer a wage scheme guaranteeing each deserving agent (within the *optimal* subset of the team) the cost of exerting effort. Such a scheme, however, is not contingent on the success of the project and, therefore, may not satisfy the budget constraint.

We then move to explore schemes that are contingent on the success of the project. Among them, we focus first on those that are *simple* and base payments only on own effort choice. We show that, for those schemes, the first best is not achievable. More precisely, we show that the optimal simple contingent (budget-balanced) wage scheme typically falls short of the first-best outcome. It turns out that such a scheme involves an endogenous hierarchy within the team. One agent is induced to exert effort assuming no other peer is doing so. Another agent is paid enough to make her exert effort when one other agent does so as well, etc. A consequent feature is that equal agents are treated unequally. That is, two deserving agents with the same qualification will end up receiving different wages (depending on the position they occupy in the endogenous hierarchy).

We also show that, if we enrich the analysis to *complex* contingent schemes that base payments on the whole effort profile, the first best is achievable. Nevertheless, all first-best schemes suffer from two shortcomings. On the one hand, they are subject to *ex post manipulations*, by which we mean that the principal might benefit from reporting a false effort decision of an agent (or group of agents) to the other members of the team. On the other hand, they do not necessarily obey the off-equilibrium path budget constraints.

The closest reference to our contribution is Winter (2004). He analyzes a very similar model of team production in which agents' (simultaneous) effort decisions are also mapped into the probability of the team's success. Two crucial differences, however, exist with respect to our model. First, effort choices are not observable, which makes the moral-hazard problem the priority of the analysis. Second, there is no utility function for the principal and it is simply assumed that the principal's aim is to make all agents exert effort, whereas our focus is to determine the optimal subset of agents (within the team) exerting effort, as well as their wages. Winter's main result is that, in order to make all agents exert effort, the principal must discriminate among identical agents, provided technology functions exhibit *increasing returns to scale*. We too get a similar result. More precisely, we show that, with simple contingent contracts (the closest specification to Winter's setting), the optimal scheme (i.e., the one maximizing the principal's expected utility) also amounts to discriminate among identical agents. As we shall show later, this feature occurs in our model without imposing additional conditions whatsoever on the technology functions. Nevertheless, our result cannot be seen as a generalization of Winter's result, as there is not moral hazard in our model.

Our work also relates to the extensive literature on optimal compensation schemes in partnerships with unobservable effort and subject to budget balance. In a seminal contribution of this literature, Holmstrom (1982) introduces a model in which a group of agents produce an output that depends deterministically on the unobservable actions taken by the agents. Given a realized output, a sharing rule divides the output among the agents. Holmstrom's main result says that, if budget balance is required, the efficient output cannot be sustained in a Nash equilibrium. However, he also shows that if the budget balancing constraint is relaxed, efficiency can be sustained through the imposition of group penalties. In the wake of Holmstrom (1982), there have been a number of attempts to challenge the conclusion that partnerships are inefficient forms of organization because the partners cannot solve their moral hazard problem. For instance, Rasmusen (1987) shows that efficient production is implementable, when agents are risk averse, by the use of random punishments. Legros and Matsushima (1991) provide necessary and sufficient conditions for sustaining efficiency by balanced transfer rules, when the output is stochastic and side payments are allowed. Legros and Matthews (1993) show that budget balanced sharing rules can implement nearly efficient outcomes in a wide variety of games through the use of mixed strategies. However, in approximating the efficient output arbitrarily closely, the punishments used to enforce the equilibrium become arbitrarily large. Thus, this equilibrium does not exhibit limited liability as the equilibrium expected output approaches the efficient output. Miller (1997) departs from Holmstrom's original model upon assuming that one (and only one) agent observes the actions taken by a subset of the other agents and issues a report conditional on that observation. Miller's main result says that whenever the observing agent can see at least one other agent's action, efficiency, limited liability, and budget balance can be achieved simultaneously. A related approach is taken by Ma (1988), who shows that when the output is stochastic and agents' actions are mutually observable (although unobservable to the principal), there exists a mechanism that implements the efficient output as the unique subgame perfect equilibrium of the game.

A partnership is characterized by joint ownership and by sharing of the final output among the partners. Thus, the issue is to design sharing rules that, ideally, are budget balanced and induce agents achieve the efficient output. In our model, however, the final output goes for the principal, who selects the compensation scheme for the members of the team. Ideally, such a scheme should also induce agents achieve the efficient (first-best) output. The budget constraint forces the principal to make payments contingent on the final output. In contrast to the above-mentioned literature, our model assumes that agents' effort is perfectly observed by the principal (or, alternatively, the monitoring cost is negligible) whereas agents do not observe their peers' actions. Thus, there is neither moral hazard nor adverse selection in our model, which allows us to place the effect of budget constraints under further scrutiny. As mentioned above, we obtain that (first-best) efficiency, limited liability, and budget balance can be achieved simultaneously in our model. Nevertheless, the three goals combined are only obtained through (complex contingent) contracts that are vulnerable to ex post manipulations and off-equilibrium path violations of the budget constraint.

Our results crucially rely on two assumptions: equilibrium uniqueness and simultaneous actions. The former amounts to consider only wage schemes whose resulting games have a unique (pure strategy) Nash equilibrium.[1] The latter amounts to assume that members of the team take their effort decisions simultaneously. As we shall show later, it turns out that dispensing with any of the two assumptions, i.e., allowing for schemes generating multiple

[1]Regarded as a standard requirement in the implementation literature, uniqueness of equilibrium has obtained surprisingly little attention in the literature on partnerships.

equilibria, or considering sequential effort decisions, permits to achieve (first-best) efficiency, limited liability, and budget balance (on and off the equilibrium path) simultaneously, through simple contracts in which payments only depend on own effort choice and the realized output, and that are immune to ex post manipulations.[2]

The rest of the paper is organized as follows. Section 2 introduces the model and basic definitions. Section 3 contains the main results of the paper. Section 4 explores two extensions of the benchmark model. Finally, Section 5 concludes.

