Advertising and Aggregate Consumption: 
A Bayesian DSGE Assessment

Benedetto Molinari (U. Pablo de Olavide)
Francesco Turino (Universitat d’Alacant)

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Advertising and Aggregate Consumption: A Bayesian DSGE Assessment

Benedetto Molinari and Francesco Turino*

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Abstract

Aggregate data reveal that advertising in the U.S. absorbs approximately 2% of GDP and has a well-defined pattern over the business cycle, being strongly procyclical and highly volatile. Because the purpose of brand advertising is to foster sales, we ask whether such spending has an appreciable effect on the pattern of aggregate consumption and, through this avenue, on economic activity. This question is addressed by developing a dynamic general equilibrium model in which households’ preferences for differentiated goods depend on the intensity of brand advertising, which is endogenously determined by profit-maximizing firms. Once the model is estimated to match the U.S. economy, it argues that the presence of advertising in the long run raises aggregate consumption and hours worked, eventually fostering economic activity. We also find that advertising has a relevant impact on fluctuations in consumption, investment and markup over the business cycle. All of the abovementioned effects are proven to depend crucially on the degree of competitiveness of advertising at the firm level.

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*Molinari: Universidad Pablo de Olavide, Sevilla (Spain), molinari@upo.es Turino: Universitat d’Alacant, Alicante (Spain), fturino@ua.es. We wish to acknowledge useful talks and suggestions about this project by Jordi Galí, Alberto Bisin, Albert Marcet, David Andolfatto, Enrique Sentana, Fabio Canova, Tim Kehoe, Andrea Caggese, Pierluigi Ciocca, and participants to the SED Annual Meeting 2009, the Macro Workshop at UPF, the XI Workshop on Dynamic Macroeconomics in Vigo, and the ASSET 2006 Conference. Albert Marcet and Ramon Marimon are gratefully acknowledged for the financial support provided to collect the data used in the paper. The database was made available by the courtesy of the staff of Johnson Graduate School of Management Library (Sage Hall, Cornell University).
"... as a matter of fact, the scale of expenditures on advertising varies positively with the general level of economic activity, so that, insofar as the effect of marginal expenditures is positive, advertising itself tends to accentuate the amplitude of economic fluctuations..."

Nicholas Kaldor (1950)

1 Introduction

In 2005, firms spent 272 billion dollars on advertising their products in the U.S. media, amounting to approximately 1,200 dollars per civilian citizen. The U.S. advertising industry accounts for 2.2% of GDP, absorbs approximately 20% of firms’ budgets for new investments and uses 16% of their corporate profits. Traditionally, the rationale for firms’ spending on advertising has been identified as the positive effect of advertisements on sales. Firms realize that the demand that they face is not an exogenous product of consumers’ preferences; instead, it can be tilted toward their own products through advertisements. The effectiveness of advertising in enhancing demand is not only revealed by firms’ willingness to spend money on it but also supported by a large number of empirical studies.1 Building on this fact, we ask whether such relationship holds in the aggregate. Because the reason for advertising is to increase consumer demand, as brand advertising increases the sales of single goods, will aggregate advertising enhance aggregate consumption? If so, will it also increase aggregate demand and production? That is, how important are the spillover effects from the advertising sector to economic activity?

To address these questions, we build and estimate a dynamic stochastic general equilibrium model in which we explicitly account for advertising. The usage of a Bayesian DSGE methodology to assess the advertising-consumption relationship is a novel approach in the literature. We believe that this method is a reasonable choice for at least two reasons. First, advertising is not merely a matter of demand and consumption, but it can affect economic activity through various other channels, for instance, by increasing the substitutability among goods and thereby influencing firms’ market power or by reducing consumers’ savings and thereby decreasing future demand. A general equilibrium framework can conveniently cope with all of these effects. Second, a DSGE model incorporates both steady-state equilibrium and aggregate dynamics, thus providing a unified framework to study the short- and long-run effects of advertising in the aggregate. The developed model is a variant of the neoclassical stochastic growth model augmented with government sector, monopolistically competitive product markets, and real rigidities in the form of habit persistence, adjustment costs and variable capital utilization. Advertising is introduced in this framework by assuming that consumers’ preferences are endogenously determined, depending on the distribution of advertising expenditures across firms. In particular, brand advertising aims to raise the marginal utility of the advertised good, and this assumption creates a positive linkage between advertising and sales. In addition, we assume that a firm’s advertising affects rivals’ sales both by attracting new customers to the market (complementary advertising) and by stealing existing customers from competitors (competitive

1A survey of empirical studies on the advertising-sales relationship can be found in Vakratsas and Ambler (1999) or Bagwell (2007).
advertising). This feature of the model captures the classical dichotomy emphasized in the industrial organization (IO) literature: a firm’s advertising may exert both positive and negative spillover effects on the demand faced by competitors (e.g., Roberts and Samuelson, 1988; Karray and Martín-Herrn, 2009).

The model is estimated using quarterly data for the U.S. economy over the 1976:I-2006:II period. The dataset includes a novel series of total advertising expenditures gathered by aggregating firms’ spending on advertisements in a number of heterogeneous U.S. media. The construction of this series was made necessary because advertising is not included among the main economic indicators of the U.S. business cycle. The results of the Bayesian estimation provide evidence in favor of a positive advertising-consumption relationship, and we show that this finding is crucially related to the relative importance of competitive versus complementary advertising at the firm level. In the long run, in addition to a 4.20% increase in consumption, we find that advertising increases worked hours by 6.9%, GDP by 3.89%, and investment by 2.62%. The underlying mechanism operates through a *work and spend* channel: because of advertising, people work more to be able to consume more, and the perceived need for additional consumption results from the advertising signals to which people are exposed. We show that this mechanism makes households unambiguously worse off because the overworking effect more than compensates for the expansion in consumption. Furthermore, we find that the average markup increases (0.64%), thereby providing evidence in favor of an anti-competitive effect of advertising in the U.S. economy. In the short run, the spillover effects of advertising are concentrated on consumption, investment, and markup. In particular, we find that the volatility of consumption and markup increases by 26.1% and 56.1%, respectively, while that of investment declines by 39%. These results are driven by a short-run intertemporal mechanism that pushes households to increase current consumption at the expense of savings. The resulting crowding effect on investment dampens the effect of advertising on output and, in general, softens the quantitative implications of the work and spending mechanism on short-run fluctuations.

The existing literature on the effects of advertising in the aggregate economy is largely empirical. Taylor and Weiserbs (1972), Ashley, Granger and Schmalensee (1980), Jacobson and Nicosia (1981), Chowdhury (1994), and Jung and Seldon (1995) estimate reduced-form specifications using aggregate time series to test for the existence of an advertising-consumption relationship based on Granger causality. Although their results are controversial and not conclusive, some evidence has emerged in favor of bi-directional Granger causality between advertising and aggregate consumption in the U.S. economy (e.g., Jung and Seldon, 1995). In this paper, we depart from this literature because we analyze the advertising-consumption relationship using a structural econometric approach. This methodology has the advantage of overcoming the potential problem of endogeneity affecting the reduced-form estimations cited above and additionally allows for a general assessment of the aggregate spillover effects of advertising. Few other papers examine the macroeconomics of advertising from a theoretical perspective. Bisin and Benhabib (2010) analyze the conditions under which neoclassical theory reproduces the so-called *post-modernist* critique of society. Grossmann (2008) analyzes the welfare implications of the complementarity between advertising and R&D expenditures in a quality-ladder model of endogenous growth. Both papers exploit a general equilibrium setup incorporating persuasive advertising *à la* Dixit-Norman as the one used in our paper. Finally, Hall (2008) analyzes the general equilibrium implications of
rent-seeking activities, showing that positive productivity shocks affect retailers’ revenues from advertising in a way that makes the latter a procyclical activity. Our results complement this finding, showing that advertising responds in step with output after a variety of shocks regardless of whether they affect productivity.

The remainder of this paper is structured as follows. Section 2 presents the novel quarterly series of aggregate advertising used in the estimation, together with a description of the advertising sector in the U.S. Section 3 provides the DSGE model with advertising, which is estimated in Section 4.1. Section 5 assesses the short- and long-run effect of advertising on the U.S. economy and evaluates its welfare implications. Section 6 concludes the paper. Details regarding the data and proofs of the propositions are presented in the Appendix.

2 Advertising Industry in the U.S.

In what follows, we define aggregate advertising as the total amount of spending by domestic and foreign firms to advertise their products in a country’s media. In Figure 1, we use the annual series of aggregate advertising constructed by Robert J. Coen of Universal McCann to assess the magnitude of aggregate advertising in the U.S. economy during the 1948-2007 period.\(^2\) Panel 1 depicts real per-capita advertising, which works as a proxy for the number of advertising messages received by each individual. The statistics indicate an average annual growth rate of 2%, showing that the intensity of advertising per consumer has grown steadily over time. Panel 2 portrays the ratio of aggregate advertising to private domestic sales, which measures the amount of resources employed in firms’ marketing

\(^2\)Advertising experts consider this to be the most reliable and complete source of data on aggregate advertising. The interested reader can find a full description of Coen’s data on Douglas Galbi’s blog: purplemotes.net/2009/05/10/robert-j-coen-advertising-data-hero/.
practise. Advertising fluctuates around a long-run mean of 3% (2.1% considering sales to both the private and public), accounting for approximately 20% of firms’ budgets for investment and absorbing almost the same amount of resources spent in R&D.\(^3\)

What remains unclear from the previous analysis is the relationship between advertising and economic activity in the short run. The ratio of advertising to domestic sales shows large fluctuations during the sample period, but annual data are not an ideal candidate to analyze short-run co-movements because they mask information on in-year contractions and expansions. To investigate this aspect, we construct a novel database of aggregate advertising at quarterly frequency and use it to perform a proper business cycle analysis.\(^4\) Data are gathered from different sources reporting firms’ expenditures on advertisements in several media: magazines and Sunday magazines, cable, spot, syndicated and network television, network and spot radio, newspapers, billboards, and outdoor advertising. The resulting series reports the unweighted sum of nominal quarterly expenditures on advertisements covering the period 1976.I-2006.II. To obtain the real series used in the business cycle analysis, nominal data are deflated using the GDP deflator, which has been shown to have the closest resemblance to an advertising-sector specific deflator (see Seldom and Jung, 1995, for further details). To the best of our knowledge, this dataset is the most complete source of quarterly data on U.S. aggregate advertising that has been used in the literature thus far.\(^5\)

\(^3\) Statistics on R&D in the U.S. are available from the National Science Foundation website (www.nsf.org).

\(^4\) The construction of a novel database is necessary because quarterly data on aggregate advertising are not available among the standard business cycle indicators. The U.S. federal administration used to collect data on advertising among the indicators used to analyze the business cycle but stopped collecting these data in 1968. Federal data were used by Blank (1962).

\(^5\) To verify whether our series is representative of aggregate advertising, we compute the cumulative yearly expenditures from our data, both in aggregate and broken down by media, and we compare them on a year-by-year basis with Coen’s annual data. Over the sample period, our data account for 30% of Coen’s data on average, with a minimum of 25% and an in-sample standard deviation of 2.95%, and adequately track the...
Table 1: Real Business Cycle Statistics

<table>
<thead>
<tr>
<th>$X_t$</th>
<th>$\frac{\sigma(X_t)}{\sigma(GDP_t)}$</th>
<th>(corr(X_t, Adv_t))</th>
<th>(corr(X_t, GDP_t))</th>
<th>(corr(X_t, X_{t-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Advertising</td>
<td>2.56</td>
<td>1</td>
<td>0.62</td>
<td>0.90</td>
</tr>
<tr>
<td>GPD</td>
<td>1</td>
<td>0.61</td>
<td>1</td>
<td>0.93</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.76</td>
<td>0.64</td>
<td>0.88</td>
<td>0.95</td>
</tr>
<tr>
<td>Non-Dur.</td>
<td>0.65</td>
<td>0.64</td>
<td>0.80</td>
<td>0.94</td>
</tr>
<tr>
<td>Durables</td>
<td>2.40</td>
<td>0.59</td>
<td>0.85</td>
<td>0.93</td>
</tr>
<tr>
<td>Investment</td>
<td>3.50</td>
<td>0.49</td>
<td>0.86</td>
<td>0.94</td>
</tr>
<tr>
<td>$\frac{Adv}{GDP}$</td>
<td>2.12</td>
<td>0.94</td>
<td>0.30</td>
<td>0.89</td>
</tr>
<tr>
<td>$\frac{Adv}{Sales}$</td>
<td>2.02</td>
<td>0.92</td>
<td>0.25</td>
<td>0.88</td>
</tr>
</tbody>
</table>

In Figure 2, we plot the cyclical component of real advertising expenditures along with real GDP (panel 1), real total consumption, and real fixed private investment (panel 2). Advertising appears procyclical, more volatile than GDP and consumption, and less volatile than investment. Table 1 reports related business cycle statistics, which confirm these findings. Advertising displays a positive correlation with GDP (0.61) and is more than twice as volatile as GDP. Also, it is very persistent over the cycle, with a point estimate of first-order autocorrelation of 0.90.

