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Approving the ISDWIR Method of Risk Measurement in Making Risk Management Decision

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ABSTRACT

This paper is devoted to risk management and risk measurement methods. The author considers methods of risk measurement and proposes the Integral Sum of Differential Weighted Indexes of Risks (or ISDWIR) method of risk measurement. The method is based on dynamic enterprise risk matrices. The matrix describes the changes of corporate risk values over the time. The method assists to choose risk management decision having good effects on corporate risk values. The ISDWIR method is also compared with other risk measurement methods.

Keywords: uncertainty; risk; risk management; risk measurement; matrix.
JEL classification: M10.
MSC2010: 40C05.

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Aprobación del método de medición del riesgo SIÍPDR en el manejo de asunción de riesgos

RESUMEN

Este artículo está dedicado a la gestión del riesgo y a los métodos de medición de riesgos. El autor considera diferentes métodos de medición de riesgo y propone el método de la Suma Integral de Índices Ponderados Diferenciales de Riesgos (o método SIÍPDR). Dicho método se basa en las matrices de riesgo empresarial dinámico. Dichas matrices describen los cambios de los valores de riesgo corporativos en el tiempo. El método ayuda a elegir la decisión de gestión del riesgo que tiene un buen efecto sobre los valores de riesgos corporativos. También se compara el método SIÍPDR con otros métodos de medición del riesgo.

Palabras clave: incertidumbre; riesgo; gestión de riesgos; medición del riesgo; matriz.
Clasificación JEL: M10.
MSC2010: 40C05.

1. INTRODUCTION

The enterprise functions under the condition of risks. To make risk management decision the company needs measure risks, the effect of decision on the risk values. A variety of risk measurement methods fall under financial or investment management. In this paper, the following methods of risk measurement are considered: coefficient of variation; Scenario Analysis; Capital Asset Pricing Model; Weighted Average Cost of Capital; Value at Risk (VaR); Monte Carlo simulation; Real Options.

Some methods of risk management come from financial fields and see risks from a financial point of view (as probability and/or value of losses), but not all risk means danger, since some risk usually gives us more profit. Thus we should consider risk not only as a danger event but also as a positive event. The company can have different risks that are measured by different units of measurement. For example, the company can have different risks at once: 1) risk of stopping manufacture because of equipment breakage (it can be measured in quantity of breakage over specified point of time); 2) the deviation of current liquidity ratio (liquidity ratio is coefficient and not has the units of measurement); 3) the risk of increase of bank interest rate (bank interest rate is measured in percentage); 4) the risk of deviation of total revenue (it is measured in money). Thus different units of risk measurement should be taken into account. It is also important to take into consideration the time period and the weights (importance) of risk values over time period for the company.

However, there is a problem of choosing the risk measurement method, because it should use methods of risk measurement considering varied parameters as time period, different units of risk measurement, the weight of risk values over time period, considering risk not only as a danger event but also as a positive event and it should not have assumption or constraints of implementation. In the article the author proposes a method of risk measurement (the integral sum of differential weighted indexes of risks, or ISDWIR method). The proposed method is compared with others to emphasize its applicability. The method is based on dynamic enterprise risk matrices. Each matrix describes the changes of corporate risk values over the time. The method assists to choose risk management decision having good effects on corporate risk values.

2. LITERATURE REVIEW

The activity of companies is connected under conditions of risk. To start the discussion about risk measurement we should know what risk is. In risk management literature, risk is seen as an expected value, probability distribution, deviation and event. Knight (1921) determines risk as conditions when the consequences of decisions and the probabilities of those outcomes are known, whereas uncertainties are conditions where the potential consequences of decisions and related probabilities may not be well known. Graham and Weiner (1995) define risk as the probability of an adverse outcome. Rosa (1998, 2003) denotes risk as a situation or event where something of human value (including humans themselves) is at stake and where the outcome is uncertain. Risk refers to uncertainty of outcome, of actions and events (Cabinet Office, 2002). Crowe, Fong, Bauman, and Zayas-Castro (2002) describe risk as the possibility of deviation in the results from expected goals. Aven (2007) reports risk is as the two-dimensional combination of events/consequences and associated uncertainties (will the events occur, what will be the consequences). The international standards become, because of theory and practice development of risk management. The most prominent standards are COSO ERM (American standard) FERMA (European standard). COSO ERM standard divide definitions of risk and opportunities. Events with a negative impact represent risks, which can prevent value creation or erode existing value. Opportunities are the possibility that an event will occur and positively affect the achievement of objectives, supporting value creation or preservation. FERMA standard has uses the definition of risk that is set out by the International Organization for Standardization (ISO) in its recent document ISO/IEC Guide 73 Risk Management - Vocabulary - Guidelines for use in standards. Risk can be defined as the combination of the probability of an event and its consequences (ISO/IEC Guide 73).