2 The model

There is a project involving n activities performed by n agents of a team, which we denote as $N = \{1, \ldots, n\}$. Each agent $i \in N$ decides simultaneously whether to exert effort (invest) or not towards the performance of her activity. We denote by $\delta_i \in \{0, 1\}$ the effort decision of agent i, where $\delta_i = 1$ (0) if agent i does (not) exert effort. The cost of exerting effort of agent i is $c_i \geq 0$. This parameter is to be interpreted as a sign of the agent's skill (i.e., the lower the cost of exerting effort, the higher the skill). We assume, without loss of generality, that $c_1 \leq c_2 \leq \cdots \leq c_n$. An agent will invest if and only if her expected benefits (i.e., her expected wage minus her cost) are non-negative. The project's technology is a non-decreasing function $p : \{0, 1, \ldots, n\} \to [0, 1]$ specifying the probability that the project succeeds, for any given number of agents exerting effort.[3] In doing so, we are implicitly assuming that agents' efforts are equally valuable for the success of the project.

A principal observes agents' skills and effort decisions, as well as the outcome of the project, and designs the wage scheme for the team with the aim of maximizing her benefits. We denote by $\gamma = 1$ (0) the event in which the project is (not) successful.[4] The value of γ will be considered as a public signal, observed not only by the principal, but also by each agent. On the other hand, agents will neither observe their peers' effort decisions nor be able to infer them ex post.

Let $\beta > 0$ denote the proceeds for the principal if the project is successful and assume that an unsuccessful project yields 0. Agents are subject to limited liability, which means that

[2]This is reminiscent of the approach taken by Strausz (1999) who shows that sequentiality can mitigate the inefficiency of partnerships. It is also reminiscent of Winter (2006) who studies sequential optimal incentive schemes in organizations where agents perform their tasks sequentially and obtains further insights to some of the features in Winter (2004).

[3]For ease of exposition, we assume that $p(1) > 0$.

[4]With this notation, we have that, for each $k \in \{0, 1, \ldots, n\}$, $p(k) = \Pr(\gamma = 1 | \sum_{i \in N} \delta_i = k)$.

the principal cannot impose negative wages on them.[5] The principal is subject to a budget constraint, which means that the (overall) offered wages could not exceed the revenues of the team.

Formally, for each $i \in N$, let $\omega_i(\delta_i, \gamma, \delta_{-i})$ denote agent i's wage. As we can infer from here, i's wage might not only depend on i's effort decision, and the success (or failure) of the project, but also on her peers' effort decisions. The resulting (expected) benefits for each agent are then constructed as follows:

$$\pi_i\left(\delta_i, \delta_{-i}\right) = p\left(\sum_{j \in N} \delta_j\right) \omega_i(\delta_i, 1, \delta_{-i}) + \left(1 - p\left(\sum_{j \in N} \delta_j\right)\right) \omega_i(\delta_i, 0, \delta_{-i}) - \delta_i c_i.$$

Note that, as effort decisions are observed by the principal, it is natural to assume that only deserving agents are rewarded. Formally,

(A1) **Agent Limited Liability**: $\omega_i(0, \gamma, \delta_{-i}) = 0$, for all $(\gamma, \delta_{-i}) \in \{0,1\}^n$.

If the budget constraint forces the principal to commit to an overall cost of the wage scheme below the proceeds obtained from the project, it is also natural to assume that no agent earns a positive wage if the project is unsuccessful. Formally,

(A2) **Team Limited Liability**: $\omega_i(\delta_i, 0, \delta_{-i}) = 0$, for all $(\delta_i, \delta_{-i}) \in \{0,1\}^n$.

Thus, (expected) benefits amount to

$$\pi_i\left(\delta_i, \delta_{-i}\right) = \delta_i \left(p\left(\sum_{j \in N} \delta_j\right) \omega_i(\delta_i, 1, \delta_{-i}) - c_i\right).$$

The issue for the principal is, ultimately, to design an optimal wage scheme $\omega = \{\omega_i(\cdot,\cdot,\cdot)\}_{i \in N}$, i.e., a scheme maximizing her benefits. Now, any wage scheme announced by the principal defines a game. In such a game, the members of the team decide whether to exert effort or not, and their benefits, as described above, will typically depend not only on their individual decisions, and the success (or failure) of the project, but also on their peers' decisions explicitly. Formally, let $\omega = \{\omega_i(\cdot,\cdot,\cdot)\}_{i \in N}$ be a given wage scheme and denote by $G^{\omega} = \{N, \{0,1\}^n, \{\pi_i\}_{i \in N}\}$ the resulting game.

We shall restrict our attention to those schemes whose resulting games have a unique (pure strategy) Nash equilibrium. In doing so, we get rid of the strategic uncertainty, univocally

[5]Limited liability of the agents may arise from workers' having the freedom to quit or from institutional constraints such as laws banning firms' exacting payments from workers. In any case, dropping this assumption would not alter the message of our results.

determining the outcome of the game and, hence, the (expected) utility for the principal derived from such a scheme.[6]

(A3) **Equilibrium Uniqueness**: G^ω has a unique (pure strategy) Nash equilibrium.