Concerning the other macroeconomic aggregates, advertising displays the strongest correlation with total consumption (0.64) and it is approximately 3 times more volatile than total consumption, 4 times more volatile than non-durable consumption, slightly more volatile than durable consumption and 25% less volatile than investment. Finally, the positive correlation between GDP and both the ratio of advertising to GDP and advertising to sales indicate that advertising is not a constant proportion of output. This evidence suggests that firms do not spend a constant amount of resources on advertising as predicted by the standard theory of optimal advertising budgeting (e.g. Dorfman and Steiner, 1954).

As last issue, we analyze the dynamic cross-correlations between advertising, GDP, consumption and investment, which provide useful evidence to support or dismiss the common idea that advertising is a leading indicator of the cycle (see Blank, 1962). According to the findings reported in Table 2, advertising only slightly leads GDP — the cross-correlation coefficients equal to 0.63 at k=1 and 0.61 at k=0 —, appears synchronized with consumption, distribution across media. This last property appears crucial for the representatives of our series because the cyclical properties of advertising are not homogeneous across media. The distributions of advertising expenditures by media and related business cycle statistics are available from the authors upon request.

Cyclical components have been extracted using a band pass filter with 6-32 as bands. For the series of aggregate advertising, we have previously eliminated the seasonal component using the X11 filter.

The pro-cyclicality of advertising at quarterly frequencies confirms the finding in recent contributions that make use of annual data (e.g. Bils, 1989; Hall, 2013).

The robustness of the findings presented in this Section is tested along several dimensions, using different data, filters, and definitions of aggregate advertising and advertising to output ratio. The main conclusions are confirmed in all of the considered cases. Aggregate advertising is always highly volatile, procyclical, and persistent, and its ratio over GDP always appear procyclical. Results are available from the authors upon request.
Table 2: Dynamic cross correlations

<table>
<thead>
<tr>
<th>$X_t$</th>
<th>k</th>
<th>(corr(X_t, GDP_{t+k}))</th>
<th>(corr(X_t, ADV_{t+k}))</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-4</td>
<td>-3</td>
<td>-2</td>
</tr>
<tr>
<td>Advertising</td>
<td>-0.05</td>
<td>0.16</td>
<td>0.36</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.16</td>
<td>0.38</td>
<td>0.59</td>
</tr>
<tr>
<td>Investment</td>
<td>0.54</td>
<td>0.77</td>
<td>0.91</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.31</td>
<td>0.41</td>
<td>0.51</td>
</tr>
<tr>
<td>(i) Non-Durable</td>
<td>0.35</td>
<td>0.47</td>
<td>0.57</td>
</tr>
<tr>
<td>(ii) Durables</td>
<td>0.17</td>
<td>0.27</td>
<td>0.38</td>
</tr>
<tr>
<td>Investment</td>
<td>0.62</td>
<td>0.69</td>
<td>0.70</td>
</tr>
</tbody>
</table>

The strongest correlation occurs at \(k=0\) (0.64) –, and strongly leads investment – the higher correlation occurs at \(k=-2\). Overall, the dynamic cross-correlation analysis denies the role of advertising as a leading indicator of the cycle.

3 A DSGE model with advertising

We consider an economy in which a continuum of differentiated goods indexed by \(i \in [0, 1]\) are produced by monopolistically competitive producers. Goods are sold by firms to households for consumption and investment purposes and to the government, which collects taxes from households to finance public expenditures. Households’ preferences for differentiated goods are not exogenous but depend on the distribution of advertising expenditures across firms. Specifically, advertisements shift consumers’ tastes toward the advertised good such that the demand for each variety increases with the amount of spending on advertising. Firms are aware of this linkage and compete in the market jointly using advertising budgeting and pricing policy. All of the interactions among firms, households and the government occur in a stochastic environment in which short-run dynamics are driven by several demand and supply shocks.

3.1 The representative household and the role of advertising

The representative household has preferences in period 0 given by

\[
U(\tilde{C}_t, H_t) = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \left[ \frac{(\tilde{C}_t - \zeta \tilde{C}_{t-1}) / T_t}{1 - \sigma} \right]^{(1-\sigma)} - 1 - \xi^h H_t^{1+\phi} \right\} \tag{1}
\]

where \(\tilde{C}_t\) is a consumption aggregate, \(H_t\) is hours worked, \(E_t\) denotes the mathematical expectations operator conditional on the information available at time \(t\), \(\beta \in (0, 1)\) represents
the subjective discount factor, \( \zeta \in [0, 1] \) controls for the degree of habit persistence, and \( \{\phi, \sigma\} \in \mathbb{R}^2_+ \) are preference parameters. \( \xi^h_t \) is a preference shock that follows a univariate autoregressive process of the form \( \log(\xi^h_t / \xi) = \rho_h \log(\xi^h_{t-1} / \xi) + \epsilon^h_t \), in which \( \rho_h \in [0, 1) \), \( \xi > 0 \) and \( \epsilon^h_t \) are i.i.d. innovations with a mean equal to 0 and a standard deviation \( \sigma_h \). As in An and Schorfheide (2007), we assume that households derive utility from the object \( \tilde{C}_t - \zeta \tilde{C}_{t-1} \) relative to the deterministic level of labor augmenting technological progress, \( \Gamma_t \), which evolves over time at the constant rate \( \tau > 0 \). This assumption is necessary to ensure that the economy evolves along a balanced growth path.

Building on Dixit and Norman (1978), we assume that the composite consumption aggregate, \( \tilde{C}_t \), takes the following form:

\[
\tilde{C}_t = \left( \int_0^1 \left( c_{i,t} + B(g_{i,t}, g_{-i,t}, \Gamma_t) \right)^\varepsilon^{-1} di \right)^{\frac{1}{\varepsilon-1}}
\]

(2)

where \( c_{i,t} \) denotes the quantity of good \( i \) consumed in period \( t \), \( g_{i,t} \) and \( g_{-i,t} \) respectively summarize the effects of a firm’s own and its competitors’ advertising outlays on preferences, and \( \varepsilon > 1 \) is the pseudo-elasticity of substitution across varieties. The influence of advertising on consumers’ brand choices is captured by the additive term \( B(g_{i,t}, g_{-i,t}, \Gamma_t) \), which is assumed to be a decreasing and convex function in its first argument and increasing and concave in its second argument, satisfying \( B(0, 0, \Gamma_t) = a \geq 0 \) for any \( \Gamma_t \).

The formulation adopted implies that advertising is persuasive in that it affects consumers’ choices by modifying their tastes. Among the main competing views of advertising proposed in the literature - persuasive, informative, and complementary (see Bagwell, 2007, section 2.2) - we rely on persuasive advertising because the marketing literature has provided convincing empirical evidence in support of this view. In the model, the persuasive effect is replicated by assuming that exposure to brand advertising creates a negative externality that induces households to feel dissatisfaction with their current levels of consumption. As a result, the marginal utility of advertised goods increases, and households devote larger

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\(^9\) The dependence of function \( B(\cdot) \) on the rate of labor-augmenting technological progress, \( \Gamma_t \), is a necessary condition to guarantee that a balanced growth path equilibrium exists. See Appendix C for further details.

\(^10\) Stern and Resnik (1991) finds that a large proportion of U.S. television advertisements are essentially uninformative with respect to prices and product attributes. Moreover, in an exhaustive empirical generalizations study, Vakratsas and Ambler (1999) argue that advertising does not need to be informative to be effective. Several pieces of empirical evidence based on both experimental economics and field studies suggest that consumers form preferences on the basis of not only product/brand attribute information but also elements such as liking, feelings and emotions triggered by mere exposure to advertisements (Chen et al., 2009). These findings are supported by structural econometric studies that analyze customers’ purchasing behaviors in the U.S. economy (e.g., Kamakura and Russell, 1993; Yoo, Donthu and Lee, 2000). The relationship between advertising and consumers’ brand perceptions has also attracted the interest of neuroscientists. Behavioral evidence provided by this literature shows that cultural messages can be incorporated into the decision-making process and yield preferences for one product or another (McClure et al., 2004). From a theoretical perspective, the behavioral economic study of Gabaix and Laibson (2006) shows that information revelation may break down in the presence of consumers who fail to discern concealed attributes, such as hidden fees or maintenance costs. As noted by Bagwell (2007), in this circumstance, there is a clear incentive for firms to conceal product information, even when advertising is costless.
fractions of their income to purchasing advertised products. We also assume that brand advertising decreases the substitutability between advertised goods and rival brands. As documented in several empirical studies, building brand equity thorough advertising is a successful strategy to differentiate firms’ own products from rival brands (Kamakura and Russell, 1993), either because advertising improves the perceived product quality (Aaker and Jacobson, 1994) or because it reinforces customers’ loyalty (Yoo, Donthu and Lee, 2000). In the theoretical literature, this characteristic is typically modeled by assuming that advertising affects the price elasticity of demand (e.g., Dixit and Norman, 1978), which in our formulation is guaranteed by the non-homotheticity of the consumption aggregate (2).

The representative household determines the optimal demand for consumption goods by minimizing consumption expenditures subject to the consumption aggregate (2), i.e.,

\[
c_{i,t} = \max \left\{ \left( \frac{p_{i,t}}{P_t} \right)^{-\varepsilon} \tilde{C}_t - B(g_{i,t}, g_{-i,t}, \Gamma_t) ; 0 \right\}
\]

(3)

where

\[
P_t = \left[ \int_0^1 p_{i,t}^{1-\varepsilon} di \right]^{\frac{1}{1-\varepsilon}}
\]

is the nominal price index. Equation (3) captures two features of the advertising-sales relationship that are often emphasized in the IO and marketing literature and documented by a large body of empirical studies (e.g., Jorgersen and Zaccour, 2004). First, sales react positively to advertising campaigns (Vakratsas and Ambler, 1999; Bagwell, 2007). In the model, an incremental investment in advertising raises \(g_{i,t}\), which in turn decreases \(B(\cdot)\), shifting the demand function (3) to the right. Additionally, the convexity of \(B(\cdot)\) implies that advertising has diminishing marginal returns on the demand in accordance with the empirical evidence reported in Deighton, Henderson and Neslin (1994) and Vakratsas et al. (2004).

