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A Decision Making Method for Educational Management Based on Distance Measures

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ABSTRACT

We develop a new approach for decision making in educational management based on the use of distance measures. We focus on the selection of a studies plan from the perspective of an academic institution. We try to develop this approach showing the benefits of establishing an ideal plan that we compare with the available alternatives. We use the Minkowski distance, the ordered weighted averaging (OWA) operator and the interval numbers. The use of the Minkowski distance allows to make comparisons between the ideal plan and the available ones in the market. The OWA operator is an aggregation operator that provides a parameterized family of aggregation operators that includes the maximum, the minimum and the average criteria, among others. And the interval numbers is a very useful technique to represent the information when the environment is very complex, because it gives all the possible results from the minimum to the maximum. We introduce a new aggregation operator called the uncertain generalized ordered weighted averaging distance (UGOWAD) operator. It is a distance aggregation operator that uses the main characteristics of the Minkowski distance, the OWA operator and the interval numbers. We develop an illustrative example where we can see the usefulness of the UGOWAD operator to select a studies plan in education management. The main advantage of using the UGOWAD is that we can consider a wide range of distance aggregation methods in the decision problem. Then, the decision maker gets a more complete view of the decision problem, being able to select the alternative that better fits the interests.

Keywords: decision making; selection of studies plan; uncertainty; Minkowski distance; aggregation operators.
JEL classification: C44; C49; D81; D89.
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Toma de decisiones en procesos de gestión de la educación basados en las medidas de distancia

RESUMEN

Se desarrolla un nuevo modelo para la toma de decisiones en procesos de gestión de la educación basados en las medidas de distancia. El análisis se enfoca en analizar un proceso de selección de plan de estudios desde la perspectiva de una institución académica. Se intenta mostrar la practicidad de utilizar un plan de estudios imaginario que sería el ideal a partir del cual se compararían las diferentes alternativas disponibles. Para realizar esto, se utilizarán diferentes técnicas disponibles en Teoría de la Decisión, como son la distancia de Minkowski, el operador de medias ponderadas (OWA) y los intervalos de confianza. La utilización de la distancia de Minkowski nos permite hacer comparaciones entre un plan de estudios ideal y los disponibles en la realidad. El operador OWA es un operador de agregación que proporciona una familia parametrizada de operadores de agregación entre los cuales se destaca el máximo, el mínimo y la media aritmética. Los intervalos de confianza son de gran utilidad para representar la información cuando el entorno es muy complejo, porque proporciona todos los resultados que se podrían producir desde un mínimo hasta un máximo. Por eso, incluye todos los posibles resultados que se pueden producir. Para realizar esto, se introduce un nuevo operador de agregación denominado como el operador de distancia media ponderada ordenada generalizada incierta (UGOWAD o UMOWAD). Es un operador de agregación de distancias que utiliza las principales características de la distancia de Minkowski, del operador OWA y de los intervalos de confianza. Se desarrolla un ejemplo ilustrativo en donde se puede ver la utilidad del operador UGOWAD para la selección de un plan de estudios en la gestión de la educación. La principal ventaja de utilizar el operador UGOWAD está en poder considerar una amplia gama de operadores de agregación de distancias en el problema decisional. Entonces, el decisor obtiene un visión mucho más completa del problema y está capacitado para seleccionar la alternativa que se acerca más a sus intereses.

Palabras clave: toma de decisiones; selección de plan de estudios; incertidumbre; distancia de Minkowski; operadores de agregación.
Clasificación JEL: C44; C49; D81; D89.
2000MSC: 90B50.

1. INTRODUCTION

Decision making problems are very common in a lot of disciplines, including educational management. Most of the decisions carried out in an educational problem are taken from an intuitive point of view or only with some very basic information. However, in the real life the problems are often not so easy and it is necessary to analyze the information in more detail. Therefore, it is necessary to establish a *decision making model* for making the decision. In the literature, there are a lot of decision making methods (Bustince *et al.*, 2008; Canós and Liern, 2008; Figueira *et al.*, 2005; Merigó, 2008; Xu, 2008b; 2008c; Yager, 1988; 1992; Yager and Kacprzyk, 1997). Some of them are based on the use of distance measures (Gil-Aluja, 1998; 1999; 2001; Gil-Lafuente, 2005; Kaufmann and Gil-Aluja, 1986; 1987; Merigó, 2008; Merigó and Casanovas, 2008; Merigó and Gil-Lafuente, 2006; 2007; 2008a; 2008b; 2008c; 2009a). The distance measures (Hamming, 1950; Kaufmann, 1975; Kaufmann *et al.*, 1994; Merigó, 2008; Szmidt and Kacprzyk, 2000) are a very useful tool for a lot of problems. One of the most known distance measures is the Minkowski distance. It generalizes a wide range of other distances such as the Hamming and the Euclidean distances.