Thus, the main feature of risk follows from all considered definitions. It is a quantity determination of risk (probability, deviation, value). To make a risk management decisions, company has to measure its risks and measure the effect of risk management decisions upon the value of risks. In practice different methods are used to measure risk. The methods come from financial fields. Later they are considered.

2.1. Coefficient of Variation

The coefficient of variation is a dimensionless number that quantifies the degree of variability relative to the mean (Kelley, 2007). The coefficient of variation is defined as:

$$k = \frac{\sigma}{\mu}, \tag{1}$$

σ –deviation

μ –mean

The coefficient of variation consists of two other risk parameters. These are mean and deviation. Thus coefficient of variation indicates the relative risk associated with the possible outcomes of a particular action. The higher ratio of variation would mean higher degree of risk (Rao, 2010). The coefficient of variation can be used to measure relative risks in finance and actuarial science (Miller and Karson, 1977). The coefficient is used as yardstick of riskiness. But the coefficient of risk measurement has some disadvantage; it sees risk only as negative phenomenon. But some risks give profit.

2.2. Scenario Analysis

Porter (1985) defines scenarios as “an internally consistent view of what the future might turn out to be –not a forecast, but one possible future outcome.” Raubitschek (1988) considers scenario analysis is a technique used to analyze future developments in situation characterized by high degree of uncertainty and complexity. Godet and Roubelat (1996) understand a scenario as a description of a future situation and the course of events, which allows one to move forward from the original situation to the future. According to Ratcliffe (1999), a primary purpose of scenarios is to create holistic, integrated images of how the future might evolve.

Damodaran (2012) emphasizes that scenario analysis set the following problems:

- Garbage in, Garbage out Garbage in, garbage out. The key to doing scenario analysis well is the setting up of the scenarios and the estimation of cash flows under each one. Not only the outlined scenarios have to be realistic, but they also have to try to cover the spectrum of possibilities. Once the scenarios have been laid out, the cash flows have to be estimated under each one; this trade-off has to be considered when determining how many scenarios to run.
- Continuous risk. Scenario analysis is best suited for dealing with risk that takes the form of discrete outcomes. When the outcomes can take on any of a very large number of potential values or the risk is continuous, it becomes more difficult to set up scenarios.
- Double counting of risk. As with the best-case/worst-case analysis, there is the danger that decision makers will double count risk when they do scenario analysis.

2.3. Capital Asset Pricing Model (CAPM)

The CAPM builds on the model of portfolio choice developed by Markowitz (1959). The model of Markowitz assumes investors are risk averse and choose “mean variance - efficient” portfolios, that 1) minimize the variance of portfolio return, given expected return, and 2) maximize expected return, given variance. Thus, the model is often called a “mean variance model.” Sharpe (1964) and Lintner (1965) add two key assumptions to the Markowitz model: complete agreement gives market clearing asset prices and complete agreement: given market clearing asset prices.

When there is risk-free borrowing and lending, the expected return on assets that are uncorrelated with the market return, E (RM), must equal the risk-free rate, Rf. The relation between expected return and beta then becomes the familiar Sharpe - Lintner CAPM equation,

$$\text{CAPM} \quad E(Ri)\ Rf + [E(RM) - Rf)]\beta iM\ ,\ i = 1, \ldots, N. \qquad (2)$$

In other words, the expected return on any asset is the risk-free interest rate, Rf , plus a risk premium, which is the asset’s market beta, βiM, times the premium per unit of beta risk, E(RM) – Rf. Glen (2005) defines several assumptions of CAPM:

1. Quality of investors:
    - they aim to maximize economic utilities;
    - they are rational and risk-averse;
    - they are broadly diversified across a range of investments;
    - they are price takers, i.e., they cannot influence prices.
2. Lend and borrow is unlimited amounts under the risk free rate of interest.
3. Trading is without transaction or taxation costs.
4. Securities are highly divisible into small parcels.
5. All information is available at the same time to all investors.

All assumptions characterize the disadvantages of CAPM. It is enough obstruct method for the dynamic nature of risk.

2.4. Weighted Average Cost of Capital

WACC is used as a means of arriving at the firm's optimal capital structure —optimal in the sense that the firm's total market value is maximized and consequently the per share price of equity is maximized—the weighted average cost of capital has been mathematically defined as:

$$w = r\left(\frac{S}{S+D}\right) + i\left(\frac{D}{S+D}\right)(1 - Tc)(3)$$

where: S = market value of common stock at instance of w calculation.