Second, in multi-product markets, advertising is typically competitive in that rival advertisements negatively affect firms’ demand (Karray and Martin-Herrn, 2009). In the model, an expansion in rival advertisements \(g_{-i,t}\) increases \(B(\cdot)\), thus shifting the demand function (3) to the left. As noted by Roberts and Samuelson (1988), however, brand advertising may also increase consumers’ awareness of the existence of a product category, thus attracting new customers to the market and therefore benefiting not only the advertiser but also its competitors. In the model, this feature of advertising — usually referred to as the complementary property — arises as a general equilibrium outcome. It is possible to grasp an intuition this result by differentiating the demand schedule (3) with respect to \(g_{-i,t}\), i.e.

\[
\frac{\partial c_{i,t}}{\partial g_{-i,t}} = \left( \frac{p_{i,t}}{P_t} \right)^{-\varepsilon} \frac{\partial \tilde{C}_t}{\partial g_{-i,t}} - \frac{\partial B(\cdot)}{\partial g_{-i,t}}
\]

The complementary effect is captured by the first term on the right-hand side of the above equation, which measures the effect of rival advertisements on firm \(i\)’s demand through changes in the object \(\tilde{C}_t\). The latter measures the magnitude of consumption expenditures or, alternatively, the size of the goods market. Hence, in those cases in which \(\partial \tilde{C}_t/\partial g_{-i,t} \succ 0\),
0, competitors’ advertising enhances the aggregate demand, thereby spilling over to firm $i$’s demand.\footnote{Note that because the utility function $\tilde{C}_t$ is continuous and represents locally non-satiated preference relations, the expenditure function is strictly increasing in $\tilde{C}_t$. Thus, the condition $\partial \tilde{C}_t / \partial g_{i,t} > 0 \forall i$ means that the household expands its total demand for consumption goods whenever $g_{i,t-1}$ increases.}

Finally, there is broad consensus in the literature regarding the carryover effect of advertising. As argued by Thepot (1983), ”marketing decisions are clearly related to dynamic processes: creating a market, building a distribution network, defining a brand image requires a considerable amount of time”. Many empirical papers show that past advertising campaigns influence current purchases, thus making advertising an inherently dynamic variable (Clarke, 1976; Leone, 1995). The dynamic characteristic of advertising is replicated in the model by assuming that current and past advertising sum to create a reputation for a good, the producer’s goodwill, which is defined as the intangible stock of advertising that affects a consumer’s tastes at time $t$. As in Nerlove and Arrow (1962), we assume that the stock of goodwill evolves according to the law of motion:

$$g_{i,t} = (1 - \delta_g) g_{i,t-1} + (1 - \phi_z(z_{i,t}/z_{i,t-1})) z_{i,t}$$  (4)

where $z_{i,t}$ is firm’s investment in new advertising at time $t$ and $\delta_g \in (0,1)$ is the goodwill depreciation rate. It easy to verify that the law of motion (4) and the demand function (3) jointly imply that both current and past advertising expenditures affect firms’ own demand, with an intensity that decays over time. The function $\phi_z(\cdot)$ captures the presence of convex adjustment costs in advertising and satisfies $\phi_z(\tau) = \phi_z'(\tau) = 0$ and $\phi_z''(\tau) > 0$. This assumption allows for a sluggish response of advertising to exogenous shocks, which may be driven by the costs that firms encounter when adjusting their current level of advertising expenditures, e.g., administrative costs or time to build new advertising campaigns.

For the rest of the demand side of the model, we assume that each household holds one asset, the capital stock $K_t$, and in each period $t$ chooses the capital utilization rate, $u_t$, to transform physical capital into effective capital according to the rule $K_t = u_t K_t$. The cost of capital utilization is defined in units of physical capital and is given by $a(u_t)$, which is assumed to be increasing and convex in $u_t$. Additionally, we assume that in the steady state, $u_t = 1$ and $a(1) = 0$. Physical capital evolves according to the law of motion

$$K_{t+1} = (1 - \delta_k) K_t + \xi^k (1 - \phi_k(I_t/I_{t-1})) I_t$$  (5)

where $I_t$ is investment in new capital, $\delta_k \in (0,1)$ is the capital depreciation rate, and $\phi^k(\cdot)$ denotes convex adjustment costs for capital, satisfying $\phi_k(\tau) = \phi'_k(\tau) = 0$ and $\phi''_k(\tau) > 0$. $\xi^k$ is an investment-specific shock that follows a univariate autoregressive process of the form $\log(\xi^k_t) = \rho_k \log(\xi^k_{t-1}) + \epsilon^k_t$, in which $\rho_k \in [0,1]$ and $\epsilon^k_t$ are i.i.d. innovations with a mean equal to 0 and standard deviation $\sigma_k$. Investment in new capital is assumed to be a composite good produced by aggregating differentiated goods via the technology

$$I_t = \left( \int_{i_{t-1}}^{i_t} di \right) ^{\frac{1}{\alpha}}$$

The resulting optimal demand for good $i$ for investment purposes is not directly affected by advertising. This assumption naturally fits in our setup because investment represents
the avenue through which consumption is postponed until the future, whereas advertising is intended to induce urge in consumption.

Each household supplies labor services per unit of time and rents physical capital to firms. Labor and capital markets are perfectly competitive, with a wage rate $W_t$ paid per unit of labor services and a rental rate $R_t$ paid per unit of capital. In addition, households receive profit $\Pi_t$ for the ownership of firms and pay lump-sum taxes $T_t$ to finance government spending. Under these assumptions, the representative household’s nominal budget constraint is

$$\int_0^1 p_{i,t} (c_{i,t} + i_{i,t}) \, di + T_t \leq W_t H_t + R_t u_t \bar{K}_t + \Pi_t - P_t a(u_t) \bar{K}_t$$

(6)

The utility maximization problem for the representative household can then be stated as choosing processes for $\tilde{C}_t$, $H_t$, and $u_t$ to maximize the inter-temporal utility function (1) subject to the constraints (5) and (6).

### 3.2 Government

In each period $t \geq 0$, the government collects lump-sum taxes from households to finance public spending. We assume that real government expenditures, $F_t$, are exogenous and stochastic, evolving according to the law of motion

$$F_t = \Gamma_t \left( \frac{F_{t-1}}{\Gamma_{t-1}} \right)^{\rho_f} \bar{F}^{(1-\rho_f)} \exp\{\epsilon^f_t\}$$

where $\rho_f \in [0,1)$, $\epsilon^f_t$ is an i.i.d. innovation with a mean of 0 and standard deviation $\sigma_f$, and $\bar{F}$ is detrended government spending ($F_t/\Gamma_t$) evaluated in the steady state. Government spending is allocated over a basket of intermediate goods, $f_{i,t}$, to maximize the amount of the composite good produced following the CES technology

$$F_t = \left( \int f_{i,t}^\epsilon \right)^{\frac{\epsilon}{\epsilon-1}}$$

(7)

The resulting demand for goods for government expenditures is not directly affected by advertising. This assumption is conservative with respect to our results. A positive linkage between advertising and government spending would in fact strengthen the effect of advertising on aggregate demand.

### 3.3 Firms

Firms produce output by combining labor and capital with a Cobb-Douglas technology of the form

$$y_{i,t} = A^f_{i,t} k_{i,t}^{1-\alpha} \left( \Gamma_t h p_{i,t} \right)$$

(8)

where $\alpha \in (0,1)$ and $y_{i,t}, k_{i,t}, h p_{i,t}$ respectively denote output, capital stock, and production-related labor. $A^f_{i,t}$ measures a purely transitory technology shock that evolves according to

http://www.upo.es/econ
Where $\phi$ the second optimality condition, the firm spends on advertising until the marginal benefit is specific and increasing in $g$, where $g$ is a price elasticity of $\epsilon_p$ consumption aggregate (2) is composed of two terms: $(i,t)$ the consumption aggregate (2). In fact, the optimal demand function resulting from the demand function (10), the law of motion of the goodwill stock (4), and the technology to replicate the brand equity effect of advertising and results from the non-homotheticity of $i$-specific productivity and evolves according to \[ z_{i,t} = A_i \Gamma_t h a_{i,t} \] (9)

where $\alpha_z \in (0,1)$ and $h a_{i,t}$ denotes advertising-related labor. $A^*_i$ measures advertising-specific productivity and evolves according to $\log(A^*_i) = \rho_z \log(A^*_{i-1}) + \epsilon^*_i + \rho_{za} \epsilon^*_a$ in which $\rho_z \in [0,1), \rho_{za} \in [0,1]$ and $\epsilon^*_i$ are i.i.d. innovations with a mean equal to 0 and standard deviation $\sigma_z$. The dependence of advertising-specific shocks on innovations $\epsilon^*_a$ is intended to capture movements in advertising spending driven by changes in the overall productivity of the economy. We therefore refer to $A^*_i$ as an economy-wide productivity shock.

Firm $i$ sells its production $y_{i,t}$ to meet the demand for consumption, investment and government purchases, i.e.,

\[ y_{i,t} = (\frac{p_{i,t}}{P_t})^{-\epsilon} \left( \tilde{C}_t + I_t + F_t \right) - B(g_{i,t}, g_{-i,t}, \Gamma_t) \] (10)

It is easy to verify that the price elasticity of demand,

\[ \eta_p(i) = \epsilon (p_{i,t}/P_t)^{-\epsilon} (\tilde{C}_t + I_t + F_t)/y_{i,t} = \epsilon (1 + B(\cdot)/y_{i,t}) \] (11)

is a decreasing function of the goodwill stock. As argued in section 3.1, this property replicates the brand equity effect of advertising and results from the non-homotheticity of the consumption aggregate (2). In fact, the optimal demand function resulting from the consumption aggregate (2) is composed of two terms: $(p_{i,t}/P_t)^{-\epsilon} (\tilde{C}_t + I_t + F_t)$ displaying a price elasticity of $\epsilon$ and $B(\cdot)$ that is independent of prices. The overall price elasticity is therefore a linear combination of the elasticities of these two terms, $\epsilon$ and 0, with the coefficient on $\epsilon$ given by the ratio of the price-elastic term to the total demand, as shown in (11). When the firm increases its goodwill stock, the weight of the price-elastic term over the total demand decreases, and the overall price elasticity $\eta_p(i)$ decreases.

Firms jointly determine pricing policy, production plans and advertising budgeting by maximizing the discounted flow of future profits subject to the constraints given by the demand function (10), the law of motion of the goodwill stock (4), and the technology to produce goods (8) and advertising (9). Optimal planning satisfies two conditions. First, firms set the optimal price by charging a positive markup over the marginal cost, i.e.,

\[ p_{i,t} = \eta_p(i)/[(\eta_p(i) - 1)] \varphi_t \equiv \mu_{i,t} \varphi_t \] (12)

where $\varphi_t$ is the marginal cost of production\(^1\) and $\mu_{i,t}$ is the optimal markup, which is firm specific and increasing in $g_{i,t}$, as result of the brand loyalty effect described above. Under the second optimality condition, the firm spends on advertising until the marginal benefit

\(^1\)Cost minimization implies that $\varphi_t = (D/A_t) W_t^a R_t^{1-a}$ with $D = [(1-a)/a]^{\alpha} [1/(1-a)]$.
from an additional unit of advertising equals its marginal cost, as stated in the following Euler equation:

\[-(p_{i,t} - \varphi_t) \frac{\partial B(\cdot)}{\partial g_{i,t}} + (1 - \delta_g) E_t \{\nu_{i,t+1} Q_{t,t+1}\} = \nu_{i,t}\]  

(13)

where \(Q_{t,t+1}\) is the stochastic discount factor and \(\nu_{i,t}\) is the marginal cost of producing advertising.\(^{13}\) Consistent with the dynamic nature of goodwill, the marginal benefit that appears in the left-hand side of equation (13) is the one-period-ahead expected payoff from a marginal unit of advertising in addition to the discounted opportunity cost of not producing tomorrow the surviving goodwill produced today. Conditions (12) and (13) jointly imply that advertising and pricing are complementary policies, and therefore, any shock that causes firms to revise their pricing policies will also cause them to adjust advertising spending in the same direction.

3.4 The general equilibrium

We restrict the analysis to symmetric equilibria in which all firms set the same price, produce the same quantity of goods, and invest the same amount of resources in advertising. In addition, we normalize the price of goods to 1 so that all of the remaining prices are defined in terms of contemporaneous consumption. The symmetric equilibrium of the model is then formally derived by imposing the following market clearing conditions:

\[H_t = \int_0^1 (h a_{i,t} + h p_{i,t}) \, di\]
\[Y_t = C_t + I_t + F_t + a(u_t) \bar{K}_t\]

where \(Y_t\) denotes total output obtained by integrating (8) over the firms’ index. In general, the presence of advertising has countervailing effects on macroeconomic aggregates, and in the next section, we employ a numerical analysis to unravel the general equilibrium results. However, in the case in which advertising is purely competitive, when the complementary effect is switched off (i.e., \(\partial \tilde{C}_t / \partial g_{-i,t} = 0 \ \forall \ i \in [0, 1]\)), the general equilibrium properties can be unambiguously characterized. The following proposition summarizes these results.

**Proposition 1.** Consider an economy in which monopolistically competitive firms can promote their products by incurring advertising expenditures. Let the households’ intertemporal utility function and the consumption aggregate be of the forms (1) and (2), respectively. Additionally, assume that the technology to produce goods and advertising is given by (8) and (9), respectively, and that capital and labor markets are perfectly competitive. Then, at the stationary perfect foresight symmetric equilibrium, purely competitive advertising reduces the equilibrium levels of consumption, output and average markup and increases hours worked and the wage rate with respect to an identical economy in which advertising is banned.