Another useful tool for decision making is the ordered weighted averaging (OWA) operator (Yager, 1988). It is an aggregation operator that provides a method for representing the attitudinal character of the decision maker (the degree of optimism) in the aggregation process. Therefore, by using the OWA we are able to consider uncertain environments according to our attitudinal character. Since its appearance, the OWA operator has been studied by a lot of authors (Beliakov *et al.*, 2007; Calvo *et al.*, 2002; Fodor *et al.*, 1995; Herrera *et al.*, 2003; Merigó, 2008; Merigó and Casanovas, 2009; Merigó and Gil-Lafuente, 2009b; Yager, 1993; 2002; 2008; Yager and Kacprzyk, 1997).

An interesting extension of the OWA is the one that uses distance measures. In general, it is known as the ordered weighted averaging distance (OWAD) operator (Merigó, 2008; Merigó and Gil-Lafuente, 2006; 2007). Further extensions of this approach include the one that uses the OWA operator in the Minkowski distance. It is known as the Minkowski OWAD (MOWAD) operator (Merigó and Gil-Lafuente, 2008b) and it uses generalized means (or the generalized OWA (Karayiannis, 2000; Yager, 2004)) in the OWAD operator. Other extensions are found in Merigó and Gil-Lafuente (2008a; 2008c; 2009a).

Sometimes, the available information can not be represented with exact numbers because the environment is very uncertain. In these cases, it is necessary to use another approach for representing the uncertainty such as the use of interval numbers (Moore, 1966).

The use of interval numbers in the OWA operator is known as the uncertain OWA (UOWA) operator (Xu and Da, 2002). Further developments of the UOWA are found in Merigó (2008), Merigó and Casanovas (2007), Xu (2008a) and Xu and Da (2003).

In this paper we suggest a generalization of the previous aggregation operators that we call the uncertain Minkowski ordered weighted averaging distance (UMOWAD) operator (or also the uncertain generalized OWAD (UGOWAD) operator). It is an aggregation operator that uses distance measures, generalized means, the OWA operator and uncertain information represented in the form of interval numbers. The main advantage of this operator is that it provides a robust formulation that includes a wide range of particular cases. Thus, the decision maker is able to consider a wide range of scenarios and select the one that is in accordance with his interests. Moreover, by using interval numbers we can represent the uncertain information in a more complete way because we can consider the best and worst result that may occur in the problem.

We apply this approach in a decision making problem about the selection of studies plan. We focus on a PhD program where the decision maker wants to select new courses to be implemented in the program. He considers some key relevant factors such as the skills of the professors and the research perspectives of the courses. By using the UMOWAD operator we can consider a wide range of methods for aggregating the information and select the one that it is closest to our interests.

This paper is organized as follows. In Section 2 we briefly describe the interval numbers and some basic distance measures and aggregation operators. Section 3 and Section 4 present the new aggregation operators (the UMOWAD and the Quasi-UOWAD). In Section 5 we briefly describe the decision making process in the selection of studies plan and in Section 6 we give a numerical example. Section 7 summarizes the main conclusions of the paper.

2. PRELIMINARIES

In this Section we briefly review the interval numbers, some basic distance measures and aggregation operators to be used in the selection process. Note that all this aggregation operators are particular cases of the general formulation that will be presented in Section 3. We consider the Minkowski distance, the OWA operator, the UOWA operator, the GOWA operator, the OWAD operator and the MOWAD operator.

2.1 Interval Numbers

The interval numbers (Moore, 1966) are a very useful and simple technique for representing the uncertainty. It has been used in an astonishingly wide range of applications.

The interval numbers can be expressed in different forms. For example, if we assume a 4-tuple (a_1, a_2, a_3, a_4), that is to say, a quadruplet, we could consider that a_1 and a_4 represents the minimum and the maximum of the interval number, and a_2 and a_3, the interval with the highest probability or possibility, depending on the use we want to give to the interval numbers. Note that $a_1 \leq a_2 \leq a_3 \leq a_4$. If $a_1 = a_2 = a_3 = a_4$, then, the interval number is an exact number; if $a_2 = a_3$, it is a 3-tuple known as triplet; and if $a_1 = a_2$ and $a_3 = a_4$, it is a simple 2-tuple interval number.

In the following, we are going to review some basic interval number operations as follows. Let A and B be two triplets, where $A = (a_1, a_2, a_3)$ and $B = (b_1, b_2, b_3)$. Then:

1) $A + B = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$.
2) $A - B = (a_1 - b_1, a_2 - b_2, a_3 - b_3)$ – the Minkowski substraction.
3) $A \cdot k = (k \cdot a_1, k \cdot a_2, k \cdot a_3)$; for $k > 0$.
4) $A \cdot B = (a_1 \cdot b_1, a_2 \cdot b_2, a_3 \cdot b_3)$; for R^+.

Note that R^+ refers to all the positive real numbers. Note also that other operations could be studied (Moore, 1966) but in this paper we will focus on these ones.

2.2 The Minkowski Distance

The normalized Minkowski distance is a distance measure that generalizes a wide range of distances such as the normalized Hamming distance and the normalized Euclidean distance. In fuzzy set theory, it can be useful, for example, for the calculation of distances between fuzzy sets, interval-valued fuzzy sets, intuitionistic fuzzy sets, etc. It can be formulated for two sets A and B as follows.