Formally, let $\delta^\omega = (\delta_1^\omega, \cdots, \delta_n^\omega)$ denote the (pure strategy) Nash equilibrium of G^ω. Let K^ω denote the set of agents exerting effort in such an equilibrium, i.e., $K^\omega = \{i \in N : \delta_i^\omega = 1\}$, and k^ω its cardinality, i.e., $k^\omega = \sum_{i\in N} \delta_i^\omega$. We then complement assumption (A2) to reflect the principal's budget constraint as follows:

(A4) **Budget Balance**: $\sum_{i\in K^\omega} \omega_i(1, 1, \delta_{-i}^\omega) \leq \beta$

Let Ω denote the set of wage schemes satisfying assumptions (A1)-(A4). Then, provided the principal obeys the vNM axioms, the principal aims to solve the program

$$\max_{\omega\in\Omega} \Pi(\omega), \tag{1}$$

where

$$\Pi(\omega) = p(k^\omega) \cdot \left(\beta - \sum_{i\in K^\omega} \omega_i(1, 1, \delta_{-i}^\omega)\right). \tag{2}$$

If ω^* denotes a solution to the above program, then ω^* is referred to as an *optimal scheme*, K^{ω^*} as the corresponding *optimal team*, and k^{ω^*} as the *optimal size of the team*. In what follows, and for ease of exposition, we shall assume that any optimal scheme ω^* satisfies $\omega_i^*(\delta_i, \gamma, \delta_{-i}) = 0$ for all $i \notin K^{\omega^*}$. As we shall see later, the optimal teams will always comprise the most skilled agents. Formally, it will always be the case that $K^{\omega^*} = \{1, \ldots, k^{\omega^*}\}$.

We now introduce some schemes that will play an important role in our analysis. For ease of exposition, we start considering schemes that are not contingent on the success of the project and, hence, might not obey (A4). Formally, let $k \in \{1, \ldots, n\}$. Then, the canonical $k-$**non-contingent wage scheme** is $\omega^{knc} = \{\omega_i^{knc}(\cdot)\}_{i\in N}$, where

$$\omega_i^{knc}(\delta_i) = \begin{cases} \delta_i c_i & \text{if } i \in \{1, \ldots, k\}, \\ 0 & \text{otherwise.} \end{cases}$$

The schemes in Ω will obviously be contingent on the success of the project. We single out first those that base payments only on own effort choice and the outcome of the project, and not on peers' effort choices. Formally, $\omega_i(\delta_i, \gamma, \delta_{-i}) = \omega_i(\delta_i, \gamma, \delta'_{-i})$ for all $i \in N$ and $\delta_{-i}, \delta'_{-i} \in \{0, 1\}^{n-1}$ for all $i \in N$. We shall refer to them as simple contingent wage schemes

[6] We shall elaborate further on this solution concept in Section 4.1.

and denote their set as $\Omega_s \subset \Omega$. An instance is the following. Let $k \in \{1, \ldots, n\}$. Then, the canonical $k-$**simple (contingent) symmetric wage scheme** is $\omega^{kss} = \{\omega_i^{kss}(\cdot,\cdot)\}_{i\in N}$, where

$$\omega_i^{kss}(\delta_i, \gamma) = \begin{cases} \gamma\delta_i \frac{c_i}{p(k)} & \text{if } \ i \in \{1, \ldots, k\}, \\ 0 & \text{otherwise.} \end{cases}$$

For reasons that will become clear later in the text, it might be necessary to provide the following alternative asymmetric scheme to the previous one. Formally, let $k \in \{1, \ldots, n\}$. Then, the canonical $k-$**simple (contingent) asymmetric wage scheme** is $\omega^{ksa} = \{\omega_i^{ksa}(\cdot,\cdot)\}_{i\in N}$, where

$$\omega_i^{ksa}(\delta_i, \gamma) = \begin{cases} \gamma\delta_i \frac{c_i}{p(i)} & \text{if } \ i \in \{1, \ldots, k\}, \\ 0 & \text{otherwise.} \end{cases}$$

The set Ω will also comprise schemes for which payments are based on the whole effort profile. We shall refer to them as complex contingent wage schemes. An instance would be the following. Let $k \in \{1, \ldots, n\}$. Then, the canonical $k-$**complex (contingent) wage scheme** is $\omega^{kcc} = \{\omega_i^{kcc}(\cdot,\cdot,\cdot)\}_{i\in N}$, where

$$\omega_i^{kcc}(\delta_i, \gamma, \delta_{-i}) = \begin{cases} \gamma\delta_i \frac{c_i}{p(\sum_{j\in N}\delta_j)} & \text{if } \ i \in \{1, \ldots, k\}, \\ 0 & \text{otherwise.} \end{cases}$$

3 The main results

3.1 The first best

In order to frame our study, we start exploring the reference case in which (A4) is not imposed. Let Ω^* denote the set of wage schemes satisfying assumptions (A1)-(A3). The following result summarizes our findings in this case.

Proposition 1 *Let $\widetilde{k}$ denote the solution to the program*

$$\max_{k=0,1,\ldots,n} \ p(k)\cdot\beta - \sum_{i=1}^{k} c_i.$$

Then $\omega^{\widetilde{k}nc}$ is an optimal scheme among those in Ω^*.