\(^{13}\)Assuming perfect financial markets, the stochastic discount factor is given by the following: \(Q_{t,t+1} = \beta(\lambda_{t+1}/\lambda_t)^{-\sigma}\), where \(\lambda_t\) is the Lagrange multiplier for the budget constraint in the utility maximization problem.
According to Proposition 1, purely competitive advertising is wasteful and pro-competitive in the long run. In the purely competitive case, advertising is a zero-sum game that in equilibrium leaves the demand function of firms’ products unaffected. However, firms do not internalize this general equilibrium effect in their optimal decisions and continue hiring workers to produce advertisements. As a result, the labor demand shifts to the right with respect to an identical economy in which advertising is banned, thereby generating upward pressure on the wage rate. The equilibrium levels of both the real wage and hours worked increase, but a fraction of production-related labor is now reallocated to an unproductive activity (advertising). As a consequence, the equilibrium level of output diminishes, which induces a negative wealth effect that eventually reduces consumption and investment. In addition, the average markup in the economy is reduced. Although purely competitive advertising has no brand equity effect, i.e., no direct effect on price elasticity, the pro-competitive effect arises in equilibrium as a result of the decline in output, which in our model makes the price elasticity of demand larger than it would otherwise be.

4 Parameters Estimates

4.1 Method and data

The model economy evolves along a balanced growth path equilibrium in which all endogenous variables grow at a constant rate. We therefore rewrite the model in terms of detrended variables to ensure that the deterministic steady state can be computed. The detrended model is then log-linearized around the steady state, and the resulting linear system of rational expectations equations is solved to obtain the state space representation used to compute the likelihood function. To estimate the model, we follow the Bayesian paradigm: a prior distribution is chosen for each structural parameter, and a Metropolis-Hastings algorithm then estimates the posterior distribution. The estimation is performed using quarterly U.S. data on real personal consumption expenditures, real output minus net exports, total hours worked, real private nonresidential fixed investment and the series of real aggregate advertising presented in Section 2. All data are in per-capita terms and are expressed as quarterly growth rates with the exception of hours worked, which is expressed as demeaned log-levels. The sample period is from the first quarter of 1976 to the second quarter of 2006, the time span for which data on aggregate advertising are available.

4.2 Functional form assumptions

The model estimation requires the functions controlling for (i) the effectiveness of advertising in preferences, (ii) the cost of capital utilization and (iii) adjustment costs to be

\[ \frac{\partial C_t}{\partial g_{i,t}} = 0 \quad \forall i \]

This follows from condition \( \frac{\partial C_t}{\partial g_{i,t}} = 0 \), which implies that \( \frac{\partial c_{i,t}}{\partial g_{i,t}} = \frac{\partial B(\cdot)}{\partial g_{i,t}} = 0 \) when \( g_{i,t} = G_t \quad \forall i \). See Appendix D for a formal analysis generating this result.

\[ \frac{\partial B(\cdot)}{\partial g_{i,t}} = 0 \quad \forall i \]

Because in the symmetric equilibrium with purely competitive advertising, \( \frac{\partial B(\cdot)}{\partial g_{i,t}} = 0 \quad \forall i \).

The work of Chen et al. (2009) reports a similar result with regard to pro-competitive advertising in a Hotelling model in which purely competitive advertising changes the distribution of consumers’ ideal consumption points.

See Appendix A for details on the data.
parameterized. First, we assume that $B(\cdot)$ satisfies the following condition:

$$B(\cdot)/\Gamma_t = S(g_{i,t}, \Gamma_t) + \gamma (1 - S(g_{-i,t}, \Gamma_t)) \quad \text{with} \quad \gamma \in [0, 1] \quad (14)$$

where

$$S(x_t, \Gamma_t) = \frac{1}{1 + x_t/\Gamma_t} \quad \text{with} \quad x_t = \{g_{i,t}; g_{-i,t}\} \quad (15)$$

and

$$g_{-i,t} = \int_0^1 g_{j,t} dj$$

This formulation conforms to the assumptions made in section 3.1 and guarantees that a balanced growth path equilibrium exists.\(^{18}\) As in Grossmann (2008), we assume that rival advertising is measured by the average stock of competitors’ goodwill. The choice of $S(\cdot)$ implies that the marginal utility of each variety is bounded. Given the upper bound, when the price is sufficiently high, the marginal utility is lower than the price for any level of consumption, and the consumer eventually drops the good from his basket of purchases.\(^{19}\)

In the absence of advertising, the upper bound is constant, whereas with advertising the bound is inversely related to the level of producer’s goodwill, and therefore, an increase in advertising corresponds to a higher marginal utility of the advertised good.

The specification of function $B(\cdot)$ nests any intermediate type of advertising between two extreme cases in which advertising either merely redistributes market shares across firms or works as a pure market-enhancing force. The mapping between these two cases is controlled by parameter $\gamma$, which measures the degree of competitiveness of advertising at the firm level. Its role is apparent if one examines equation (14). When $\gamma = 1$, the effectiveness of brand advertising in enhancing sales depends on the intensity of rivals’ advertising. At the symmetric equilibrium, advertising is a zero-sum game because the negative effect of rival goodwill precisely compensates for the effect of firms’ own goodwill, thus leaving the demand function unaffected. We refer to this case as purely competitive advertising. Conversely, when $\gamma = 0$, the demand-stealing mechanism is switched off (i.e., $\partial B(\cdot)/\partial g_{-i,t} = 0 \ \forall \ i$), and brand advertising expands the producer’s sales regardless of the intensity of competitors’ advertising. Because of this effect, the advertising of each individual firm jointly exerts upward pressure on the aggregate demand, which eventually spills over to all firms. Finally, when $\gamma \in (0, 1)$, the parametrization yields a combination of the two extreme cases in which firms’ advertising both steals customers from competitors and expands the size of the market. We refer to this general case as market-enhancing advertising.

With regard to adjustment costs, we adopt standard quadratic functions satisfying $\phi_z''(\tau) = \psi_z > 0$ and $\phi_k''(\tau) = \psi_k > 0$. Finally, the cost of capital utilization is parametrized by assuming that the function $a(\cdot)$ takes the following form:

$$a(u_t) = R_{ss} \left( \omega u_t^2/2 + (1 - \omega)u_t + \omega/2 - 1 \right)$$

\(^{18}\)See Appendix C for a formal proof of this result.

\(^{19}\)See Melitz and Ottaviano (2008) for another application featuring bounded marginal utility.
where $R_{ss}$ denotes the rental rate of capital evaluated in the steady state and $\omega > 0$ is a parameter that controls for the curvature of $a(\cdot)$. In the estimation of the model, we adopt the normalization used in Iacoviello and Neri (2010) and Smets and Wouters (2007) by specifying the prior distribution for the curvature parameter in terms of $\psi_u = \omega/(1 + \omega)$.

### 4.3 Prior distributions

Prior distributions for structural parameters are summarized in Table 3. A rather dispersed inverse gamma distribution is assumed for the standard errors of exogenous shocks, whereas beta distributions are used for the autoregressive parameters with mean values ranging from 0.5 to 0.9 and relatively high standard deviations. Prior means for these parameters are chosen to match the actual volatility and persistence of the observable variables. The common quarterly growth rate $\tau^*$ is assumed to follow a dispersed normal distribution centered at 0.005. This value is chosen to match the average quarterly growth rate of actual per-capita GDP. The preference parameter $\sigma$ follows a gamma distribution with a mean equal to 2 and a standard deviation equal to 0.5; the inverse of the Frisch elasticity of the labor supply $\phi$ is described by a gamma random variable with a mean of 0.77 and a standard deviation of 0.5, while the subjective discount factor $\beta$ follows a beta distribution with a mean of 0.995 and a standard deviation of 0.002. The prior means for these parameters are in the range of values typically used in RBC models. The labor elasticity of output $\alpha$ is beta distributed with a mean equal to 0.70 and a standard deviation equal to 0.02. This prior mean implies a steady-state labor share ($WH/Y$) of 0.65 that matches the long-run average of its counterpart in the data. Following Iacoviello and Neri (2010), the habit persistence parameter $\zeta$ and the curvature of the capital utilization cost $\psi_u$ are both assumed to follow a beta distribution with a mean of 0.5 and a standard deviation of 0.2. For the capital adjustment costs, we follow Justiniano, Primiceri and Tambalotti (2010) by assuming that $\psi_k$ is gamma distributed with a mean equal to 4 and standard deviation equal to 1.

Regarding the advertising-related parameters, we assume that the elasticity of labor in the production of advertising, $\alpha z$, is beta distributed with a mean of 0.73 and a standard deviation of 0.02. Conditional on the remaining parameters, this prior mean implies a steady-state ratio of advertising-related labor to total worked hours ($Ha/H$) of 0.02, which corresponds to the average fraction of the labor force employed in the advertising industry during the 2002-2012 period, as reported by the Bureau of Labor Statistics. The prior for $\gamma$ is a uniform [0,1] distribution that reflects our neutral stance between competitive and market-enhancing advertising. Finally, the parameter controlling for advertising adjustment costs, $\phi z$, is assumed to be gamma distributed with a mean of 4 and a standard error of 1. This prior mean implies equivalent elasticities of adjustment costs in the laws of motion of capital and advertising, which reflects our a priori lack of knowledge about these adjustment cost functions.

Five parameters are fixed in the estimation procedure, either because they are typically difficult to identify in estimated DSGE models or to improve identification of the remaining parameters. The gross elasticity of substitution across varieties, $\varepsilon$, and the depreciation rate of capital, $\delta_k$, are fixed at 4.33 and 0.010, respectively, to match the long-run average markup (1.11) and investment share (20%). The depreciation rate of the goodwill stock $\delta_g$ is calibrated at 0.11, implying a long-run ratio of advertising expenditures to GDP of
Table 3: Prior and Posterior Distributions

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Density</th>
<th>Domain</th>
<th>Mean</th>
<th>90% interval</th>
<th>Mean</th>
<th>90% interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Beta</td>
<td>[0, 1)</td>
<td>0.995</td>
<td>[0.991 0.998]</td>
<td>0.994</td>
<td>[0.991 0.997]</td>
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<tr>
<td>$\sigma$</td>
<td>Gamma</td>
<td>$R_+$</td>
<td>2.000</td>
<td>[1.255 2.887]</td>
<td>1.952</td>
<td>[1.283 2.645]</td>
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<tr>
<td>$\phi$</td>
<td>Gamma</td>
<td>$R_+$</td>
<td>0.770</td>
<td>[0.167 1.732]</td>
<td>2.662</td>
<td>[1.588 3.721]</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Uniform</td>
<td>[0, 1]</td>
<td>0.500</td>
<td>[0.050 0.950]</td>
<td>0.189</td>
<td>[0.000 0.376]</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Beta</td>
<td>[0, 1]</td>
<td>0.708</td>
<td>[0.674 0.740]</td>
<td>0.739</td>
<td>[0.771 0.765]</td>
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<tr>
<td>$\alpha_z$</td>
<td>Beta</td>
<td>[0, 1]</td>
<td>0.734</td>
<td>[0.700 0.766]</td>
<td>0.721</td>
<td>[0.668 0.752]</td>
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<tr>
<td>$\tau^*$</td>
<td>Normal</td>
<td>$R_+$</td>
<td>0.005</td>
<td>[0.0017 0.0083]</td>
<td>.0035</td>
<td>[.0032 .0037]</td>
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<tr>
<td>$\psi_k$</td>
<td>Gamma</td>
<td>$R_+$</td>
<td>4.000</td>
<td>[2.509 5.774]</td>
<td>2.420</td>
<td>[1.579 3.224]</td>
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<tr>
<td>$\psi_z$</td>
<td>Gamma</td>
<td>$R_+$</td>
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<td>[2.509 5.774]</td>
<td>1.327</td>
<td>[0.653 1.913]</td>
</tr>
<tr>
<td>$\psi_u$</td>
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<td>0.201</td>
<td>[0.066 0.327]</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>Beta</td>
<td>[0, 1]</td>
<td>0.500</td>
<td>[0.171 0.828]</td>
<td>0.620</td>
<td>[0.532 0.707]</td>
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<td>$\rho_a$</td>
<td>Beta</td>
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<td>[0.615 0.939]</td>
<td>0.889</td>
<td>[0.826 0.961]</td>
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<tr>
<td>$\rho_h$</td>
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<td>(0, 1)</td>
<td>0.500</td>
<td>[0.172 0.828]</td>
<td>0.951</td>
<td>[0.924 0.988]</td>
</tr>
<tr>
<td>$\rho_f$</td>
<td>Beta</td>
<td>(0, 1)</td>
<td>0.900</td>
<td>[0.807 0.967]</td>
<td>0.979</td>
<td>[0.971 0.990]</td>
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<td>$\rho_k$</td>
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<td>(0, 1)</td>
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<td>[0.172 0.828]</td>
<td>0.537</td>
<td>[0.396 0.689]</td>
</tr>
<tr>
<td>$\rho_z$</td>
<td>Beta</td>
<td>(0, 1)</td>
<td>0.900</td>
<td>[0.807 0.967]</td>
<td>0.707</td>
<td>[0.573 0.831]</td>
</tr>
<tr>
<td>$\rho_{za}$</td>
<td>Beta</td>
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<td>[0.171 0.828]</td>
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<td>[0.130 0.756]</td>
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<tr>
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<td>InvGamma</td>
<td>$R_+$</td>
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<td>[0.002 0.022]</td>
<td>0.006</td>
<td>[0.006 0.007]</td>
</tr>
<tr>
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<td>InvGamma</td>
<td>$R_+$</td>
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<td>[0.001 0.017]</td>
<td>0.024</td>
<td>[0.016 0.053]</td>
</tr>
<tr>
<td>$\sigma_f$</td>
<td>InvGamma</td>
<td>$R_+$</td>
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<td>[0.002 0.029]</td>
<td>0.048</td>
<td>[0.043 0.053]</td>
</tr>
<tr>
<td>$\sigma_k$</td>
<td>InvGamma</td>
<td>$R_+$</td>
<td>0.008</td>
<td>[0.002 0.022]</td>
<td>0.025</td>
<td>[0.015 0.035]</td>
</tr>
<tr>
<td>$\sigma_z$</td>
<td>InvGamma</td>
<td>$R_+$</td>
<td>0.006</td>
<td>[0.001 0.017]</td>
<td>0.049</td>
<td>[0.030 0.059]</td>
</tr>
</tbody>
</table>