Definition 1. A normalized Minkowski distance of dimension n is a mapping d_m: $R^n \times R^n \rightarrow R$ such that:

$$d_m(A,B) = \left(\frac{1}{n} \sum_{i=1}^{n} | a_i - b_i |^{\lambda} \right)^{1/\lambda}, \quad (1)$$

where a_i and b_i are the i^{th} arguments of the sets A and B and λ is a parameter such that $\lambda \in (-\infty, \infty)$.

Note that $\lambda \neq 0$ and if $\lambda \leq 0$, we can only use positive numbers R^+. If we give different values to the parameter λ, we can obtain a wide range of special cases. For example, if $\lambda = 1$, we obtain the normalized Hamming distance (NHD). If $\lambda = 2$, the normalized Euclidean distance (NED).

Sometimes, when normalizing the Minkowski distance, we prefer to give different weights to each individual distance. Then, the distance is known as the weighted Minkowski distance. It can be defined as follows.

Definition 2. A weighted Minkowski distance of dimension n is a mapping d_{wm}: $R^n \times R^n \rightarrow R$ that has an associated weighting vector W of dimension n such that the sum of the weights is 1 and $w_j \in [0, 1]$. Then:

$$d_{wm}(A,B) = \left(\sum_{i=1}^{n} w_i \, | a_i - b_i |^{\lambda} \right)^{1/\lambda}, \tag{2}$$

where a_i and b_i are the i^{th} arguments of the sets A and B and λ is a parameter such that $\lambda \in (-\infty, \infty)$.

Note that $\lambda \neq 0$ and if $\lambda \leq 0$, we can only use positive numbers R^+. In this case, we can also obtain a wide range of special cases by using different values in the parameter λ.

2.3 The OWA Operator

The OWA operator was introduced by Yager (1988) and it provides a parameterized family of aggregation operators that include the arithmetic mean, the maximum and the minimum. It can be defined as follows.

Definition 3. An OWA operator of dimension n is a mapping OWA: $R^n \rightarrow R$ that has an associated weighting vector W of dimension n such that the sum of the weights is 1 and $w_j \in [0, 1]$, then:

$$OWA(a_1, a_2, \ldots, a_n) = \sum_{j=1}^{n} w_j b_j, \tag{3}$$

where b_j is the j^{th} largest of the a_i.

From a generalized perspective of the reordering step, we can distinguish between the descending OWA (DOWA) operator and the ascending OWA (AOWA) operator [24]. The OWA operator is commutative, monotonic, bounded and idempotent. For further information on the OWA and its applications, see for example Beliakov *et al.* (2007), Bustince *et al.* (2008), Calvo *et al.* (2002) and Merigó (2008).

2.4 The UOWA Operator

The UOWA operator (Xu and Da, 2002) is an extension of the OWA operator. Essentially, its main difference is that it uses interval numbers in the arguments to be aggregated. The reason for using this aggregation operator is that sometimes the environment is very uncertain and the information is not clear. Thus, it can only be assessed by using interval numbers. The UOWA operator provides a parameterized family of aggregation operators that include the uncertain maximum, the uncertain minimum and the uncertain average (UA), among others. It can be defined as follows.

Definition 4. Let Ω be the set of interval numbers. An UOWA operator of dimension n is a mapping $UOWA$: $\Omega^n \rightarrow \Omega$ that has an associated weighting vector W of dimension n with the following properties:

1) $\sum_{j=1}^{n} w_j = 1$,

2) $w_j \in [0, 1]$,

and such that:

$$UOWA(\tilde{a}_1, \tilde{a}_2 \ldots, \tilde{a}_n) = \sum_{j=1}^{n} w_j b_j , \quad (4)$$

where b_j is the j^{th} largest of the $\tilde{a}_i$, and the $\tilde{a}_i$ are interval numbers.

From a generalized perspective of the reordering step, we can distinguish between the descending UOWA (DUOWA) operator and the ascending UOWA (AUOWA) operator. The weights of these operators are related by $w_j = w^*_{n-j+1}$, where w_j is the jth weight of the DUOWA and w^*_{n-j+1} the jth weight of the AUOWA operator.

The UOWA operator is commutative, monotonic, bounded and idempotent. Different families of UOWA operators can be found by choosing a different manifestation in the weighting vector such as the median-UOWA, the olympic-UOWA or the centered-UOWA operator.

2.5 The GOWA Operator

The GOWA operator (Karayiannis, 2000; Yager, 2004) represents a generalization of the OWA operator by using generalized means. Then, it is possible to include in the same formulation, different types of OWA operators such as the OWA operator or the ordered weighted geometric (OWG) operator. It can be defined as follows.

Definition 5. A GOWA operator of dimension n is a mapping $GOWA: R^n \rightarrow R$ that has an associated weighting vector W of dimension n such that $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$, then:

$$GOWA(a_1,\ldots,a_n) = \left(\sum_{j=1}^{n} w_j b_j^{\lambda} \right)^{1/\lambda}, \tag{5}$$

where b_j is the j^{th} largest of the a_i and λ is a parameter such that $\lambda \in (-\infty, \infty)$.