Proof. It is straightforward to show that $K^{\omega^{\widetilde{k}nc}} = \{1, \ldots, \widetilde{k}\}$ and, thus, $\Pi(\omega^{\widetilde{k}nc}) = p(\widetilde{k})\cdot\beta - \sum_{i=1}^{\widetilde{k}} c_i$. Assume, by contradiction, that there is a scheme $\omega \in \Omega^*$ such that $\Pi(\omega) > \Pi(\omega^{\widetilde{k}nc})$. Then, as the agents in K^ω should be granted (in expected terms) at least their costs of exerting effort, we have

$$\Pi(\omega) \le p(k^\omega)\cdot\beta - \sum_{i\in K^\omega} c_i.$$

Now, by definition of $\widetilde{k}$,

$$p(k^{\omega}) \cdot \beta - \sum_{i \in K^{\omega}} c_i \leq p(k^{\omega}) \cdot \beta - \sum_{i=1}^{k^{\omega}} c_i \leq p(\widetilde{k}) \cdot \beta - \sum_{i=1}^{\widetilde{k}} c_i = \Pi(\omega^{\widetilde{k}nc}),$$

a contradiction. ■

The previous proposition tells us that if the principal faces no budget constraint, then she cannot do better than by rewarding agents within the optimal team with their cost of exerting effort. We will refer to $\Pi(\omega^{\widetilde{k}nc}) = p(\widetilde{k}) \cdot \beta - \sum_{i=1}^{\widetilde{k}} c_i$ as the first-best outcome for the principal, and to a scheme guaranteeing such outcome as a scheme achieving first-best efficiency or, simply, a *first-best scheme.*

It is worth mentioning that $\omega^{\widetilde{k}nc}$ induces a unique Nash equilibrium thanks to our tie-breaking rule by which agents exert effort when they are indifferent between doing so and shirking. An alternative option to imposing this tie-breaking rule would have been to consider the following formal definition of an optimal mechanism: ω is an optimal mechanism if (1) there exists no other mechanism yielding more expected benefits for the principal and (2) for any $\varepsilon > 0$, $\{\omega + \varepsilon\}_{i \in K^{\omega}}$ is an investment-inducing mechanism, i.e., the unique Nash equilibrium of the corresponding game is the profile in which all agents in K^{ω} exert effort. This innocent technical caveat is needed because rewards take continuous values.[7]

In what follows, we consider schemes that reflect the existence of budget constraints upon making wages contingent on the success of the project, i.e., we move from Ω^* to Ω. Our aim is to explore whether first-best schemes might exist among them. We start focusing on what we called simple contingent schemes (Ω_s) for which the wage of an agent is only determined by the effort decision of such agent, and the outcome of the team (to be interpreted as a public signal), but not (explicitly) by her peers' effort decisions. We then move to the general case in which contingent schemes might explicitly include peers' effort decisions.

3.2 The simple contingent case

Our main result within this section is the following.

Theorem 1 *Let $\bar{k}$ denote the solution to the program*

$$\max_{k=0,1,\ldots,n} \; p(k) \cdot \left(\beta - \sum_{i=1}^{k} \frac{c_i}{p(i)} \right). \tag{3}$$

Then, $\omega^{\bar{k}sa}$ is the optimal scheme among those in Ω_s.

[7] A similar proposal is made by Winter (2004).

Proof. Let us first consider a simple contingent wage scheme for which the unique Nash equilibrium of the corresponding game is the profile in which all agents exert effort. Formally, let $\omega = (\omega_1(\delta_1, \gamma), \omega_2(\delta_2, \gamma), \ldots, \omega_n(\delta_n, \gamma)) \in \Omega_s$ be such that $(\delta_1^\omega, \delta_2^\omega, \ldots, \delta_n^\omega) = (1, 1, \ldots, 1)$. Then, in particular, $(\delta_1, \delta_2, \ldots, \delta_n) = (0, 0, \ldots, 0)$ is not a Nash equilibrium for that game, which implies that there exists, at least, an agent $i_1 \in N$ wanting to deviate by investing. In other words, there exists $i_1 \in N$ for which $p(1)\omega_{i_1}(1, 1) \geq c_{i_1}$.[8] Equivalently,

$$\omega_{i_1}(1, 1) \geq \frac{c_{i_1}}{p(1)}.^{9} \tag{4}$$

Let us consider now the profile $(\delta_1, \delta_2, \ldots, \delta_n)$ in which $\delta_{i_1} = 1$ and $\delta_i = 0$ otherwise.[10] As this profile cannot be an equilibrium either, it follows that there exists an agent $i_2 \in N \setminus \{i_1\}$ wanting to deviate by investing. In other words, there exists $i_2 \in N \setminus \{i_1\}$ for which $p(2)\omega_{i_2}(1, 1) \geq c_{i_2}$.[11] Equivalently,

$$\omega_{i_2}(1, 1) \geq \frac{c_{i_2}}{p(2)}.$$

This argument can be subsequently repeated for the remaining profiles to show, in the end, that there exists some permutation π of the set $\{1, \ldots, n\}$, for which

$$(\omega_1(1, 1), \omega_2(1, 1), \ldots, \omega_n(1, 1)) \geq (\omega_1^\pi(1, 1), \ldots, \omega_n^\pi(1, 1)) = \left(\frac{c_1}{p(\pi(1))}, \ldots, \frac{c_n}{p(\pi(n))}\right).$$

Now, as p is non-decreasing, and $c_1 \leq c_2 \leq \cdots \leq c_n$, it is straightforward to show that

$$\sum_{i=1}^{n} \frac{c_i}{p(i)} \leq \sum_{i=1}^{n} \omega_i^\pi(1, 1),$$

for any permutation π, which shows that

$$(\omega_1(1, 1), \omega_2(1, 1), \ldots, \omega_n(1, 1)) = \left(\frac{c_1}{p(1)}, \ldots, \frac{c_n}{p(n)}\right),$$

is *the* optimal scheme among those in Ω_s to guarantee that all agents in N exert effort (together with the schemes obtained by permuting indices corresponding to agents with a same cost, which would also generate the same overall wage).