D = market value of debt at instance of w calculation,

i = yield on debt, hereafter called the interest rate,

r = expected or required rate of return on common stock.

Tc = tax on profits

Tc in the above formula reflects the existence of tax benefits due to the deductibility of interest costs (Modigliani and Miller, 1958; 1963; Scott, 1976). The WACC calculation is based on the assumptions (Pratt and Grabowsky, 2011) that the capital structure will remain unchanged over the time period of the valuation. It means:

- The proportional mix of debt and equity in the capital structure, in terms of market value, would remain constant over the investment horizon.
- The cost of capital would remain unchanged over the investment period.
- Corporation tax rate and interest rate on debt are constant.
- The common disadvantage of CAPM and WACC (it goes from their formulas) is that they are one time period methods or risk measurement. In practice the discounter rate of net present value (NPV) is calculated from CAMP and WACC but at this case there is assumption that the risk value does not change over the time period (but risk value is not constant). Also they have assumptions of their implementation.

2.5. Value at Risk – VaR

Frey and McNeil (2002) define VaR as follows: Given some confidence level $\alpha \in (0; 1)$, the value-at-risk (VaR) of our portfolio at the confidence level α is given by the smallest number l such that the probability that the loss L exceeds l is no larger than $(1 - \alpha)$. According to Hull (2005), VaR is denoted as "the loss corresponding to the $(100 - \mathrm{X})^{th}$ percentile of the distribution of the change in the value of the portfolio over the next N days" when N days is the time horizon and X% is the confidence level. Value at risk has some disadvantages of implementation (Damodaran, 2007):

- VaR measures the likelihood of losses to an asset or portfolio due to market risk, thus risk is almost considered to be negative. VaR are built around market risk effects, hence there is no reason to look at the VaR, relative to all risks, practicality forces us to focuse on just market risks and their effects on value;
- VaR is calculated for a short time period (over a day, week, or a few weeks);
- VaR is not appropriate for the firms that are focused on comparing investments with different scales and returns, because VaR gives a certain value of losses in terms of probability (for these firms more conventional scaled measure of risk can be standard deviation).

2.6. Monte Carlo Method

The Monte Carlo method (Metropolis and Ulam, 1949) is a widely accepted risk analysis technique and is deemed to be an effective way of analyzing the uncertainty associated with cost and schedule risks. Monte Carlo is defined as a distributional simulation technique. In Monte Carlo analysis, instead of using single point-estimates as inputs, distributions, defined as probability density functions (PDFs), are used as inputs. As a result, distributions of output variables are produced. The output distributions are simulated by drawing random values from the distribution of input variables. The input variables relate to the output variables according to a mathematical model (Lipton, Shaw, Holmes, and Patterson, 1995).

Probability functions must be defined for all uncertain parameters, regardless the information available. In many cases, there is not enough data available to determine an accurate probability density distribution. Another disadvantage of the Monte Carlo method is the need to determine dependencies and correlations between the input variables if an accurate result is desired (Bardossy and Fodor, 2004).

2.7. Real Options

The real options framework suggests that holding a real option on a strategically important opportunity, after making an initial investment, allows firms to postpone further commitment until part of the uncertainty about the opportunity has been resolved (Dixit and Pindyck, 1994;

Amram and Kulatilaka, 1999). Trigeorgis (1996) categorized the various types of real options. They are the option to defer investment, the longer the time horizon, the time-to-build option, the option to abandon, the option to switch, growth options. Groups of options may interact, leading to different total valuation than the simple sum of each individual option.

Teach (2003) and Miller and Park (2002) exclaim that real option only works for tradable assets, i.e. when the asset price over time can be observed in the financial market. They point out that the key parameter in a real option is volatility, and that to estimate volatility, you need appropriate and sufficient data, such as historical data and actuarial information. Scholleová (2008) suggests that on certain types of situations it cannot be applied. These situations can be summarized as follows:

- decision making under certainty or zero risk;
- decision making that cannot be postponed or modified, the real option that measures flexibility does not have sense when flexibility is not possible;
- twin options, when the option value would be assigned to more interdependent projects, in such a case, the real option would over-evaluate the flexibility;
- low budget projects where the estimated option value would exceed the total costs of the projects.

It goes from the literature of real options that asset must has liquidity and the company can refuse asset or project (it is as assumption of real options).

In risk management, risk matrices are used. Cox and Anthony (2008) write that risk can be described by risk matrix that has several categories of “probability,” “likelihood,” or “frequency” for its rows (or columns) and several categories of “severity,” “impact,” or “consequences” for its columns (or rows, respectively). It allows emphasizing at list three levels (values) of risks –low, medium and high level of risk (Levine, 2012; Meacham, 2010). It means the level of risk is not equal for each company and it should take into account when you choose the form of corporate restructuring.