0.029, as in the data. Following Ravn, Schmitt-Grohe and Uribe (2006), the government spending-GDP ratio, $\bar{F}/Y$, is set to 0.12, whereas the preference parameter $\xi$ is chosen such that the steady-state value of hours worked, $H$, is equal to 0.25, thus implying that households devote 1/4 of their time to labor activities.

### 4.4 Posterior distributions and estimation results

Table 3 reports posterior means and 90% probability intervals for structural parameters jointly with the corresponding prior values. According to the reported results, all

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20The algorithm for the Bayesian estimation works as follows. First, the posterior kernel is maximized to find the mode of the posterior distribution. Second, starting from a random perturbation around the mode, a random-walk Metropolis-Hastings algorithm is used to sample from the posterior distribution. We run this algorithm 4 times from different perturbation points, eventually building 4 chains of 200,000 draws each. This strategy seems to ensure a relatively rapid convergence of the Markov chains. The convergence diagnostics indicate that approximately 60,000 draws are sufficient to attain convergence for all of the parameters. Finally, we report selected statistics for the posterior distributions by computing the average of correspondent moments from all of the chains, wherein we discard the initial 40% of observations from each chain to remove the dependence from the initial condition.
advertising-related parameters are estimated quite accurately. The data appear to be particularly informative regarding the competitiveness parameter, $\gamma$, whose posterior probability interval is approximately 60% smaller than its prior counterpart. Its posterior mean is 0.189, and the upper bound $\gamma = 1$ lies well outside the 90% posterior probability interval, thereby indicating that the market-size channel of advertising is active in the U.S. economy. The data are also substantially informative with respect to the parameter that controls for the adjustment costs of advertising, $\psi_z$. Its posterior mean is equal to 1.327, suggesting a much faster response of advertising to shocks than what was assumed a priori.

As for the stochastic processes, we find that government shocks, labor supply shocks and economy-wide productivity shocks are quite persistent, with coefficients of autocorrelation ranging between 0.89 and 0.98. The advertising and investment-specific shocks show a milder persistence with autocorrelation coefficients equal to 0.71 and 0.54, respectively. This result is likely to be driven by the presence of adjustment costs, which generate endogenous persistence and therefore reduce the degree of exogenous autocorrelation needed to match the actual persistence in investment and advertising data. The estimates of the other structural parameters are consistent with the evidence available in the literature, with the exception of the habit persistent parameter, $\zeta$, which is somewhat lower than existing estimates (e.g., Smets and Wouters, 2007).

As Section 5 clarifies, the competitiveness parameter $\gamma$ plays a central role in determining the effect of advertising on consumption, and therefore, we conduct a number of robustness checks on its estimates. First, we re-estimate the model using a series of hours worked in first differences instead of demeaned log levels. Second, we allow for measurement error in aggregate advertising. Finally, we estimate an alternative version of the model in which advertising is assumed to be purely competitive ($\gamma = 1$). The results are summarized in Table 4. For each specification, we report the estimated $\gamma$, its 90% posterior probability interval, and the marginal log data density. For the sake of convenience, the baseline estimation is reported in the last line.

As the table illustrates, the estimate $\hat{\gamma}$ is stable across specifications, and the 90% posterior probability interval never includes the upper bound $\gamma = 1$. These findings, together with the fact that the restricted model with $\gamma = 1$ yields the lowest marginal likelihood, confirm

\begin{table}[h]
\centering
\begin{tabular}{|l|c|c|c|}
\hline
Model & Parameter Value ($\hat{\gamma}$) & 90% interval & Log Data Density \\
\hline
Hours in difference & 0.19 & [0.00 0.39] & 1837.44 \\
Measurement Error & 0.18 & [0.00 0.37] & 1833.43 \\
$\gamma = 1$ & – & – & 1829.91 \\
Baseline & 0.19 & [0.00 0.38] & 1836.97 \\
\hline
\end{tabular}
\caption{Robustness}
\end{table}

21The first robustness exercise controls for the assumption that the economy evolves along a balanced growth path in which hours worked are stationary. As noted by Chang, Doh and Schorfheide (2007), this theoretical restriction might be inconsistent with the data, potentially leading to biases in the estimates of the structural parameters and the shock processes. The second exercise controls for potential noise in our partial series of aggregate advertising. The last exercise is useful to compare the likelihood of the baseline model with the alternative specification in which the complementary property of advertising is switched off.
### Table 5: Second-order moments: model vs. data

<table>
<thead>
<tr>
<th></th>
<th>Model</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Standard deviation (percentage)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GDP Growth ($\Delta Y$)</td>
<td>0.97</td>
<td>0.77</td>
</tr>
<tr>
<td>Investment Growth ($\Delta I$)</td>
<td>1.99</td>
<td>1.90</td>
</tr>
<tr>
<td>Demeaned Hours Worked ($H$)</td>
<td>0.97</td>
<td>1.15</td>
</tr>
<tr>
<td>Consumption Growth ($\Delta C$)</td>
<td>0.71</td>
<td>0.54</td>
</tr>
<tr>
<td>Advertising Growth ($\Delta Z$)</td>
<td>4.83</td>
<td>3.85</td>
</tr>
<tr>
<td><strong>Correlations</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta C, \Delta Y$</td>
<td>0.56</td>
<td>0.59</td>
</tr>
<tr>
<td>$\Delta I, \Delta Y$</td>
<td>0.57</td>
<td>0.67</td>
</tr>
<tr>
<td>$H, \Delta Y$</td>
<td>0.06</td>
<td>-0.01</td>
</tr>
<tr>
<td>$\Delta Z, \Delta Y$</td>
<td>0.15</td>
<td>0.11</td>
</tr>
<tr>
<td>$\Delta Z, \Delta C$</td>
<td>0.26</td>
<td>0.25</td>
</tr>
<tr>
<td>$\Delta Z, \Delta I$</td>
<td>0.01</td>
<td>0.12</td>
</tr>
<tr>
<td>$\Delta Z, H$</td>
<td>-0.07</td>
<td>-0.07</td>
</tr>
</tbody>
</table>

The results of the baseline estimation, thereby providing further evidence in support of the complementary effect of advertising. Our intuition suggests that this result is driven by the effect of advertising on the marginal utility of consumption. In fact, advertising provides an additional mechanism that generates endogenous consumption persistence, which helps the model to match actual persistence in the consumption data. Because this mechanism is active only if $\gamma \neq 1$, the large decline in the log-data density of the restricted model can be interpreted as evidence in favor of advertising as an additional source of consumption persistence.

Finally, we assess the ability of the model to fit the data by comparing the volatilities and correlations of the observable variables in the model with their counterparts in the data.

---

22 To illustrate this point, note that the optimal goodwill stock in equilibrium can be written as $G_t = \Psi_t C_t$, where $\Psi_t$ is a positive and time-varying coefficient that is obtained by combining the optimal advertising policy (13) with the demand for consumption goods (3). Using this result, we can show that the marginal utility of present consumption is the following:

$$\frac{\partial U}{\partial C_t} = \{C_t - \zeta [C_{t-1} + B(\Psi_{t-1} C_{t-1}) - B(z_t + (1 - \delta_g)\Psi_{t-1} C_{t-1})/\zeta]\}^{-\sigma}$$

The last two terms on the right-hand side of the above equation capture the effect of advertising on consumption persistence. In particular, the equation shows that as long as $\gamma \neq 1$, advertising may either strengthen or soften the effect of $C_{t-1}$ on the marginal utility of current consumption, depending on the sign of $B(G_{t-1}) - B(G_t) \zeta$. A possible interpretation of this effect is that advertising, by modifying the demand for consumption goods over time, gives rise to customers’ brand loyalties, which may either reinforce or dissolve preexisting consumption habits.
data.23 According to Table 5, the model adequately replicates the main features of the data. All of the actual statistics fall into the corresponding 90\% probability interval, with the exception of the standard deviations of consumption and advertising, which are slightly over-estimated. The fit of co-movements between advertising and the other aggregates is particularly accurate, showing that the model adequately replicates the pattern of cross-correlations measured in the data.

5 Quantitative Results

In what follows, we use the estimated model to dwell on the long- and short-run implications of advertising by analyzing the steady-state equilibrium (Section 5.1) and aggregate dynamics (Section 5.2). The effect of advertising is disentangled by comparing the properties of the estimated model with those of an identical economy in which advertising is banned. We refer to this case as the benchmark economy without advertising.

5.1 Long-run effects

The top panel of Table 6 displays the steady states of selected endogenous variables in percentage deviations from their benchmark counterparts.24 According to the reported results, the estimated model predicts that advertising stimulates consumption (4.15\%), GDP (3.89\%) and investment (2.62\%); raises the markup (0.61\%); and leads to a more consumption-based economy (\( \Delta C/Y = 0.29\% \)). The strong increase in hour worked (6.90\%) and the moderate decline in the wage rate (-1.62\%) suggest that the underlying mechanism operates through a labor supply channel. In the presence of advertising, the labor supply schedule evaluated in the steady state is

\[
H = \left( 1 - \beta \frac{\zeta}{\tau} \right)^{\frac{1}{\phi}} \left( \frac{W}{\xi h} \right)^{\frac{1}{\phi}} \left( C + B(G) - \frac{\xi}{\phi} \right)
\]

where, according to equations (14) and (15),

\[
B(G) = \frac{1 + \gamma G}{1 + G}
\]

Hence, as long as \( \gamma \neq 1 \), advertising tilts the marginal rate of substitution between consumption and leisure toward the former. The labor supply schedule thus shifts to the right

---

23 Model-based moments involved in the computation of the statistics reported in the table are estimated using the following procedure. First, we construct a random selection of 1,000 draws from the posterior distribution. Second, for each vector of parameters, we generate 100 artificial time series of the main variables of length equal to that of the data. Third, we detrend the artificial data with the HP filter(1600) and use the resulting series to compute second-order moments. Finally, summary statistics for the posterior distribution of moments are computed by pooling together all simulations.

24 The non-stochastic steady state is derived by setting each shock equal to its mean value and assuming that endogenous variables are constant over time. To account for parameter uncertainty, we construct a random section of 50,000 draws from the posterior distribution and compute the steady-state effects for each parameter vector. The summary statistics reported in Table ?? are obtained by pooling together all simulations.
and exerts downward pressure on real wages. In equilibrium, this mechanism triggers a twofold effect. On the one hand, firms expand their production, which induces a positive wealth effect that eventually increases consumption and investment. On the other hand, firms substitute capital with labor, thus reducing the optimal ratio of capital to labor. Eventually, the investment share in equilibrium decreases, explaining the observed increase in the share of consumption. Additionally, market-enhancing advertising is effective in shifting consumers’ preferences, and firms successfully exploit the wedge between perceived and true product differentiation by charging higher prices. As a result, the average markup also increases in equilibrium.