[8] Note that if i_1 deviates there would only be one agent exerting effort, which would make $p(1)$ the probability of success and therefore the probability for agent i_1 of getting a positive wage $\omega_{i_1}(1, 1)$.

[9] Note that this condition guarantees that exerting effort is a dominant strategy for agent i_1.

[10] Note that (4) not only guarantees that the profile in which no agent exerts effort is not a Nash equilibrium, but also that no profile in which only one agent (different from i_1) exerts effort constitutes a Nash equilibrium either.

[11] Note that if i_2 deviates there would be only two agents exerting effort, which would make $p(2)$ the probability of success and therefore the probability for agent i_2 of getting a positive wage $\omega_{i_2}(1, 1)$.

Thus, it follows that the optimal scheme among those in Ω_s to guarantee that agents in a certain subset $K \subseteq N$ (and only them) exert effort is given by

$$\omega_i(1,1) = \frac{c_i}{p(\sigma(i))},$$

for all $i \in K$, where $\sigma(i)$ denotes the rank of i in K, and $\omega_i(1,1) = 0$ for all $i \notin K$. Therefore, among the sets with the same cardinality of K, the *optimal* one for the principal would be $\{1, 2, \ldots, k\}$. Consequently, the objective function (2) translates into

$$p(k) \cdot \left(\beta - \sum_{i=1}^{k} \frac{c_i}{p(i)} \right),$$

as desired. ■

A straightforward consequence from the statement of the above theorem is the following:

Corollary 1 *In general, no simple contingent (budget-balanced) wage scheme is first best.*

Proof. It suffices to compare the objective functions from the statements of Proposition 1 and Theorem 1. Formally, it is straightforward to show that

$$p(k) \cdot \left(\beta - \sum_{i=1}^{k} \frac{c_i}{p(i)} \right) \leq p(k) \cdot \beta - \sum_{i=1}^{k} c_i,$$

with a strict inequality for any $k > 1$. Thus, unless $\bar{k} = \widetilde{k} \leq 1$,

$$p(\bar{k}) \cdot \left(\beta - \sum_{i=1}^{\bar{k}} \frac{c_i}{p(i)} \right) < p(\widetilde{k}) \cdot \beta - \sum_{i=1}^{\widetilde{k}} c_i,$$

from where the statement follows.[12] ■

It also follows from Theorem 1 that the optimal way of managing a team by a budget-balanced principal, restricted to simple contingent schemes, would be designing a cascade that generate a sort of endogenous hierarchy within the team. More precisely, one agent is induced to exert effort assuming no other peer is doing so. Another agent is paid enough to make her exert effort when one other agent does so as well, etc. A consequent feature of such a scheme is that it treats equal agents unequally. That is, two deserving agents with the same qualification

[12] One might argue that the scheme $\omega^{\bar{k}sa}$ is reminiscent of an incentive compatible contract with hidden action: as the budget constraint prevents the principal from offering a compensation to the agent if the team is not successful, the principal is forced to offer a higher compensation in case of success of the project, so that in expectation it is worth it for the agent to exert effort.

will end up receiving different wages (actually depending on the position they occupy in the endogenous hierarchy just described). In other words, the optimal (budget-balanced) simple contingent wage scheme is extremely inegalitarian, as it violates the most fundamental notion of impartiality.

Corollary 2 *The optimal simple contingent (budget-balanced) wage scheme violates equal treatment of equals.*

The previous feature is also obtained by Winter (2004) who shows that, when effort is not observable, and the principal's aim is to induce all agents exert effort, then the optimal way to do so would be upon discriminating among agents, even when they are identical and act simultaneously. Alternative explanations as to why organizations might treat equal agents unequally have also been recently considered (e.g., Yildrim, 2007; Dhillon and Herzog-Stein, 2009). The intuition behind the role of discrimination in the above scheme is also related to some common strategies in trade contracts (e.g., Cabral et al., 1999; Segal, 2003).

3.3 The complex contingent case

Simple contingent wage schemes might partially be justified on the grounds that agents neither observe their peers' effort decisions nor are able to infer them ex post, and therefore would not find credible contingent contracts which depend on additional aspects to their individual effort choices and the outcome of the project (which is indeed observed by all agents and thus could be interpreted as a public signal). Nevertheless, one might think of alternative contexts of team production in which more flexibility is allowed while designing wage schemes, and additional information (e.g., private signals that only the principal observes) might also be considered. If so, complex contingent wage schemes would be meaningful. It turns out that, contrary to what happened with simple contingent schemes, complex contingent schemes might be first-best schemes.

Proposition 2 *The scheme $\omega^{\widetilde{k}cc}$ is a first-best (contingent) scheme.*

Proof. It is straightforward to show that $K^{\omega^{\widetilde{k}cc}} = \{1, \ldots, \widetilde{k}\}$. Then, the statement simply follows from the fact that the program

$$\max_{k=0,1,\ldots,n} p(k) \cdot \left(\beta - \sum_{i=1}^{k} \frac{c_i}{p(k)} \right)$$

coincides with the program of Proposition 1. Thus, $\Pi(\omega^{\widetilde{k}cc}) = \Pi(\omega^{\widetilde{k}nc})$, as desired. ■

One would be tempted to infer from the above proposition that the effect of budget constraints is indeed negligible for a principal with enough flexibility to design contingent schemes (in particular, to consider the scheme $\omega^{\widetilde{k}cc}$). The rest of the section conveys two caveats to such a statement.