3. THE ISDWIR METHOD OF RISK MEASUREMENT

Risk is an event having its likelihood of occurrence and consequences that changes the performance indicators of company (improving or worsening them) when the external and / or internal environment varies, and occurs both by reason of management decision and independently of it.

This definition emphasizes that:

- risk is measured and there are many units of risk measurement because the consequences of risk (its impact on the company) can be measured by different units of measurement;

- risk must have impact on company that is reflected in performance indicators of company (it means that sometimes we can measure only consequences of events), if company is not exposed to risk, it does not have risk;
- risk is considered as positive or negative event (negative and positive phenomenon);
- risk occurs due to changes in the external or internal environment - with a static enterprise environment there is no risk;
- risk is an event and occurs both by reason of management decision and independently of it.

This article proposes the use of a Dynamic Enterprise Risk Matrix to describe the change of corporate risk values over the time period. This matrix describes the value of several risks in dynamic. Table 1 shows the Dynamic Enterprise Risk Matrix.

Table 1. Dynamic Enterprise Risk Matrix

Period / Risk	T_1	……….	T_i	……….	T_n
R_1	α_{11}	……….	α_{1j}	……….	α_{1n}
……….	……….	……….	……….	……….	……….
R_i	α_{i1}	……….	α_{ij}	……….	α_{in}
R_m	α_{m1}	……….	α_{mj}	……….	α_{mn}

Source: author

Each line corresponds to the value of particular kind of risk (R), and the column (T) correlate with time period, for example –week, month and year. Each risk is considered in the dynamic. This matrix shows the value of each corporate risk according to time period. This matrix is filled on the basis of the prediction of risk value changes.

At the beginning, the company makes Basic Enterprise Risk Matrix (original matrix). The matrix describes the value corporate risks dynamically as if there is no any process of risk management. Risk management decision (for example, corporate restructuring) affects the value of corporate risks. The company can made various risk management decision and each kind has different effects on the values of corporate risks. The effect is described by Resulting Enterprise Risk Matrices (resulting matrices). It shows the changed values of corporate risks resulting from risk management decisions. It is important to compare the influence of each solution option to select an appropriate risk management decision (for example, kind/type of corporate restructuring).

It is necessary to compare Basic Enterprise Risk Matrix and Resulting Enterprise Risk Matrices of each variant of risk management decision. Thus our task is to compare two matrices. It is impossible to make a comparison on the basis of matrix norms, because: the values of matrix elements in a row can reduce (it means that risk values reduce), and this reduction is positive, but it is not included in the calculation of matrix norms.

The author of the article proposes the Integral Sum of Differential Weighted Indexes of Risks method (ISDWIR method). This method measures the risk of decision: the decision of risk management improves original matrix of risks or makes it worse. The process of risk measurement of ISDWIR method proceeds as follows:

1) At the beginning there are several matrices: Basic Enterprise Risk Matrix - A matrix (the original matrix) and some Resulting Enterprise Risk Matrices - B, C, D matrices.

2) Then, the Risk Index Matrices are formed - I 1, I 2, I3 matrices. The elements of B, C and D matrices are divided by the elements of A matrix. This mathematical operation assists to compare changes of risk values assessed in different units of measurement (money units of measurement, per cent, coefficients or others). Also the Basic Unit Matrix (E matrix) is set (the elements of basic Enterprise Risk Matrix are divided by themselves). The setting of Basic Unit permits to assess the changes of risk values of Risk Index Matrices).

3) Next, it is need to take into account the importance of each kind of risk and the time period on the basis of pair comparisons methods, in practice many methods of pair comparisons are used (Louis Thurstone's method is the first method of pair comparison). On the basis of chosen pair comparisons method weighting coefficients of each kind of risk and each time period are defined. Then the weights coefficients of each kind of risk and time periods are multiplied and Matrix of Weighting Coefficients is obtained (W - matrix). The matrix describes the importance of each kind of risk according to time period. The sum of all the elements of weight coefficients matrix is equal to 1. In practice the company cannot take into account the importance of each kind of risk and the point of time period, it means to miss 3 item of the process (in that case the method is called the Integral Sum of Differential Indexes of Risks – ISDIR method).

4) Later, each element of Risk Index Matrices (I 1, I 2, I3 matrices) and the Basic Unit Matrix (E matrix) is multiplied by the elements of Weight Coefficients Matrix. Thus, Weighted Base Unit Matrix (E º) and Weighted Risk Index Matrices (I 1 º, I 2 º, I 3 º matrices) are set.