The mechanism linking advertising to consumption through the labor supply is known in the literature as the work and spend cycle and has been empirically supported by Baker and George (2010) for the U.S. and by Fraser and Paton (2003) for the U.K. Our results complement and extend this empirical evidence showing that, contrary to conventional wisdom, advertising does not necessarily stimulate consumption at the expense of savings. In fact, the estimated model argues that in equilibrium, consumption and investment both increase and contribute to fostering economic activity.

In general, the comparison between the estimated model economy and the economy with purely competitive advertising characterized in Proposition 1 highlights the key role of the complementary property in determining both the sign and scale of the aggregate effects of advertising. In sharp contrast to the purely competitive case, we find that (i) market-enhancing advertising is beneficial for GDP and its components and (ii) the effect of the work and spend cycle on hours worked is much larger (approximately 18 times larger) than the labor demand effect induced by purely competitive advertising.25 In the literature, the importance of complementary advertising was already postulated by Galbraith (1958), arguing that monopolistically competitive firms use advertising as a way to manipulate preferences with the aim of fostering high levels of consumption to increase profits. This process would affect agents’ decisions regarding consumption and leisure, eventually increasing the supply of labor and the monopolistic power of firms and inducing a greater need for material consumption that makes the economy more consumption based. This view was subsequently criticized by Solow (1968) and Simon (1970). They argued that because of its competitive nature, advertising affects the composition but not the size of aggregate consumption, therefore being essentially irrelevant in the aggregate.26 According to our estimated results, once both the complementary and competitive properties of advertising are accommodated in the model, the data indicate that the first property dominates the second and that the nature of aggregate spillover effects of advertising eventually closely resembles that conjectured by Galbraith.

In the previous analysis, it remains unclear whether advertising is beneficial for households. Advertising reduces welfare by inducing a higher markup that exacerbates the distortions of monopolistic competition, but it may also improve welfare by fostering consumption and hours worked, which potentially bring the economy closer to competitive equilibrium. We assess the overall effect of advertising by pursuing a welfare analysis that compares the

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25 Side simulations show that the effect of advertising on hours worked (consumption) is in fact monotonically decreasing (increasing) in the degree of advertising competitiveness among firms.

26 See Bagwell (2007) for references on the long-standing debate about aggregate advertising among classical economists.
estimated model economy with the benchmark model without advertising. In the presence of persuasive advertising, however, it is not obvious which reference welfare criterion should be used. As advertising changes consumers’ tastes, there are at least two natural yardsticks for welfare comparisons: the pre-advertising and post-advertising preference relations. We follow Benhabib and Bisin (2002) by relying on a welfare criterion that considers both preferences. Formally, let $G$ and $U(C(G), H(G), G)$ respectively denote the equilibrium level of the goodwill and the utility function associated with that equilibrium allocation. Then, a welfare criterion can be defined as follows.

**Definition 1.** The household is better off in the presence of advertising if and only if the welfare increases with respect to post-advertising preferences,

$$U(C(G), H(G), G) \geq U(C(0), H(0), G)$$

and it also increases with respect to pre-advertising preferences,

$$U(C(G), H(G), 0) \geq U(C(0), H(0), 0)$$

with at least one inequality holding strictly.

Definition 1 states that the consumer is better off with advertising ($G > 0$) if and only if he prefers the allocation $\{C(G), H(G)\}$ to the allocation $\{C(0), H(0)\}$, regardless of which welfare yardstick is used. To simplify the exposition, we compute the welfare costs as the

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27 This issue is rather controversial in the literature. See Dixit and Norman (1978) and Benhabib and Bisin (2002) for a detailed discussion of this topic.

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Table 6: Long-run effects

<table>
<thead>
<tr>
<th></th>
<th>Median</th>
<th>90% interval</th>
<th>$\gamma = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Steady state effects</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Consumption</td>
<td>4.20</td>
<td>[0.96 9.29]</td>
<td>-1.58</td>
</tr>
<tr>
<td>Total Hours Worked</td>
<td>6.90</td>
<td>[3.34 12.4]</td>
<td>0.39</td>
</tr>
<tr>
<td>Wage</td>
<td>-1.62</td>
<td>[-2.34 -0.92]</td>
<td>0.13</td>
</tr>
<tr>
<td>Production-related Labor</td>
<td>4.34</td>
<td>[1.05 9.48]</td>
<td>-1.51</td>
</tr>
<tr>
<td>Output</td>
<td>3.89</td>
<td>[0.73 8.88]</td>
<td>-1.56</td>
</tr>
<tr>
<td>Investment</td>
<td>2.62</td>
<td>[-0.05 7.15]</td>
<td>-1.47</td>
</tr>
<tr>
<td>Markup</td>
<td>1.22</td>
<td>[0.70 1.75]</td>
<td>-0.09</td>
</tr>
<tr>
<td>Consumption share</td>
<td>0.29</td>
<td>[0.16 0.44]</td>
<td>-0.02</td>
</tr>
<tr>
<td><strong>Welfare</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\tau_0$ (percentages)</td>
<td>-3.12</td>
<td>[-4.60 -2.29]</td>
<td></td>
</tr>
<tr>
<td>$\tau_G$ (percentages)</td>
<td>-2.20</td>
<td>[-1.68 -0.70]</td>
<td></td>
</tr>
<tr>
<td>$H_G/H_{ce}$</td>
<td>1.04</td>
<td>[0.90 1.07]</td>
<td></td>
</tr>
<tr>
<td>$C_G/C_{ce}$</td>
<td>0.93</td>
<td>[0.78 1.11]</td>
<td></td>
</tr>
</tbody>
</table>
fraction of steady-state consumption that the household would be willing to give up and the costs under a policy intervention that completely bans advertising \((G = 0)\), i.e.,

\[
U((1 - \tau_s)C(G), H(G), s) = U(C(0), H(0), 0) \text{ with } s = \{0, G\}
\]

where \(\tau_0\) and \(\tau_G\) denote the welfare costs associated with pre-advertising and post-advertising preferences, respectively. Hence, Definition 1 holds when both \(\tau_0 \geq 0\) and \(\tau_G \geq 0\), with at least one inequality holding strictly. The results are reported in the bottom panel of Table ???. We find that advertising is unambiguously detrimental to welfare because the household is worse off according to both ex ante and ex post preferences (\(\tau_G\) and \(\tau_0\) are both negative). Households are willing to reduce up to 3\% of their long-run consumption level to embrace a policy that bans advertising. The origin of this result can be traced back to the overworking effect generated by advertising that offsets the positive effect on consumption. To illustrate this point, in the last two lines of Table 6, we compare the baseline allocation with the perfectly competitive allocation \(\{C_{ce}, H_{ce}\}\).\(^{28}\) Advertising pushes households to work 4\% more hours while enjoying 7\% less consumption. Overworking and frugality are thus summed in determining the observed welfare losses.

### 5.2 Short-run effects

Panel A of Table 7 presents the results of the asymptotic variance decomposition for the estimated model (panel A.1) and for the counter-factual economy with purely competitive advertising (panel A.2).\(^{29}\) The reported contribution of the advertising-specific shock \((\epsilon_{zt})\) provides direct information on the nature of the spillover effects from the advertising sector to the broader economy. The result shows that exogenous shifts in advertising productivity contribute sensibly to fluctuations in consumption, investment and markup, respectively accounting for 16.5\%, 12.1\% and 7.5\% of their volatility. On the contrary, their contribution to the volatility of total hours worked appears small, and the contribution to the volatility in GDP is almost null. These findings suggest that in the short run, the spillover effects of advertising are concentrated on consumption, investment and, to a lesser extent, markup. The results reported in panel A.2 confirm the long-run analysis: the complementary property and the associated market-enhancing effect are crucial for advertising to influence aggregate dynamics. When this property is switched off, in fact, the contribution of advertising-specific shock to the volatility of all considered aggregates falls virtually to zero.

To shed light on the mechanisms through which advertising affects aggregate dynamics, Figure 3 reports the impulse response functions of some selected endogenous variables with respect to a 1\% increase in the advertising-specific shock. A transitory boost in the productivity of advertising (i) raises consumption, hours worked, markup and advertising itself; (ii) induces a decline in investment and in the wage rate; and (iii) leaves the output virtually unaffected. The increase in advertising at the impact is due to the reduced marginal cost, which, in the presence of adjustment costs, implies that changes in advertising gradually

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\(^{28}\)Perfectly competitive allocations are determined from the benchmark model without advertising by setting the average markup equal to 1.

\(^{29}\)The results for the counter-factual economy are obtained by setting \(\gamma = 1\) and holding all other parameters at their posterior mean values.
Table 7: Short-run Spillovers

<table>
<thead>
<tr>
<th>Variables</th>
<th>Consumption</th>
<th>Hours Worked</th>
<th>Investment</th>
<th>GDP</th>
<th>Markup</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A) Asymptotic Variance Decomposition</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(A.1) Baseline Estimation</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Productivity ($\epsilon_y$)</td>
<td>32.9</td>
<td>7.80</td>
<td>24.3</td>
<td>55.4</td>
<td>51.0</td>
</tr>
<tr>
<td>Labor supply ($\epsilon_h$)</td>
<td>21.0</td>
<td>78.5</td>
<td>15.3</td>
<td>37.2</td>
<td>35.8</td>
</tr>
<tr>
<td>Government ($\epsilon_f$)</td>
<td>22.0</td>
<td>9.30</td>
<td>6.30</td>
<td>5.50</td>
<td>4.30</td>
</tr>
<tr>
<td>Investment ($\epsilon_k$)</td>
<td>12.0</td>
<td>2.90</td>
<td>37.5</td>
<td>1.80</td>
<td>1.40</td>
</tr>
<tr>
<td>Advertising ($\epsilon_z$)</td>
<td>12.1</td>
<td>1.60</td>
<td>16.5</td>
<td>0.10</td>
<td>7.50</td>
</tr>
<tr>
<td>(A.2) Purely competitive advertising ($\gamma = 1$)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Productivity ($\epsilon_y$)</td>
<td>36.7</td>
<td>7.30</td>
<td>27.8</td>
<td>56.3</td>
<td>56.5</td>
</tr>
<tr>
<td>Labor supply ($\epsilon_h$)</td>
<td>23.5</td>
<td>81.3</td>
<td>19.0</td>
<td>37.0</td>
<td>37.1</td>
</tr>
<tr>
<td>Government ($\epsilon_f$)</td>
<td>24.9</td>
<td>8.60</td>
<td>7.30</td>
<td>4.80</td>
<td>4.50</td>
</tr>
<tr>
<td>Investment ($\epsilon_k$)</td>
<td>14.7</td>
<td>2.80</td>
<td>45.7</td>
<td>1.60</td>
<td>1.80</td>
</tr>
<tr>
<td>Advertising ($\epsilon_z$)</td>
<td>0.20</td>
<td>0.10</td>
<td>0.20</td>
<td>0.40</td>
<td>0.40</td>
</tr>
<tr>
<td>(B) Historical contribution</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Change in volatility (%)</td>
<td>26.1</td>
<td>4.78</td>
<td>-39.2</td>
<td>-7.13</td>
<td>56.1</td>
</tr>
</tbody>
</table>

occur thus inducing the observed hump-shaped response. With this increase in advertising expenditures, the price elasticity of the demand declines because of the brand equity effect, and this decline triggers a persistent and countercyclical movement in the average markup. At the same time, advertising increases the marginal utility of current consumption, and this occurs not only at the impact but also in all of the sub-sequential quarters in which the goodwill stock increases. This effect gives rise to an inter-temporal mechanism that makes households less willing to smooth consumption through savings (urge in consumption) and triggers a persistent decline in investment accompanied by a positive and hump-shaped response of consumption.\(^{30}\) Finally, the increase in total hours worked and the decline in the real wage indicate that the work and spend mechanism is also active in the short run. As Figure 3 clarifies, however, production-related labor decreases for several quarters after the shock as a result of the reallocation of labor to the advertising sector. This effect dampens the overall effect of advertising on total hours worked, thereby softening the quantitative implications of the work and spend channel on output fluctuations.