3.3.1 Ex post manipulations

As mentioned above, the corresponding optimal team for the scheme $\omega^{\widetilde{k}cc}$ is $K^{\omega^{\widetilde{k}cc}} = \{1, \ldots, \widetilde{k}\}$. Let us assume that an agent $i \in K^{\omega^{\widetilde{k}cc}}$ deviates from the equilibrium and ends up shirking. Assume, too, that the project ends up being successful. Then, the scheme $\omega^{\widetilde{k}cc}$ would grant the principal the following payoffs:

$$\beta - \sum_{j \in K^{\omega^{\widetilde{k}cc}} \setminus \{i\}} \frac{c_j}{p(\widetilde{k}-1)}.$$

Had no such deviation occurred, however, the principal would be obtaining

$$\beta - \sum_{j=1}^{\widetilde{k}} \frac{c_j}{p(\widetilde{k})}.$$

Thus, if the technology function p is such that

$$\sum_{j \in K^{\omega^{\widetilde{k}cc}} \setminus \{i\}} \frac{c_j}{p(\widetilde{k}-1)} > \sum_{j=1}^{\widetilde{k}} \frac{c_j}{p(\widetilde{k})},$$

then the principal would benefit from hiding the shirking decision of i to the other members of the optimal team. In other words, there would be an incentive to manipulate the outcome ex post.[13]

In general, we say that a wage scheme ω is **subject to (ex post) manipulation** if, for some technologies, the principal could benefit from hiding the shirking of an agent (or group of agents) within the optimal team. Formally, if there exists $K \subset K^{\omega}$, such that

$$\beta - \sum_{j \in K^{\omega} \setminus K} \omega_j(\delta'_j, 1, \delta'_{-j}) \leq \beta - \sum_{j \in K^{\omega}} \omega_j(\delta^{\omega}_j, 1, \delta^{\omega}_{-j}),$$

where $\delta'_j = 1$ if $j \in K^{\omega} \setminus K$ and 0 otherwise.

[13] This is, by no means, a specific case. Think, for instance, of the natural case in which $p(k) = \alpha^{n-k}$ for some $\alpha \in (0, 1)$ (e.g., Winter, 2004; 2006) and $c_1 = c_n$. It turns out that if $\alpha < 1/e$ then $\omega^{\widetilde{k}cc}$ is ex post manipulable.

The next result shows that $\omega^{\widetilde{k}cc}$ is, by no means, unique in being subject to ex post manipulation.[14]

Theorem 2 *All first-best contingent wage schemes are subject to ex post manipulation.*

Proof. Let ω be a first-best contingent scheme. It is straightforward to show that the optimal team for this scheme must coincide with the optimal team for the non-contingent scheme $\omega^{\widetilde{k}nc}$. Formally, $K^{\omega} = \{1, \ldots, \widetilde{k}\}$ and $\delta^{\omega} = (\overbrace{1, \ldots, 1}^{\widetilde{k}}, 0, \ldots, 0)$. It follows from here that

$$\omega_i(1, 1, \delta^{\omega}_{-i}) = \frac{c_i}{p(\widetilde{k})},$$

for all $i \in K^{\omega}$.[15]

Similarly, the profile in which all agents shirk is not an equilibrium of the game G^{ω}. Thus, there exists an agent $i_0 \in K^{\omega}$ such that

$$\omega_{i_0}(1, 1, 0_{-i_0}) \geq \frac{c_{i_0}}{p(1)}.$$

Let p be a technology satisfying that

$$\frac{c_1}{p(1)} > \sum_{i=1}^{\widetilde{k}} \frac{c_i}{p(\widetilde{k})}.$$

Then, it is straightforward to show that

$$\beta - \omega_{i_0}(1, 1, 0_{-i_0}) \leq \beta - \sum_{j \in K^{\omega}} \omega_j(1, 1, \delta^{\omega}_{-j}).$$

This shows that ω is subject to ex post manipulations, as expected. ■

A caveat to Theorem 2 is worth mentioning. The theorem is indeed showing that, for some technologies, the first-best (constrained) wage schemes are subject to ex post manipulations. Equally true is that for other technologies, the first-best (constrained) wage schemes are immune from that feature. This suggests a natural procedure to *rank* efficient (and budget-balanced) schemes by comparing the set of technologies for which they are immune to ex post manipulations.

To conclude with this section, it is worth commenting about the contractibility assumptions behind the concept of ex post manipulation, which are somehow at odds with the standard

[14]On the other hand, it is straightforward to show that all simple contingent schemes are immune to ex post manipulations.

[15]Note that an inequality would either prevent that profile from being an equilibrium, or the scheme ω from being first best.

approach in contract theory. In contract theory, a contractible event is generally assumed to be an event that is verifiable in a court. In this view, either the agent's effort is contractible or it is not. We, nevertheless, make a distinction between verifying in a court a low or a high effort decision. More precisely, we assume that an agent can show evidence in a court of her (high) effort decision, which would prevent the principal from cheating about it. On the other hand, as agents' decisions are not mutually observable in our model, agent i could not stand in a court to denounce a false report by the principal of agent j's decision. In other words, an agent can only resort to a court to defend her effort decision, but not to accuse another agent of a shirking decision. Thus, the principal cannot falsely report than an agent has chosen low effort (shirking), but she might well report (falsely) that an agent has chosen high effort.