5) Next, the Integral Sum of Differential Weighted Indexes of Risks (ISDWIR) between Weighted Base Unit Matrix (E º) and each Weighted Risk Index Matrix (I 1 º, I 2 º, I 3 º matrices) is calculated in accordance with following formula:

$$In^o = \begin{pmatrix} i_{11} & i_{12} & i_{1n} \\ i_{21} & i_{22} & i_{mn} \end{pmatrix}$$

$$E^o = \begin{pmatrix} e_{11} & e_{12} & e_{1n} \\ e_{21} & e_{22} & e_{mn} \end{pmatrix}$$

$$\text{R(A,B)} = (i_{11} - e_{11} + i_{12} - e_{12} + i_{1n} - e_{1n}) * \lambda + (i_{21} - e_{21} + i_{11} - e_{11} + i_{mn} - e_{mn}) * \lambda \qquad (4)$$

where:

R(A,B) - the Integral Sum of Differential Weighted Indexes of Risks

In^o - Weighted Risk Index Matrix

E^o - Weighted Base Unit Matrix

i_{mn} – elements of Weighted Index Matrices

e_{mn} – elements of Weighted Base Unit Matrix

λ – parameter having the value of +1 or -1

If it is necessary for the enterprise to increase the value of risk (for example, the enterprise wants to increase the deviation of profit), λ is equal to +1. If the aim is to decrease the value of risk, λ is equal to -1. Thus several values of ISDWIR are obtained. The values of ISDWIR are integral not having the units of measurement. If the value of ISDWIR is equal to 0, it means that the value of Basic Enterprise Risk Matrix is not changed. The value of Integral Sum of Differential Weighted Indexes of Risks must be greater than 0. If the value is less than 0, this risk management decision is refused, because the risk management decision increases the values of risks (the values of elements of Basic Enterprise Risk Matrix become badly). If the value of ISDWIR is greater than 0 it means that risk management decision reduces the values of risks (the values of elements of Basic Enterprise Risk Matrix become well). The Integral Sum of Differential Weighted Indexes of Risks (ISDWIR) is a synthetic indicator (no having unit of measurement), which measures the change of values of several risks at certain risk management decision that allows to determine its impact.

6) At the end, the values of the Integral Sums of Differential Weighted Indexes of Risks for each variant of risk management decision are compared. Decision rule is to select the variant of risk management decision with the maximum Integral Sum of Differential Weighted Indexes of Risks.

The example of carrying out of calculations of ISDWIR method is given in the Appendix.

Thus the Integral Sum of Differential Weighted Indexes of Risks defines the effect of risk management decision on risk values. In this article previously considered methods of risk measurement are compared according with following criteria:

• the existence of assumptions or constraints of implementation of method;

• risk is considered as negative and/or as negative and positive event;

• the units of risk measurement are homogeneous or heterogeneous (homogeneous units of measurement means that risks have similar units of measurement, for example financial risks and market risks, etc.; heterogeneous units of measurement means that risks have different units of measurement, for example method considers mixture of financial and market risks and social risks);

• method takes into account the time in risk measuring;

• the difficulty of risk measurement and interpretation of risk measurement results (low, middle, high).

Table 2. The comparison of risk measurement methods
A: The assumptions or constraints of implementation of method
B: Risk is considered as a negative phenomenon (N) and/or as a negative and positive event (NP)
C: The units of risk measurement are homogeneous or heterogeneous
D: The method takes into account the time in risk measuring
E: The difficulty of risk measurement and interpretation of risk measurement results (low, middle, high)

Methods / Criteria	A	B	C	D	E
Coefficient of Variation	No	N	Homogeneous	No	Low
Scenario Analysis	No	NP	Heterogeneous	Yes	High
Capital Asset Pricing Model - CAPM	Yes	N	Homogeneous	No	Middle
Weighted Average Cost of Capital - WACC	Yes	N	Homogeneous	No	Middle
Value at Risk -VaR	No	N	Homogeneous	No*	Low
Monte Carlo method	No	NP	Heterogeneous	No	High
Real Options	Yes	NP	Heterogeneous	Yes	High
The Integral Sum of Differential Weighted Indexes of Risks	No	NP	Heterogeneous **	Yes	Low

Remarks:
* No, within a short period of time, several days
** Heterogeneous, the result of calculation have not unit of measurement
Source: author