For the sake of completeness, Figure 3 also depicts the impulse response functions in the counter-factual economy with purely competitive advertising. As the figure illustrates, the market-enhancing effect is a necessary condition for advertising to trigger a positive and persistent response of consumption. Absent this effect ($\gamma = 1$), the response of consumption

\(^{30}\)Note that this effect is in contrast to the outcome of the long-run analysis, in which the increase in consumption does not occur at the expense of savings. The reason is that the inter-temporal mechanism is absent in the long-run equilibrium, and therefore, the wealth effect induced by the work and spend mechanism fuels both consumption and investment.
to a boost in advertising productivity is negative and short-lived. In this case, the model predicts a counter-factual negative correlation between consumption and advertising, which is a property that hinges exclusively on the presence of advertising-specific shocks. This finding suggests that the mechanism triggered by this shock is most likely the source of identifying information for parameter $\gamma$. In this respect, note that the response of advertising is not affected by the value of $\gamma$ (see Figure 3), meaning that this parameter is not identified by the cyclical properties of advertising in the data (i.e., volatility and persistence). Additionally, we find that all of the other advertising-related parameters affect the magnitude but not the sign of the response of consumption to advertising-specific shocks. Hence, $\gamma$ is the only parameter that determines the sign of the predicted co-movements between advertising and consumption.

To quantify the overall spillover effects, we provide in-sample estimates of the historical role played by advertising in shaping U.S. aggregate dynamics. To this end, Figure 4 presents the predicted consumption and investment (continuous lines) along with their counterparts implied by the benchmark model without advertising (dashed lines), whereas Panel B of Table 7 reports the in-sample variance of selected endogenous variables expressed as percentage deviations from their benchmark counterparts. As the figure illustrates, fluctuations in consumption are generally amplified in the estimated model. Because of these effects, we find that consumption volatility increases by 26% with advertising. Investment shows an opposite pattern, and in fact, in-sample volatility decreases by 39.2%, suggesting that

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Footnotes:

31 Impulse-response analysis shows that an advertising-specific shock is the only case in which changing the value assigned to parameter $\gamma$ reverts the sign of the response of consumption. These results are available from the authors upon request.

32 The reported results are obtained by setting the parameters to their posterior mean values and using the resulting smoothed estimates of the shocks to simulate both the baseline and benchmark models. To facilitate comparisons, the series reported in Figure 4 are normalized to equal 1 in the initial quarter.
the intertemporal mechanism described above is the main driving force behind the spillover effects on consumption. We also find that (i) the volatility of markup substantially increases (56.1%) and (ii) the spillover effects on total hours worked (4.8%) and GDP (-7.1%) are quantitatively less important in comparison with the remaining aggregates. These findings confirm that the short-run spillover effects of advertising are concentrated on consumption, investment and markup.

6 Conclusions

This paper assesses the role of marketing practices in stimulating consumption and economic activity. By estimating a variant of the neoclassical growth model that accounts for firms’ spending on advertising with U.S. data, we show that advertising has a relevant long-run effect on consumption and work activities. The mechanism operates through a work and spend channel: in the presence of advertising, people work more to afford larger purchases of goods, and the perceived need for higher consumption results from the advertising signals to which consumers are exposed. Because of this effect, advertising enhances the production of output, eventually increasing consumption and investment. We also show that in the short run, the effect of the work and spend channel on output and hours worked is offset by the counterwork of an intertemporal substitution mechanism. Households substitute savings with current consumption, and the resulting reduction in aggregate investment offsets the increase in consumption, maintaining production close to the that observed in a counterfactual economy in which advertising is banned. In fact, we find that advertising affects the business cycle primarily through the dynamics of consumption, investment and prices.

Finally, it should be noted that the flexible-price one-sector model developed in this paper uses the worst-case scenario for advertising to influence economic activity. In fact,
in a model with nominal rigidities, the work and spend channel can be enhanced because the lower wage variability might determine a larger increase in consumers’ supply of labor in response to advertising shocks. Moreover, in a one-sector model, any increase in markup due to advertising makes investment more expensive, therefore reducing the real return on capital and decreasing savings. As a result, the effect of advertising on investment is negative in the short run, further smoothing the overall effect of advertising on economic activity.

References


Appendix

A Data

<table>
<thead>
<tr>
<th>Source</th>
<th>Media</th>
<th>Frequency</th>
<th>Sample</th>
</tr>
</thead>
<tbody>
<tr>
<td>Robert J. Coen</td>
<td>All Media</td>
<td>Annual</td>
<td>1948-2007</td>
</tr>
<tr>
<td>Ad$Summary (quarterly issues)</td>
<td>All Media</td>
<td>Quarterly</td>
<td>1976.I-2006.II</td>
</tr>
<tr>
<td>Newspapers Association of America</td>
<td>Newspapers</td>
<td>Quarterly</td>
<td>1976.I-2006.II</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Macroeconomic Variable</th>
<th>Code</th>
<th>Freq.</th>
<th>Sample</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real GDP</td>
<td>GDPC96</td>
<td>Quarterly</td>
<td>1976.I-2006.II</td>
</tr>
<tr>
<td>Real Export</td>
<td>EXPGSC96</td>
<td>Quarterly</td>
<td>1976.I-2006.II</td>
</tr>
<tr>
<td>Real Import</td>
<td>IMPGSC96</td>
<td>Quarterly</td>
<td>1976.I-2006.II</td>
</tr>
<tr>
<td>Real Personal Consumption Exp.</td>
<td>PCECC96</td>
<td>Quarterly</td>
<td>1976.I-2006.II</td>
</tr>
<tr>
<td>Real Private Fixed Investment</td>
<td>FPIC96</td>
<td>Quarterly</td>
<td>1976.I-2006.II</td>
</tr>
</tbody>
</table>

*All Media* refers to: Magazines, Sunday Magazines, Outdoor, Network Television, Spot Television, Cable Television, Syndicated Television, Network Radio, Spot radio, Newspapers. Data sources: Coen’s data (https://spreadsheets.google.com/pub?key=p9LENaiKJey0BXYyRIFZEEw); NAA data (www.naa.org); Business Cycle Indicators (research.stlouisfed.org/fred2). *Code* refers to the coding system of the *FRED II* database issued by the Federal Reserve Bank of St. Louis.

B The model equations

This appendix summarizes the system of equations describing the decentralized equilibrium of the economy. The equilibrium price of goods is normalized to 1 in each period and, accordingly, the price index is: \( P_t = 1 \ \forall \ t \). Then, using the optimal demand for investment and consumption goods, the budget constrain for the representative household can be written as

\[
C_t + I_t + T_t = W_t H_t + R_t u_t K_t + \Pi_t - a(u_t) K_t
\]

where the capital utilization cost is given by

\[
a(u_t) = R \left( \omega u_t^2 / 2 + (1 - \omega) u_t + \omega / 2 - 1 \right)
\]
with $R$ denoting the steady state rental rate of capital. The representative household chooses sequence of consumption ($C_t$), worked hours ($H_t$) and capital utilization ($u_t$) by maximizing the inter-temporal utility function under the budget constraint and the capital law of motion

$$K_{t+1} = (1 - \delta_k) \bar{K}_t + \xi_t^k (1 - \phi_{k,t}) I_t \quad \text{(B1)}$$

where the capital adjustment cost is specified as

$$\phi_{k,t} = \frac{\psi_k}{2} \left( \frac{I_t}{I_{t-1}} - \gamma \right)^2$$

with $\gamma$ denoting the growth rate of investment in the balanced growth path equilibrium.

The first order conditions for an interior solution of the utility maximization problem are:

$$\lambda_t = \Gamma_t^{\sigma} \left( \bar{C}_t - \zeta \bar{C}_{t-1} \right)^{-\sigma} - \Gamma_{t+1}^{\sigma} \beta E_t \left( \bar{C}_{t+1} - \zeta \bar{C}_t \right)^{-\sigma} \quad \text{(B2)}$$

$$\lambda^k_t = \beta E_t \left\{ (1 - \delta_k) \lambda^k_{t+1} + (R_{t+1} u_{t+1} - a'(u_{t+1})) \lambda_{t+1} \right\} \quad \text{(B3)}$$

$$\lambda_t = \lambda^k_t \left[ \xi_t^h (1 - \phi_{k,t}) - \frac{\partial \phi_{k,t}}{\partial I_t} I_t \right] - \beta E_t \left\{ \lambda^k_{t+1} \xi_{t+1} \frac{\partial \phi_{k,t+1}}{\partial I_t} I_{t+1} \right\} \quad \text{(B4)}$$

$$\lambda_t W_t = \xi_t^h H_t^\phi$$

$$R_t = a'(u_t) \quad \text{(B5)}$$

where $\lambda_t$ and $\lambda^k_t$ respectively denote the Lagrange multipliers associated with the budget constraint and the capital law of motion. Using equation (14), the consumption aggregate (2) can then be written as

$$\bar{C}_t = C_t + B(G_t, G_t, \Gamma_t) \equiv C_t + \Gamma_t \left[ 1 + \gamma (G_t / \Gamma_t) \right] \quad \text{(B7)}$$

At the symmetric equilibrium, total output ($Y_t$) and aggregate advertising ($Z_t$) are, respectively, given by

$$Y_t = A_t^y K_{t,t}^{1-\alpha} (\Gamma_t H p_t)^\alpha \quad \text{(B8)}$$

$$Z_t = A_t^z \Gamma_t H a_t^{\alpha z} \quad \text{(B9)}$$

where $K_t$ denotes effective capital that evolves according to

$$K_t = u_t \bar{K}_t \quad \text{(B10)}$$

Aggregate advertising accumulates into the goodwill stock ($G_t$) according to the law of motion

$$G_t = (1 - \delta_g) G_{t-1} + (1 - \phi_{z,t}) Z_t \quad \text{(B11)}$$

where the advertising adjustment cost is specified as

$$\phi_{z,t} = \frac{\psi_z}{2} \left( \frac{Z_t}{Z_{t-1}} - \gamma_z \right)^2$$
with $\gamma_z$ denoting the growth rate of advertising in the balanced growth path. The first order conditions for profit-maximizing firms are:

$$\mu_t = \frac{\varepsilon (1 + B(\cdot)/Y_t)}{1 + \varepsilon (1 + B(\cdot)/Y_t)} \quad \text{(B12)}$$

$$\varphi_t = \mu_t^{-1} \quad \text{(B13)}$$

$$R_t = \varphi_t (1 - \alpha) \frac{Y_t}{K_t} \quad \text{(B14)}$$

$$W_t = \varphi_t \alpha \frac{Y_t}{H_{p,t}} \quad \text{(B15)}$$

$$\nu_t^z = \alpha_z^{-1} \left( \frac{W_t}{Z_t} \right) H_{a,t} \quad \text{(B16)}$$

$$\nu_t^z = \nu_t \left[ (1 - \phi_z, \cdot) - \frac{\partial \phi_z, t}{\partial Z_t} Z_t \right] - \beta E_t \left\{ \nu_{t+1} \frac{\partial \phi_z, t+1}{\partial Z_t} Z_{t+1} \right\}$$

$$-(1 - \varphi_t) B_{g_t}(G_{t+1}, G_{t+1}, \Gamma_{t+1}) + (1 - \delta_g) E_t \{ \nu_{t+1} Q_{t+1} \} = \nu_t \quad \text{(B17)}$$

where $Q_{t,t+1} = \beta(\lambda_{t+1}/\lambda_t)^{-\sigma}$ is the stochastic discount factor, while $B_{g_t}(G_t, G_t, \Gamma_t)$ denotes the partial derivative of $B(\cdot)$ with respect to its first argument evaluated at the aggregate goodwill stock, $G_t$.\(^\text{33}\)

Market clearing requires that the following conditions hold

$$H_t = H_{a,t} + H_{p,t} \quad \text{(B19)}$$

$$Y_t = C_t + I_t + F_t + a(u_t) \tilde{K}_t \quad \text{(B20)}$$

while fiscal revenues are determined on a balanced basis, according to which $T_t = F_t \forall t$. Furthermore, in equilibrium dividends paid to households read as

$$\Pi_t = (1 - \varphi_t) Y_t - \alpha_z \nu_t^z Z_t$$

while GDP for this economy is defined as

$$GDP_t = C_t + I_t + F_t \quad \text{(B21)}$$

Letting $X_t = \{G_t, \mu_t, Z_t, H_t, H_{a,t}, H_{p,t}, C_t, \bar{K}_{t+1}, I_t, Y_t, R_t, W_t, \bar{C}_t, u_t, K_t, \nu_t^z, \nu_t^z \lambda_t, \lambda_t^k, GDP_t, \varphi_t\}$ denote the vector of endogenous variables and $\vartheta_t = \{A_t^y, A_t^z, \xi_t^k, \xi_t^h, F_t\}$ be the vector of exogenous state variables, a symmetric equilibrium can then be formally defined as a pair of initial conditions $(\tilde{K}_0, G_0) \in R^2_+$ and a sequence $\{X_t\}_{t=0}^\infty$ that solves equations (B1)-(B21) given the stochastic process $\{\vartheta_t\}_{t=0}^\infty$.\(^\text{33}\)

Equation (B18) thus implies that firms do not take into account the effect on the aggregate goodwill stock while choosing their optimal investment in advertising.