Note that $\lambda \neq 0$ and if $\lambda \leq 0$, we can only use positive numbers R^+. As we can see, if $\lambda = 1$ we get the OWA operator. If $\lambda \rightarrow 0$ the OWG operator and if $\lambda = 2$ the ordered weighted quadratic averaging (OWQA) operator. Note that it is possible to further generalize the GOWA operator by using quasi-arithmetic means. The result is the Quasi-OWA operator (Fodor *et al.*, 1995).

2.6 The OWAD Operator

The OWAD (or Hamming OWAD) operator (Merigó, 2008; Merigó and Gil-Lafuente, 2006; 2007) is an extension of the traditional normalized Hamming distance by using OWA operators. The main difference is the reordering of the arguments of the individual distances according to their values. Then, it is possible to calculate the distance between two elements, two sets, two fuzzy sets, etc., modifying the results according to the interests of the decision maker. It can be defined as follows.

Definition 6. An OWAD operator of dimension n is a mapping OWAD: $[0, 1]^n \times [0, 1]^n \rightarrow [0, 1]$ that has an associated weighting vector W, with $\sum_{j=1}^{n} w_j = 1$ and $w_j \in [0, 1]$ such that:

$$OWAD(\langle \mu_1, \mu_1^{(k)} \rangle, \ldots, \langle \mu_n, \mu_n^{(k)} \rangle) = \sum_{j=1}^{n} w_j D_j, \tag{6}$$

where D_j represents the j^{th} largest of the pairs $\langle \mu_i, \mu_i^{(k)} \rangle$ represented in the form of individual distances $|\mu_i - \mu_i^{(k)}|$, $\mu_i \in [0, 1]$ for the i^{th} characteristic of the ideal P, $\mu_i^{(k)} \in [0, 1]$ for the i^{th} characteristic of the k^{th} alternative P_k, and $k = 1, 2, \ldots, m$.

Note that this definition can be generalized to all the real numbers R by using OWAD: $R^n \times R^n \rightarrow R$. Note also that it is possible to distinguish between ascending and descending orders. The weights of these operators are related by $w_j = w^*_{n-j+1}$, where w_j is the j^{th} weight of the descending OWAD (DOWAD) operator and w^*_{n-j+1} the j^{th} weight of the ascending OWAD (AOWAD) operator.

2.7 The MOWAD Operator

The Minkowski OWAD (MOWAD) operator (Merigó, 2008; Merigó and Gil-Lafuente, 2008b) represents an extension of the traditional normalized Minkowski distance by using OWA operators. The difference is that we reorder the arguments of the individual distances according to their values. It can be defined as follows.

Definition 7. A Minkowski OWAD operator of dimension n is a mapping $MOWAD$: $R^n \times R^n \rightarrow R$ that has an associated weighting vector W of dimension n such that the sum of the weights is 1 and $w_j \in [0, 1]$. Then, the distance between two sets is:

$$MOWAD(\langle \mu_1, \mu_1^{(k)} \rangle, \ldots, \langle \mu_n, \mu_n^{(k)} \rangle) = \left(\sum_{j=1}^{n} w_j D_j^{\lambda} \right)^{1/\lambda}, \qquad (7)$$

where D_j represents the j^{th} largest of the $|\mu_i - \mu_i^{(k)}|$, μ_i is the i^{th} characteristic of the ideal P, $\mu_i^{(k)}$ is the i^{th} characteristic of the k^{th} alternative P_k, $k = 1, 2, \ldots, m$, and λ is a parameter such that $\lambda \in (-\infty, \infty)$.

Note that $\lambda \neq 0$ and if $\lambda \leq 0$, we can only use positive numbers R^+. Note that it is possible to distinguish between descending and ascending orders by using $w_j = w^*_{n-j+1}$, where w_j is the j^{th} weight of the descending MOWAD (DMOWAD) operator and w^*_{n-j+1} the j^{th} weight of the ascending MOWAD (AMOWAD) operator.

3. THE UNCERTAIN MINKOWSKI ORDERED WEIGHTED AVERAGING DISTANCE OPERATOR

The uncertain Minkowski OWAD (UMOWAD) operator is an extension of the MOWAD operator for situations where the available information can not be assessed with exact numbers but it is possible to use interval numbers. The interval numbers are useful for representing uncertain information giving the best and worst possible result that may occur and some knowledge about the most possible results. It can be defined as follows.

Definition 8. Let Ω be the set of interval numbers. An UMOWAD operator of dimension n is a mapping $UMOWAD$: $\Omega^n \times \Omega^n \rightarrow \Omega$ that has an associated weighting vector W of dimension n such that the sum of the weights is 1 and $w_j \in [0, 1]$. Then, the distance between two sets is:

$$UMOWAD(\langle \mu_1, \mu_1^{(k)} \rangle, \ldots, \langle \mu_n, \mu_n^{(k)} \rangle) = \left(\sum_{j=1}^{n} w_j D_j^{\lambda} \right)^{1/\lambda}, \qquad (8)$$

where D_j represents the j^{th} largest of the $|\mu_i - \mu_i^{(k)}|$, μ_i and $\mu_i^{(k)}$ are interval numbers, μ_i is the i^{th} characteristic of the ideal P, $\mu_i^{(k)}$ is the i^{th} characteristic of the k^{th} alternative P_k, $k = 1, 2, \ldots, m$, and λ is a parameter such that $\lambda \in (-\infty, \infty)$.