3.3.2 Off-path budget constraint

Another caveat to first-best (contingent) schemes comes from the fact that we can only guarantee they fully obey the budget constraint on the equilibrium path. More precisely, think of the scheme $\omega^{\widetilde{k}cc}$, which, as we know from Proposition 2, is a first-best (contingent) scheme. Let us assume that an agent $i \in K^{\omega^{\widetilde{k}cc}}$ deviates from the equilibrium and ends up shirking. Assume, too, that the project ends up being successful. Then, the scheme $\omega^{\widetilde{k}cc}$ would grant the principal the following payoffs:

$$\beta - \sum_{j \in K^{\omega^{\widetilde{k}cc}} \setminus \{i\}} \frac{c_j}{p(\widetilde{k}-1)},$$

which might well be negative, for some technology functions p. Thus, $\omega^{\widetilde{k}cc}$ does not always obey the budget constraint off the equilibrium path. The next result shows that $\omega^{\widetilde{k}cc}$ is not unique in this respect either.[16]

Theorem 3 *All first-best contingent wage schemes are subject to off-path violations of the budget constraint.*

Proof. Let ω be a first-best (contingent) scheme. As shown in the proof of Theorem 2, there exists an agent $i_0 \in K^\omega$ such that

$$\omega_{i_0}(1, 1, 0_{-i}) \geq \frac{c_{i_0}}{p(1)}.$$

[16] On the other hand, it is straightforward to show that all simple contingent schemes obey the budget constraint off the equilibrium path too. This is actually a consequence of the fact that those schemes are immune to ex post manipulations.

Let p be a technology satisfying that

$$\beta < \frac{c_1}{p(1)}.$$

If, for such a technology, all agents from the optimal team, except for i_0, would deviate (from the equilibrium path) then the principal would face the payoffs $\beta - \frac{c_{i_0}}{p(1)} < 0$. Thus, ω would violate at least one of the off-path budget constraints, as expected. ■

To summarize, the purpose of this section was twofold: first, to show that first-best efficiency can indeed be attained with complex contingent schemes, hence challenging the lessons derived from simple contingent schemes;[17] and second, to show that there exists a toll in adopting those first-best contingent schemes.

4 Further insights

We explore in this section how our analysis would change by relaxing two crucial assumptions over which our benchmark model relies.

4.1 Strategic uncertainty

As mentioned above, our benchmark model only considers wage schemes whose resulting games have a unique (pure strategy) Nash equilibrium (assumption A3). The reason for that was to get rid of the strategic uncertainty that would arise with the presence of more than one equilibrium (and, thus, more than one option to materialize agents' coordination).

An alternative (less demanding) modeling option would be to allow for wage schemes whose resulting games have at least one (pure strategy) Nash equilibrium. An interpretation for this alternative option is that the principal is *optimistic* and believes that agents will coordinate on the *right* equilibrium. In other words, the principal is not concerned with the strategic uncertainty induced by the presence of multiple equilibria. Possible rationales for this feature would be to assume that the principal is able to pick her preferred equilibrium by acting as a mediator who coordinates the agents expectations, or that one equilibrium out of multiple can be naturally focal (e.g., taking into consideration potential pre-play communication).

We show next that this alternative modeling option alters our results substantially, as summarized in the following proposition.

[17] A similar feature has been recently shown in the context of a multitask moral hazard problem with partial effort observation (e.g., Zhao, 2008; Chen, 2010).

Proposition 3 *If strategic uncertainty is permitted, then there exists a simple contingent (budget-balanced) wage scheme which is first best, impartial, and immune to ex post manipulations.*

Proof. We start by noting that the first best would not change in this new setting in which strategic uncertainty is permitted. Consider then the simple contingent (budget-balanced) scheme $\omega^{\widetilde{k}ss}$. It is not difficult to show that $(\overbrace{1, \ldots, 1}^{\widetilde{k}}, 0, \ldots, 0)$ is an equilibrium of the game $G^{\omega^{\widetilde{k}ss}}$.[18] Thus, the principal would achieve the first-best outcome, provided agents coordinate in such equilibrium. It is also straightforward to show that $\omega^{\widetilde{k}ss}$ is impartial and immune to ex post manipulations. ■

It follows from Proposition 3 that the effect of the budget constraint could be mitigated at the cost of assuming strategic uncertainty in the design of contingent wage schemes. Nevertheless, this seemingly innocuous aspect actually constitutes a substantial cost: the alternative equilibrium (in which all agents shirk) that would typically arise under $\omega^{\widetilde{k}ss}$ is risk dominant and, thus, a more likely candidate to describe the potential coordination of the agents (e.g., Cabrales et al., 2010).

4.2 Hierarchical structures

Our benchmark model also assumes a flat organization for the team, i.e., all agents take their effort decisions simultaneously. This could be interpreted as assuming that communication among agents does not exist, perhaps reflecting geographical constraints. An alternative option would be to assume a hierarchical organization in which agents instead of performing their tasks simultaneously do so sequentially and, therefore, knowing the effort choices of their predecessors in the hierarchy. In such a case, the natural equilibrium notion to be used, while designing schemes, would be the *subgame perfect Nash* equilibrium. It turns out, as it happened with the case of strategic uncertainty, that this change also alters our results substantially, as summarized in the following proposition.

Proposition 4 *If the team has a hierarchical structure, then there exists a simple contingent (budget-balanced) wage scheme which is first best, impartial, and immune to ex post manipulations.*

[18] Typically, the profile in which all agents shirk will also be an equilibrium of this game and hence the strategic uncertainty. This was not the case for scheme $\omega^{\bar{k}sa}$, thanks to our tie-breaking rule.