\(^{33}\)http://www.upo.es/econ
C  Balanced growth path equilibrium

The following proposition summarizes sufficient conditions to guarantee that balanced growth path equilibria exist.

**Proposition 2.** Consider an economy in which monopolistically competitive firms may promote their products by incurring advertising expenditures. Suppose that consumer preferences are defined as in (1), that technologies for producing goods and advertising are respectively given as in (8) and (9), and that government spending evolves according to (7). Then, denoting with \( G_t \) the aggregate goodwill stock at the symmetric equilibrium (i.e. \( g_{i,t} = g_{-i,t} = G_t \quad \forall \quad i \in [0,1] \) and \( t \geq 0 \)), a sufficient condition for a balanced growth path equilibrium to exist is that:

\[
\frac{G_{t+1}}{G_t} = \tau
\]

implies both:

\[
\frac{B(G_{t+1}, G_{t+1}, \Gamma_{t+1})}{B(G_t, G_t, \Gamma_t)} = \tau \quad (i)
\]

and

\[
B_{g_i}(G_{t+1}, G_{t+1}, \Gamma_{t+1}) = B_{g_i}(G_t, G_t, \Gamma_t) \quad (ii)
\]

**Proof.** The assumptions of labor-augmenting technologies together with the law of motions (B1) and (B11) and the market clearing conditions (B19)-(B20) imply that along the balanced growth path, consumption, output, investment, capital, goodwill stock and advertising grow at the common rate \( \tau \), while capital utilization rate, worked hours, production-related and advertising related labor stay constant.\(^{34}\) In addition, these conditions imply that the marginal product of capital \( Y_t/K_t \) stays constant, while the marginal product of labor \( Y_t/H_{p,t} \) grows at the rate \( \tau \). Thus, denoting one plus the growth rate of a variable \( x \) as \( \gamma_x \), in the balanced growth path equilibrium it must be true that:

\[
\gamma_y = \gamma_k = \gamma_c = \gamma_a = \gamma_z = \gamma_g = \gamma_f = \gamma_y/H_p = \gamma > 1
\]

(C1)

and

\[
\gamma_h = \gamma_{h_a} = \gamma_{h_p} = \gamma_{y/k} = \gamma_u = 1
\]

(C2)

Conditions (C1)-(C2) describe the technologically feasible steady state. To prove the proposition, we need to show that any function \( B(\cdot) \) that satisfies the sufficient conditions (i) and (ii) also guarantees that technological feasible steady state verifies the first order conditions for households and firms. To this end, let us assume that function \( B(\cdot) \) conforms to proposition 2. Then, because of property (i), in the balanced growth path equilibrium \( B(\cdot) \) grows at the same rate of output, thereby implying that the average markup (B12) and the output marginal cost (B13) stay constant (i.e. \( \gamma_{\mu} = \gamma_{\phi} = 1 \)). Moreover, the real wage (B14) grows at the rate \( \tau \) (\( \gamma_w = \tau \)) while the interest rate (B15) stays constant (\( \gamma_r = \)

\(^{34}\)The proof of these results is standard. The interested reader is referred to King, Plosser and Rebelo (1988) for further details.

http://www.upo.es/econ
1). This last condition together with \( \gamma_a = 1 \) implies that the optimality condition (B6) is verified. Consider now the consumption aggregate \( \tilde{C}_t \) and rewrite its growth rate as follows

\[
\frac{\tilde{C}_t}{\tilde{C}_{t-1}} = \left( \frac{C_t}{C_{t-1}} - \frac{B(G_t, G_t, \Gamma_t)}{B(G_{t-1}, G_{t-1}, \Gamma_{t-1})} \right) \frac{C_{t-1}}{C_{t-1}} + \frac{B(G_t, G_t, \Gamma_t)}{B(G_{t-1}, G_{t-1}, \Gamma_{t-1})}
\]

Therefore if \( C_t \) and \( B(G_t, G_t, \Gamma_t) \) grow at the common rate \( \tau \), then also \( \tilde{C}_t \) does \( (\gamma_c = \tau) \), implying from equation (B2) that the Lagrange multiplier \( \lambda_t \) grows at the rate \( \tau^{-1} \). This last condition together with the fact that \( \gamma_w = \tau, \gamma_r = 1 \) and \( \phi_{k,t} = \phi'_{k,t} = 0 \) imply that the optimality conditions (B3)-(B5) are verified. It remains to prove that also the equations characterizing the optimal advertising policy of each firm are satisfied along the balanced growth path. To this end, notice first that because of \( \phi_{z,t} = \phi'_{z,t} = 0 \), equation (B17) implies that \( \nu_t^2 = \nu_t \), with the result that the optimality condition (B18) is verified only if function \( B_{g_t}(\cdot) \) stays constant. But since \( \gamma_g = \tau \), this condition is guaranteed by item (ii) of proposition 2.

It is easy to verify that the parametrization chosen for \( B(\cdot) \) (see equation 14) conforms to Proposition 2, and therefore our model economy features steady state growth.

D Proof of Proposition 1

Throughout the proof we refer to \( X_{CA} \) and \( X_{NA} \) as the steady states of \( X \), respectively, in the model economy with purely competitive advertising and in the benchmark model without advertising. Without loss of generality, we assume that the steady state level of government spending, \( F \), is equal to 0 in both economies. We first prove by contradiction that \( Y_{CA} < Y_{NA} \) and then we show that this implies \( \mu_{CA} < \mu_{NA}, C_{CA} < C_{NA}, W_{CA} > W_{NA} \) and \( H_{CA} > H_{NA} \).

In the case of purely competitive advertising, equation (2) implies

\[
\frac{\partial \tilde{C}_t}{\partial g_{-i,t}} = \tilde{C}_t^2 \int_0^1 (c_{i,t} + B(\cdot))^2 \left[ \frac{\partial B(\cdot)}{\partial g_{i,t}} \frac{dg_{i,t}}{dg_{-i,t}} + \frac{\partial B(\cdot)}{\partial g_{-i,t}} \right] \, di = 0
\]

Because at the symmetric equilibrium \( g_{i,t} = g_{-i,t} = G_t \) \( \forall i \), and therefore \( dg_{i,t}/dg_{-i,t} = 1 \), the above condition is verified only if

\[
\frac{\partial B(G_t, G_t, \Gamma_t)}{\partial g_{i,t}} = -\frac{\partial B(G_t, G_t, \Gamma_t)}{\partial g_{-i,t}} \quad \forall i \in [0, 1]
\]

(D1)

Now, because the derivative of (3) with respect to \( g_{i,t} \) is equal to

\[
\frac{\partial c_{i,t}}{\partial g_{i,t}} = \frac{\partial \tilde{C}_t}{\partial g_{i,t}} - \left[ \frac{\partial B(\cdot)}{\partial g_{i,t}} + \frac{\partial B(\cdot)}{\partial g_{-i,t}} \frac{dg_{-i,t}}{dg_{i,t}} \right]
\]

then condition (D1) implies that \( \partial c_{i,t}/\partial g_{i,t} = 0 \), which meansthat at the symmetric equilibrium purely competitive advertising leaves the demand function of each firm unaffected.
This property can equivalently be stated as $B_{CA} = B(G_t, G_t, \Gamma_t) = B(0, 0, \Gamma_t) = B_{NA} \forall G_t \geq 0$. In addition, equations (B2) and (B3) prove that the equilibrium interest rate is the same in two economies ($R_{CA} = R_{NA} = \tau/\beta - (1 - \delta_k)$).

Suppose now that $Y_{CA} = Y_{NA}$. Given that $B_{CA} = B_{NA}$, equal output implies equal markup as apparent from equation (B12). The fact that $R_{CA} = R_{NA}$ and $\mu_{CA} = \mu_{NA}$, joint with the equilibrium condition (B13), imply $K_{CA} = K_{NA}$ because of equation (B14), which in turns leads to $(H_p)_{CA} = H_{NA}$ because of (B8), and thus to $W_{CA} = W_{NA}$ because of (B15). Using the law of motion of capital (B1), the optimality condition (B2) and the resources constraint (B19), these results imply $I_{CA} = I_{NA}$, $C_{CA} = C_{NA}$ and $\lambda_{CA} = \lambda_{NA}$. But then the labor supply schedule (B5) implies $H_{CA} = H_{NA}$, and therefore $H_{CA} = (H_p)_{CA} + (H_a)_{CA} = H_{NA}$. Given that $(H_p)_{CA} = H_{NA}$, the last condition holds only if $(H_a)_{CA} = 0$, implying no advertising ($Z = 0$) in the CA economy. Thus, if $Z > 0$ then it must be true that $Y_{CA} \neq Y_{NA}$.

Suppose now that $Y_{CA} > Y_{NA}$. Because of (B12), this assumption would imply $\mu_{CA} > \mu_{NA}$, which in turn determines $(H_p/K)_{CA} > (H/K)_{NA}$ in equation (B14), and $W_{CA} < W_{NA}$ in equation (B15). Using the law of motion of capital to work out $I$ in the resources constraint, dividing the resulting equation by $K$ and solving for $(C/K)$ yields:

\[(C/K)_S = (Y/K)_S - [\tau - (1 - \delta_k)]/\tau \text{ with } S = \{NA, CA\} \tag{16}\]

Since $(H_p/K)_{CA} > (H/K)_{NA}$ equivalently implies $(Y/K)_{CA} > (Y/K)_{NA}$, condition (16) implies

\[(C/K)_{CA} > (C/K)_{NA} \tag{17}\]

Suppose now that $C_{CA} \geq C_{NA}$. Given equation (B5), this assumption and the fact that $W_{CA} < W_{NA}$ jointly imply that $H_{CA} < H_{NA}$. Then, the clearing condition in the labor market implies $(H_p)_{CA} < H_{NA}$, which in turn leads to $K_{CA} < K_{NA}$ because of (17). A smaller amount of both production factors would imply $Y_{CA} < Y_{NA}$, which contradicts the assumption $Y_{CA} > Y_{NA}$. Thus, it must be true that $C_{CA} < C_{NA}$. Given (17), this inequality would imply that $K_{CA} < K_{NA}$, or equivalently, $I_{CA} < I_{NA}$. But then, from the resources constraint, we would obtain $Y_{CA} = (C_{CA} + I_{CA}) < (C_{NA} + I_{NA}) = Y_{NA}$ which again contradicts the assumption $Y_{CA} > Y_{NA}$, thereby proving that $Y_{CA} < Y_{NA}$.

The condition $\mu_{CA} < \mu_{NA}$ follows immediately by applying $Y_{CA} < Y_{NA}$ to equation (B12). Then, the optimality conditions (B14) and (B15) and the inequalities $\mu_{CA} < \mu_{NA}$ and $Y_{CA} < Y_{NA}$ jointly determine (i) $(H_p/K)_{CA} < (H_p/K)_{NA}$ and (ii) $W_{CA} > W_{NA}$. The resource constraint thus implies:

\[(C/Y)_{CA} = 1 - \delta_k (K/Y)_{CA} < 1 - \delta_k (K/Y)_{NA} = (C/Y)_{NA} \tag{D2}\]

where $\delta_k = [\tau - (1 - \delta_k)]/\tau$. Because $Y_{CA} < Y_{NA}$, equation (D2) in turn implies that $C_{CA} < C_{NA}$, which in turn determines $\lambda_{CA} > \lambda_{NA}$ in equation (B2). This last condition together with $W_{CA} > W_{NA}$ implies $H_{CA} > H_{NA}$ because of equation (B5).