Note that $\lambda \neq 0$ and if $\lambda \leq 0$, we can only use positive numbers R^+. Note also that the reordering of the individual distances (the arguments) has an additional difficulty because now we are using interval numbers. Then, in some cases, it is not clear which interval number is higher, so we need to establish an additional criteria for reordering the interval numbers. For simplicity, we recommend the following criteria. For 2-tuples, calculate the arithmetic mean of the interval: $(a_1 + a_2) / 2$. For 3-tuples and more, calculate a weighted average that gives more importance to the central values; that is, $(a_1 + 2a_2 + a_3) / 4$. Then, for 4-tuples we could calculate: $(a_1 + 2a_2 + 2a_3 + a_4) / 6$. And so on. In the case of tie, we will select the interval with the lowest increment $(a_2 - a_1)$. For 3-tuples and more we will select the interval with the highest central value. Note that for 4-tuples and more we need to calculate the average of the central values following the initial criteria.

Moreover, in more complex analysis it is possible to consider that the weights w_j and the parameter λ are interval numbers. Moreover, it is possible to consider other types of uncertain information such as the fuzzy numbers, the linguistic variables (linguistic representations of numerical problems), etc.

Furthermore, it is also possible to distinguish between ascending and descending orders. The weights of these operators are related by $w_j = w^*_{n-j+1}$, where w_j is the j^{th} weight of the descending UMOWAD (DUMOWAD) operator and w^*_{n-j+1} the j^{th} weight of the ascending UMOWAD (AUMOWAD) operator.

Let B be a vector corresponding to the ordered arguments D_j, we call this the ordered argument vector, and W^T is the transpose of the weighting vector. Then the UMOWAD operator can be expressed as:

$$UMOWAD(\langle\mu_1, \mu_1^{(k)}\rangle, \ldots, \langle\mu_n, \mu_n^{(k)}\rangle) = W^T B . \tag{9}$$

Note that if the weighting vector is not normalized, i.e., $W^* = \sum_{j=1}^{n} w_j \neq 1$, then, the UMOWAD operator can be expressed as:

$$UMOWAD(\langle\mu_1, \mu_1^{(k)}\rangle, \ldots, \langle\mu_n, \mu_n^{(k)}\rangle) = \frac{1}{W^*} \sum_{j=1}^{n} w_j D_j . \tag{10}$$

The UMOWAD operator is commutative, monotonic, bounded and idempotent. It is commutative because any permutation of the arguments has the same evaluation. That is,

$UMOWAD(\langle\mu_1, \mu_1^{(k)}\rangle, \ldots, \langle\mu_n, \mu_n^{(k)}\rangle) = UMOWAD(\langle u_1,d_1\rangle, \langle u_2,d_2\rangle\ldots, \langle u_n,d_n\rangle)$, where $(\langle u_1,d_1\rangle, \langle u_2,d_2\rangle\ldots, \langle u_n,d_n\rangle)$ is any permutation of the arguments $(\langle\mu_1, \mu_1^{(k)}\rangle, \ldots, \langle\mu_n, \mu_n^{(k)}\rangle)$.

It is monotonic because if $\langle\mu_i, \mu_i^{(k)}\rangle \geq \langle u_i,d_i\rangle$, for all i, then, $UMOWAD(\langle\mu_1, \mu_1^{(k)}\rangle, \ldots, \langle\mu_n, \mu_n^{(k)}\rangle) \geq UMOWAD(\langle u_1,d_1\rangle, \langle u_2,d_2\rangle\ldots, \langle u_n,d_n\rangle)$.

It is bounded because the UMOWAD aggregation is delimitated by the minimum and the maximum distance. That is, $\text{Min}\{|\mu_i - \mu_i^{(k)}|\} \leq UMOWAD(\langle\mu_1, \mu_1^{(k)}\rangle, \ldots, \langle\mu_n, \mu_n^{(k)}\rangle) \leq \text{Max}\{|\mu_i - \mu_i^{(k)}|\}$.

It is idempotent because if $|\mu_i - \mu_i^{(k)}| = d$, for all i, then, $UMOWAD(\langle\mu_1, \mu_1^{(k)}\rangle, \ldots, \langle\mu_n, \mu_n^{(k)}\rangle) = d$.

Note that the proofs of these theorems are straightforward. For similar proofs on other types of OWA, see for example, Merigó (2008), Merigó and Casanovas (2009) and Merigó and Gil-Lafuente (2009b).

Another interesting issue to analyze are the measures for characterizing the weighting vector W. Following a similar methodology as it has been developed for the OWA (Yager, 1988; 1996; 2002) and the GOWA operator (Yager, 2004), we can formulate the attitudinal character, the entropy of dispersion, the divergence of W and the balance operator.