Proof. We start by noting that the first best would not change in this new setting in which a hierarchical (instead of flat) structure is assumed for the team. Consider then, as in the proof of Proposition 3, the simple contingent (budget-balanced) scheme $\omega^{\widetilde{k}ss}$. It is not difficult to show that the unique subgame perfect Nash equilibrium outcome of the (sequential) game $G^{\omega^{\widetilde{k}ss}}$ is $(\overbrace{1, \ldots, 1}^{\widetilde{k}}, 0, \ldots, 0)$.[19] It follows from here that $\omega^{\widetilde{k}ss}$ is first best, impartial and immune to ex post manipulations. ■

Proposition 4 then shows that the effect of budget constraints would also be mitigated if principals were allowed to freely design the architecture of their teams.

5 Discussion

We have studied in this article the design of optimal wage schemes, under the presence of budget constraints, in a simple model of organization in which agents' effort decisions are mapped into the probability of the team's success. We have shown that the first best a principal would attain with no budget constraint can only be recovered under complex contracts, and at the price of vulnerability to *ex post* manipulations and to violations of the budget constraint off the equilibrium path. Such a finding crucially relies on two modeling assumptions regarding the structure of the team and the (lack of) strategic uncertainty. More precisely, if one assumes a hierarchical (rather than a flat) organization for the team, or permits strategic uncertainty, then first-best efficiency can be recovered with a simple contract, immune to ex post manipulations. Now, even though sequential production has been a common practice for the traditional production of most durable goods (e.g., Winter, 2006), it seems to be a less realistic assumption nowadays, where hierarchies are transforming themselves from top-down structures into more horizontal and collaborative ones (e.g., Friedman, 2007). As for permitting strategic uncertainty, which is a natural course of action in the literature on partnerships, the problem arises with the fact that the equilibrium in which all agents shirk would typically emerge as a coordination device among all existing equilibria (e.g., Cabrales et al., 2010).

Our model of team production departs from the standard literature on partnerships in its observability assumptions. In our case, and as opposed to that literature, the principal of the team observes each agent's effort decision, which are, on the other hand, not mutually observed by the agents themselves. In that sense, our model would fit better the case of firms subject to

[19]Note that the location of the agents in the hierarchy is irrelevant for this fact.

fragmentation, i.e., the breakdown of technology for producing some good into discrete parts that can be separated in space. Nowadays, and thanks to the Internet, many fragmentation processes might involve agents who do not communicate among themselves, but only report to a principal, who is monitoring each step of the process. In those cases, it is natural to assume that workers within the firm do not observe the effort choices of their peers, although they might observe the outcome of the team (the goods that are eventually produced and delivered by the firm). That is indeed the case of our model.

We have shown that, in our context, contracts relating only to an agent's individual effort decision and to the final outcome of the team (the public signal) never attain first-best efficiency (unless we change the two modeling assumptions mentioned above). On the other hand, they are immune to ex post manipulations (and hence obey off-path budget constraints too). It turns out that the optimal scheme under this specification involves an endogenous hierarchy, which treats equal agents unequally and thus violates impartiality. Imposing non-discrimination as a requirement of wage schemes (as it happens in most advanced democracies) would therefore add to the deadweight (efficiency) loss, as the optimal scheme would amount to modify the endogenous hierarchy by equalizing (to the top) the wages of equally deserving agents. Imposing further compensations, such as requiring a positive (albeit limited) discrimination in favor of the disabled (something also frequent in advanced democracies; e.g., the Americans with Disabilities Act), would increase even more the deadweight (efficiency) loss. For instance, requiring that the wage scheme be prioritarian (e.g., Moreno-Ternero and Roemer, 2006) which would mean in this context that no agent dominates another both in the wage obtained, as well as in the expected benefits earned.

Our analysis also provides rationale for the so-called "rich get richer" hypothesis. In a market economy, there is no clear implication as to whether economic activities will tend to reduce or else to widen initial wealth disparities (e.g., Durham et al., 1998). The so-called Paradox of Power (e.g., Hirshleifer, 1991) is the observation that poorer or weaker contestants improve their position relative to richer or stronger opponents. Nevertheless, in some social and economic contexts the reverse occurs, i.e., initially richer and/or more powerful contestants do exploit weaker rivals and thus the *rich get richer*. Our model and results take a side on the debate between the Paradox of Power and the "rich get richer" hypothesis upon endorsing the latter one. As we have outlined above, if ex post manipulation wants to be ruled out from the outset, then budget constraints prevent a principal from obtaining first-best efficiency. Hence initial wealth disparities between principals will tend to increase after the production phase, making rich principals richer. One might argue that the principal could buy insurance to get around the budget constraints. A plausible way to explore this option in our model would be to enrich the analysis to intermediate types of management strategies in which wages are only *partially* contingent on the project's success. More precisely, assume that the principal has some stock of resources, although maybe not enough to face the salaries of all workers if the project is not successful. If so, it can be shown (e.g., López-Pintado and Moreno-Ternero, 2009) that the optimal scheme would guarantee each deserving agent a fraction from the stock, as well as a bonus contingent on the project's success, and that the most talented agents would be fully paid. This is reminiscent of some other discrimination processes in the labor market (e.g., Milgrom and Oster, 1988).

To conclude, it is also worth mentioning that our results are based on the assumption that agents are all risk neutral. The reason for this was simply to provide the most conservative framework regarding the effect of a budget constraint. It is straightforward to show that if agents were risk averse, then the deadweight (efficiency) loss associated to the budget constraints would be even higher.

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