According to Table 2, some ideas can be summarized. The following methods of risk measurement have assumptions or constraints of implementation: Capital Asset Pricing Model (CAPM); Weighted Average Cost of Capital (WACC); Real Options. Other methods have not assumptions or constraints: it is Coefficient of Variation, Scenario Analysis, Value at Risk (VaR), Monte Carlo method, and The Integral Sum of Differential Weighted Indexes of Risks. Risk is considered as negative event by: Coefficient of Variation, Capital Asset Pricing Model (CAPM), Weighted Average Cost of Capital (WACC), and Value at Risk (VaR). Risk is regarded as negative and positive event by: Scenario Analysis, Monte Carlo method, Real Options and The integral sum of differential weighted indexes of risks. The units of risk measurement are homogeneous in such methods as: Coefficient of Variation, Capital Asset Pricing Model (CAPM), Weighted Average Cost of Capital (WACC) and Value at Risk (VaR). The units of risk measurement are heterogeneous in: Scenario Analysis, Monte Carlo method and Real Options. The Integral Sum of Differential Weighted Indexes of Risks is method allowing to use heterogeneous units of risk measurement, the result of calculation have not unit of measurement (it is integral parameter). The subsequent methods do not take into account the time period in risk measuring: Coefficient of Variation, Capital Asset Pricing Model (CAPM), Weighted Average Cost of Capital (WACC), Monte Carlo method. The following methods permit to take into account the time in risk measuring: Scenario Analysis and Real Options (within a short period of time, several days), the Integral Sum of Differential Weighted Indexes of Risks. High difficulty of risk measurement and interpretation of risk measurement results is characteristically for: Scenario Analysis, Real Options and Monte Carlo method. Middle difficulty of risk measurement and interpretation of results is typical for: Capital Asset Pricing Model (CAPM), Weighted Average Cost of Capital (WACC). Low difficulty of risk measurement and interpretation of results is characteristic of: Coefficient of Variation, Value at Risk (VaR), the Integral Sum of Differential Weighted Indexes of Risks. The integral sum of the differences weighted indexes is a method assessing the influence (effect) of risk management decision on risk values.

4. DISCUSSION

The author method (the Integral Sum of Differential Weighted Indexes of Risks method) is near to scenario analysis, but the form of representing (it is a matrix form) and the way of calculation is different. Entrywise division of matrices is used in the ISDWIR method. Entrywise operation on matrices is known as Schur product or also Hadamard product (named after I. Schur and J. Hadamard, respectively). It should not be confused with the more common matrix product. Such matrix operation is realized in many computing environments (MATLAB and Mathematica, for example) and that is why there is no obstacle of making calculations.

Entrywise division in the ISDEWIR method allows comparing changes of risk values assessed in different units of measurement (money units of measurement, percentage, coefficients, or others). The ISDWIR method of risk measurement does not exclude the results of other methods of risk assessment. The main task of all risk measurement methods is to approve the source of data and answer on such questions as the following: “Why do we use these data?”, “Why we have chosen this time period?”, etc. It is too important to understand all our assumptions concerning our calculations.

5. CONCLUSION

In this paper, the definitions of risk are considered and the following definition is proposed: risk is an event having its likelihood of occurrence and consequences that changes the performance indicators of company (improving or worsening them) when the external and/or internal environment varies, and appears both by reason of management decision and independently of it.

This definition emphasizes that:

- Risk is measured and there are many units of risk measurement because the consequences of risk (its impact on the company) can be measured by different units of measurement.
- Risk must have the impact on company that is reflected in performance indicators of company (it means that sometimes we can measure only consequences of events), if company is not exposed to risk, it does not have risk.
- Risk is considered as positive or negative event (negative and positive phenomenon).
- Risk occurs due to changes in the external or internal environment –with a static enterprise environment there is no risk.
- Risk is an event and occurs both by reason of management decision and independently of it.

The Integral Sum of Differential Weighted Indexes of Risks method is proposed by the author of the article. The method is based on assessment and comparison of the values of Integral Sums of Differential Weighted Indexes of Risks (the values of ISDWIR) at certain risk management decision (the decision with maximum value of ISDWIR is chosen). The Integral Sum of Differential Weighted Indexes of Risks (ISDWIR) is a composite generalizing indicator (no having unit of measurement), which measures the change of values of several risks at certain risk management decision that allows to determine the common impact of the decision on risks of the company.

ISDWIR method has ensuing advantages:

- it has no assumptions or constraints of implementation;
- risk is regarded as negative and as positive event by the method;
- it uses heterogeneous units of risk measurement;
- it takes into account the time in risk measuring;
- it is characterized by low difficulty of risk measurement and interpretation of results;
- it takes into account the importance (weight) of each kind of risks over the time period;
- it does not exclude the results of other methods of risk assessment.