The first measure $\alpha(W)$, the attitudinal character, is defined as:

$$\alpha(W) = \sum_{j=1}^{n} \left(\frac{n-j}{n-1}\right) w_j . \tag{11}$$

It can be shown that $\alpha \in [0, 1]$. The more weight is located near the top of W, the closer α is to 1, while the more weight is located toward the bottom of W, the closer α is to 0.

The entropy of dispersion measures the amount of information being used in the aggregation.

$$H(W) = -\sum_{j=1}^{n} w_j \ln(w_j) . \tag{12}$$

For example, if $w_j = 1$ for some j, known as step-UMOWAD, then $H(W) = 0$, and the least amount of information is used.

The balance operator measures the balance of the weights against the orness or the andness, that is, the tendency to the maximum or to the minimum.

$$Bal(W) = \sum_{j=1}^{n} \left(\frac{n+1-2j}{n-1}\right) w_j . \tag{13}$$

It can be shown that $Bal(W) \in [-1, 1]$. Note that for the optimistic criteria, $Bal(W) = 1$, and for the pessimistic criteria, $Bal(W) = -1$.

The divergence of W measures the divergence of the weights against the attitudinal character measure. It is useful in some exceptional situations when the attitudinal character and the entropy of dispersion are not enough to correctly analyze the weighting vector of an aggregation.

$$Div(W) = \sum_{j=1}^{n} w_j \left(\frac{n-j}{n-1} - \alpha(W) \right)^2 . \tag{14}$$

Another interesting issue to consider is the different families of UMOWAD operators that are found in the weighting vector W and the parameter λ. If we analyze the parameter λ, we get the following particular cases.

- The uncertain OWAD (UOWAD) operator if λ = 1 (arithmetic).
- The uncertain ordered weighted geometric averaging distance (UOWGAD) operator if λ approaches to 0.
- The uncertain ordered weighted quadratic averaging distance (UOWQAD) operator if λ = 2.
- The uncertain ordered weighted harmonic averaging distance (UOWHAD) operator if λ = −1 (harmonic).
- Etc.

And if we analyze the weighting vector W, we get the following ones.

- The uncertain maximum distance ($w_1 = 1$ and $w_j = 0$, for all $j \neq 1$).
- The uncertain minimum distance ($w_n = 1$ and $w_j = 0$, for all $j \neq n$).
- The uncertain Minkowski distance ($w_j = 1/n$, for all $\tilde{a}_i$).
- The uncertain weighted Minkowski distance ($w_j = 1/n$, for all $\tilde{a}_i$).
- The MOWAD operator (when the interval numbers are reduced to exact numbers).
- The uncertain Hurwicz distance criteria ($w_1 = \alpha$, $w_n = 1 - \alpha$ and $w_j = 0$, for all $j \neq 1, n$).
- The step-UMOWAD ($w_k = 1$ and $w_j = 0$, for all $j \neq k$).
- The olympic-UMOWAD operator ($w_1 = w_n = 0$, and $w_j = 1/(n-2)$ for all others).
- The general olympic-UMOWAD operator ($w_j = 0$ for $j = 1, 2, \ldots, k, n, n-1, \ldots, n-k+1$; and for all others $w_{j*} = 1/(n-2k)$, where $k < n/2$).

- The S-UMOWAD ($w_1 = (1/n)(1 - (\alpha + \beta) + \alpha$, $w_n = (1/n)(1 - (\alpha + \beta) + \beta$, and $w_j = (1/n)(1 - (\alpha + \beta)$ for $j = 2$ to $n - 1$ where $\alpha, \beta \in [0, 1]$ and $\alpha + \beta \leq 1$).
- The centered-UMOWAD (if it is symmetric, strongly decaying from the center to the maximum and the minimum, and inclusive).
- Etc.

Note that these families are based on the methodology explained by Karayiannis (2000), Merigó (2008), Yager (1988; 1992; 1993; 1996; 2004; 2007) and Yager and Filev (1994). Other families of UMOWAD operators may be used following a similar methodology as it has been developed for the OWA operator and some of its extensions (Ahn and Park, 2008; Beliakov *et al.*, 2007; Emrouzejad, 2008; Liu, 2008; Xu, 2005; Yager, 2002).

4. THE QUASI-UOWAD OPERATOR

The UMOWAD operator can be generalized by using quasi-arithmetic means in a similar way as it was done in Beliakov *et al.* (2007), Calvo *et al.* (2002), Fodor *et al.* (1995), Karayiannis (2000), Merigó (2008), Merigó and Casanovas (2007; 2008b) and Merigó and Gil-Lafuente (2009a; 2009b). We will call it the Quasi-UOWAD operator. It is defined as follows.