The Integral Sum of Differential Weighted Indexes of Risks method (ISDWIR method) is a simple method of risk measurement to choose appropriate risk management decisions on the basis of integral indicator. It allows identifying the effect of risk management decision on the values of several risks measured by different units: risk decision increases or reduce the values of elements of Dynamic Enterprise Risk Matrix. It is possible to use risk values of different units of risk measurement (money units of measurement, per cent, coefficients or others) because the method convert the values of risks in indexes.

This method is universal and can be used (not only in risk management), when it is necessary to choose the solution that changes the performance of the enterprise on the basis of comparison of the values of indicators before and after the decision.

REFERENCES

Amram, M. and Kulatilaka, N. (1999). "Real options: Managing strategic investment in an uncertain world", Boston: *Harvard Business School Press*.

Aven, T. (2007). "A unified framework for risk and vulnerability analysis covering both safety and security", *Reliability Engineering & System Safety* 92, pp. 745–754

Bardossy, G. and Fodor, J. (2004). "Evaluation of Uncertainties and Risks in Geology", Berlin: *Springer*.

Cabinet Office (2002). "Risk: Improving government's capability to handle risk and uncertainty", Strategy Units, Cabinet Office, *HM Government*, London.

Cox, Jr. and Louis Anthony (Tony) (2008). "What's Wrong with Risk Matrices?", *Risk Analysis: An International Journal*, 28 (2), pp. 497–512.

Crowe, T.J.; Fong, P.M.; Bauman, T.A. and Zayas-Castro, J.L. (2002). "Quantitative risk level estimation of business process reengineering efforts", *Business Process Management Journal*, 8 (5), pp. 490–512.

Damodaran, A. (2007). "Strategic Risk Taking: A Framework for Risk Management", New Jersey: *Pearson Prentice Hall*.

Damodaran, A. (2012). "Investment Valuation: Tools and Techniques for Determining the Value of Any Asset", New Jersey: *John Wiley & Sons*.

Dixit, A. and Pindyck, R. (1994). "Investment under uncertainty", New Jersey: *Princeton University Press*.

Frey, R. and McNeil, A.J. (2002). "VAR and expected shortfall in portfolios of dependent credit risks: Conceptual and practical insights", *Journal of Banking and Finance*, 26, pp.1317–1334.

Glen, A. (2005). "Corporate financial management" (3rd ed.), Harlow: Pearson Prentice Hall.

Godet, M. and Roubelat, F. (1996). "Creating the future: the use and misuse of scenarios", *Long Range Planning*, 29(2), pp.164–171.

Graham, J.D. and Weiner, J.B. (eds.) (1995). "Risk versus risk: Tradeoffs in protecting health and the environment", Cambridge: *Harvard University Press*.

Hull, J.C. (2005). "Options, Futures, and Other Derivatives" (6th edition), New Jersey: *Prentice Hall*.

Kelley, K. (2007). "Sample size planning for the coefficient of variation from the accuracy in parameter estimation approach", *Behavior Research Methods*, 39(4), pp. 755–766.

Knight, F.H. (1921). "Risk, uncertainty and profit", New York: Harper & Row.

Levine, E.S. (2012). "Improving risk matrices: the advantages of logarithmically scaled axes", *Journal of Risk Research*, 15 (2), pp. 209–222.

Lintner, J. (1965). "The valuation of risky assets and the selection of risky investments in stock portfolio and capital budgets", *Review of Economics and Statistics*, 47, pp. 13–37

Lipton, J., Shaw, W.D., Holmes, J., and Patterson, A. (1995). “Short communication: selecting input distributions for use in Monte Carlo simulations”, *Regulatory Toxicology and Pharmacology*, 21, pp. 192–198.

Markowitz, H. (1959). “Portfolio Selection: Efficient Diversification of Investments”, New York: *John Wiley & Sons*.

Meacham, B.J. (2010). “Risk-informed performance-based approach to building regulation”, *Journal of Risk Research*, 13(7), pp. 877–893.

Metropolis, N. and Ulam, S. (1949). “The Monte Carlo Method”, *Journal of the American Statistical Association*, 44, pp. 335–341.

Miller, E.G. and Karson, M.J. (1977). “Testing the equality of two coefficients of variation”, *American Statistical Association*: Proceedings of the Business and Economics Section, Part 1, pp. 278–283.

Miller, L.T. and Park, C.S. (2002). “Decision making under uncertainty –Real options to the rescue?”, *The Engineering Economist*, 47(2), pp. 105–150.

Modigliani, F. and Miller, M. (1958). “The cost of capital, corporation finance and the theory of investment”, *American Economic Review*, 48(3), pp. 261–297.