Definition 9. Let Ω be the set of interval numbers. A Quasi-UOWAD operator of dimension n is a mapping $QUOWAD$: $\Omega^n \times \Omega^n \rightarrow \Omega$ that has an associated weighting vector W of dimension n such that the sum of the weights is 1 and $w_j \in [0, 1]$. Then, the distance between two sets is:

$$QUOWAD(\langle \mu_1, \mu_1^{(k)} \rangle, \ldots, \langle \mu_n, \mu_n^{(k)} \rangle) = g^{-1}\left(\sum_{j=1}^{n} w_j g\left(b_{(j)}\right) \right), \tag{15}$$

where D_j represents the j^{th} largest of the $|\mu_i - \mu_i^{(k)}|$, μ_i and $\mu_i^{(k)}$ are interval numbers, μ_i is the i^{th} characteristic of the set $\mu = \{\mu_1, \ldots, \mu_n\}$, $\mu_i^{(k)}$ is the i^{th} characteristic of the k^{th} alternative P_k, $k = 1, 2, \ldots, m$, and g is a strictly continuous monotonic function.

As we can see, when $g(b) = b^{\lambda}$, then, the Quasi-UOWAD becomes the UMOWAD operator. Note that it is also possible to distinguish between descending (Quasi-DUOWAD) and ascending (Quasi-AUOWAD) orders.

Note that if the weighting vector is not normalized, i.e., $W^* = \sum_{j=1}^{n} w_j \neq 1$, then, the UMOWAD operator can be expressed as:

$$QUOWAD(\langle \mu_1, \mu_1^{(k)}\rangle, \ldots, \langle \mu_n, \mu_n^{(k)}\rangle) = g^{-1}\left(\frac{1}{W^*}\sum_{j=1}^{n} w_j g\left(b_{(j)}\right)\right). \quad (16)$$

Note that all the properties and particular cases commented in the UMOWAD operator are also applicable in the Quasi-UOWAD operator. Thus, we can use a wide range of interval numbers such as triplets and quadruplets; we have to establish a criterion for ranking interval numbers, and so on.

5. DECISION MAKING PROCESS IN THE SELECTION OF STUDIES PLAN

Decision making problems are very common in the scientific literature. They can be implemented in a lot of environments such as in statistics, engineering, economics and politics. In this paper, we focus on a decision making problem about the selection of studies plan in a university. The process to follow in the selection of studies plan is similar to the process developed in Gil-Aluja (1998; 1999; 2001), Gil-Lafuente (2005), Gil-Lafuente and Merigó (2006), Merigó (2008), Merigó and Gil-Lafuente (2006; 2007; 2008a; 2008b; 2008c), with the difference that now we are considering an educational management problem. The 5 steps to follow can be summarized in the following way:

Step 1: Analysis and determination of the significant characteristics of the available alternatives. Theoretically, it will be represented as: $C = \{C_1, C_2, \ldots, C_i, \ldots, C_n\}$, where C_i is the i^{th} characteristic to consider of the alternative and we suppose a limited number n of required characteristics.

Step 2: Establishment of the ideal levels of each characteristic in order to form the ideal study plan.

Table 1. Ideal study plan.

	C_1	C_2	…	C_i	…	C_n
$P =$	μ_1	μ_2	…	μ_i	…	μ_n

In Table 1, P is the ideal study plan expressed by a fuzzy subset, C_i is the i^{th} characteristic to consider, and μ_i is the valuation for the i^{th} characteristic.

Step 3: Establishment of the real level of each characteristic for all the alternatives considered.

Table 2. Available alternatives.

	C_1	C_2	…	C_i	…	C_n
$P_k =$	$\mu_1^{(k)}$	$\mu_2^{(k)}$	…	$\mu_i^{(k)}$	…	$\mu_n^{(k)}$

In Table 2, k = 1, 2, …, m, P_k is the kth alternative expressed by a fuzzy subset, C_i is the i^{th} characteristic to consider, and $\mu_i^{(k)}$ is the valuation for the i^{th} characteristic of the k^{th} alternative.

Step 4: Comparison between the ideal study plan and the different alternatives considered, and determination of the level of removal using the UMOWAD operator. That is, changing the neutrality of the results to over estimate or under estimate them. In this step, the objective is to express numerically the removal between the ideal study plan and the different alternatives considered. Note that by using the UMOWAD operator, we can use all the particular cases mentioned in Section 3.

Step 5: Adoption of decisions according to the results found in the previous steps. Finally, we should take the decision about which study plan select. Obviously, our decision will consist in choosing the study plan with the best results according to the method used that is in accordance with the interests of the decision maker.

6. ILLUSTRATIVE EXAMPLE

In this Section, we present an illustrative example of the new approach in a decision making problem. We will study a problem of selection of studies plan. We are going to consider a PhD program in business administration that is considering which courses to offer the next year. Note that it is possible to consider other applications in educational management or in other business decision making problems.

Assume that a PhD program that wants to increase its quality is planning the creation of some new courses in order to be more efficient for the PhD students. They consider five possible alternatives.

- A_1 = Increase the number of courses in mathematics.
- A_2 = Increase the number of courses in statistics.
- A_3 = Increase the number of courses in decision theory and operational research.
- A_4 = Increase the number of courses in management.
- A_5 = Increase the number of courses in research orientation.

In order to evaluate these alternatives, the board of directors of the PhD program considers five main characteristics that are relevant for the selection process.

- C_1 = Knowledge of the available professors. This characteristic analyzes the skills of the available professors and how they can help the students in this research area in order to develop a good research with relevant publications, etc.
- C_2 = Number of courses in this field. This characteristic analyzes the number of courses in similar topics and if it is necessary to add more courses based on the research specialization of the available professors, the potential research that can be expected from the students, etc.
- C_3 = Usefulness for future research of the students. This characteristic analyzes if these courses give very interesting topics that can help the research of the students in the future. This aspect depends on the present situation of the research in this area, if there are a lot of new topics appearing, etc.
- C_4 = General evaluation of the course. It considers if the course itself seems to be interesting to the PhD program in general.
- C_5 = Other variables. It includes other variables to be taken into account such as the motivation of the student for this course, competitive advantage against other PhD programs, research methods to be used, etc.

The board of directors of the PhD program evaluates the courses given marks to each characteristic from 0 to 100, being 100 the best result. The results obtained depending on the characteristic C_i and the course A_k are shown in Table 3.

Table 3. Evaluation of the results.

	C_1	C_2	C_3	C_4	C_5
A_1	(70, 80)	(50,60)	(80,90)	(60,70)	(50,60)
A_2	(60,70)	(70,80)	(50,60)	(40,50)	(80,90)
A_3	(70,80)	(60,70)	(60,70)	(40,50)	(80,90)
A_4	(50,60)	(60,70)	(40,50)	(70,80)	(80,90)
A_5	(70,80)	(70,80)	(50,60)	(50,60)	(60,70)

The board of directors establishes the ideal results that the new courses should have in order to be included in the PhD program.

Table 4. Ideal alternative.

	C_1	C_2	C_3	C_4	C_5
Ideal	(80,100)	(70,90)	(80,100)	(80,100)	(80,100)

In this problem, the experts of the selection process assume the following weighting vector: W = (0.1, 0.2, 0.2, 0.2, 0.3). With this information, we can aggregate the expected results for each student in order to take a decision. In Table 5, we present different results obtained by using different types of UMOWAD operators. We consider the maximum and the minimum distance, the uncertain averaging distance (UAD), the uncertain weighted averaging distance (UWAD) (or uncertain weighted Hamming distance) and the UOWAD operator.

Table 5. Aggregated results.

	Max	Min	UAD	UWAD	UOWAD
A_1	(30,40)	(0,10)	(16,26)	(18,28)	(13,23)
A_2	(40,50)	(0,10)	(18,28)	(16,26)	(14,24)
A_3	(40,50)	(0,10)	(16,26)	(15,25)	(12,22)
A_4	(40,50)	(0,10)	(18,28)	(15,25)	(14,24)
A_5	(30,40)	(10,20)	(20,30)	(21,31)	(18,28)

If we establish an ordering of the alternatives, a typical situation if we want to consider more than one alternative, then, we get the following results shown in Table 6.

Table 6. Ordering of the studies plan.

	Ordering		*Ordering*
Max	$A_1=A_5 \succ A_2=A_3=A_4$	*UWAD*	$A_3=A_4 \succ A_2 \succ A_1 \succ A_5$
Min	$A_1=A_2=A_3=A_4 \succ A_5$	*UOWAD*	$A_3 \succ A_1 \succ A_2=A_4 \succ A_5$
UAD	$A_1=A_3 \succ A_2=A_4 \succ A_5$		

As we can see, depending on the aggregator operator used, the ordering of the studies plan may be different. Note that the main advantage of using the UMOWAD operator is that we can consider a wide range of particular distance measures such as the UAD, the UWAD and the UOWAD operator. Due to the fact that each particular family of UMOWAD operator may give different results, the decision maker will select for his decision the one that is closest to his interests. However, by using this analysis he will be able to see the results and optimal decisions in other potential situations that may occur in the future.

Note that these types of methods are very useful for dealing with uncertainty, because under uncertainty we do not know the optimal choice because we do not know the future. Therefore, we can only give recommendations according to the particular interests of the decision maker such a being risk averse or not but our results can not predict the future.

7. CONCLUSIONS

We have presented the UMOWAD operator and we have analyzed its applicability in decision making problems about educational management. We focussed on the selection of studies plan in a PhD program that it is considering to add new courses in its program and they are looking for the optimal one. We have seen that by using the UMOWAD we are able to provide a general formulation in a decision process where we can compare the available alternatives with an ideal one. The main advantage of this approach is that we can consider a wide range of future scenarios according to our interests and select the one that it is closest to our real interests. We have studied the UMOWAD and have found a lot of particular cases such as the UOWAD operator, the UOWQAD operator, the uncertain Minkowski distance, the uncertain weighted Minkowski distance, the uncertain Hamming distance, the uncertain Euclidean distance, the S-UMOWAD operator, and a lot of other cases. We have further generalized the UMOWAD operator by using quasi-arithmetic-means in order to obtain a more general formulation that includes the UMOWAD as a particular case. We have called it the Quasi-UOWAD operator. The main advantage of this generalization is that it is more robust and general that the UMOWAD operator.

In future research, we expect to present further extensions to this approach by using other factors that should be relevant in the decision problem such as the use of order inducing variables and other approaches such as the ones used in Merigó (2008). We will also analyze other potential problems in other educational management situations and in other business decision making applications.

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