Modigliani, F. and Miller, M. (1963). “Corporate income taxes and the cost of capital: A correction”, *American Economic Review*, 53, pp. 433–443.

Porter, M. (1985). “Competitive Advantage”, New York: *Free Press*.

Pratt P. and Grabowsky J. (2011). “Cost of Capital: Workbook and Technical Supplement”, New Jersey: *John Wiley and Sons*.

Rao, P.M. (2010). “Financial Statement Analysis and Reporting”, New Delhi: *PHI Learning*.

Ratcliffe, J. (1999). “Scenario building: a suitable method for strategic property planning”, The Cutting Edge 1999, *The Property Research Conference of the RICS St. John’s Collage*, Cambridge, 5th-7th September.

Raubitschek, R.S. (1988). “Multiple Scenario Analysis and Business Planning”, *Advances in Strategic Management*, 5, pp. 181–205.

Rosa, E.A. (1998). “Metatheoretical foundations for post-normal risk”, *Journal of Risk Research*, 1, pp. 15–44.

Rosa, E.A. (2003). “The logical structure of the social amplification of risk framework (SARF): Metatheoretical foundation and policy implications”, in: N. Pidegeon, R.E. Kaspersen, and P. Slovic (eds.). The social amplification of risk (pp. 47–76). Cambridge: *Cambridge University Press*.

Scholleová, H. (2008). “Basic Areas of Utilization of Real Options to Manage a Firm”, *Acta Oeconomica Pragensia*, 16 (4), pp. 3–11.

Scott, J.H. (1976). “A theory of optimal capital structure”, *Bell Journal of Economics*, 7, pp. 33–54.

Sharpe, W.F. (1964). “Capital asset prices: A theory of market equilibrium under conditions of risk”, *Journal of Finance*, 19, pp. 425–442.

Teach, E. “Will real options take root? Why companies have been slow to adopt the valuation technique”, *CFO Magazine*, 1 July 2003. Web: 10th Feb., 2014.

Trigeorgis, L. (1996). “Real Options: Managerial Flexibility and Strategy in Resource Allocation”, Massachusetts: *MIT Press*.

APPENDIX

The company has three main risks: the value of bank interest rate (the risk of increasing of bank interest rate, it is measured in per cents); the mean of total revenue over the time period (the risk of demand deviation, it is measured in money, thousands of Euros); and the value of spoilt production (the risk of increasing of quantity of spoilt production, it is measured in pieces).

The company aims to increase the mean of total revenue over the time period and to reduce the value of bank interest rate, the value of spoilt production.

1) The company assesses the risk values in dynamic to set the Basic Enterprise Risk Matrix (A matrix) as if there is no any process of risk management. Line one describes the changes of bank interest rate value, line two reports the mean of total revenue over the time period, line three presents the value of spoilt production. Columns refer to the time period one, two and three.

$$A=\begin{pmatrix} 0,09 & 0,1 & 0,11 \\ 50\ 000 & 45\ 000 & 47\ 000 \\ 15\ 000 & 15\ 500 & 15\ 700 \end{pmatrix}$$

The company works out two risk management decisions. Each decision has different effects on the values of corporate risks. The company sets Resulting Enterprise Risk Matrices (B, C matrices); they show the new values of risks according to risk management decision.

$$B=\begin{pmatrix} 0,09 & 0,1 & 0,105 \\ 50\ 000 & 49\ 000 & 51\ 000 \\ 15\ 000 & 15\ 400 & 15\ 200 \end{pmatrix}$$

$$C=\begin{pmatrix} 0,09 & 0,11 & 0,115 \\ 50\ 000 & 52\ 000 & 53\ 000 \\ 15\ 000 & 15\ 300 & 15\ 000 \end{pmatrix}$$

It can be seen that decision one reduces the value of bank interest rate more than decision two (at time period two and three is increased in comparison with the value of original matrix), Decision one increases the mean of total revenue less over the time period and reduces the value of spoilt production than decision two. It is necessary to compare Basic Enterprise Risk Matrix and Resulting Enterprise Risk Matrices of each variant of risk management decision.

2) Late, Risk Index Matrices are formed I 1, I 2 and Basic Unit Matrix (E matrix). The elements of B, C matrices are divided by the elements of A matrix to define Risk Index Matrices.

$$I1=\begin{pmatrix} 0,09/0,09 & 0,1/0,1 & 0,105/0,11 \\ 50000/\ 50\ 000 & 49\ 000/45000 & 51\ 000/47\ 000 \\ 15\ 000/15000 & 15\ 400/15\ 500 & 15\ 200/15700 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0,954545 \\ 1 & 1,088889 & 1,085106 \\ 1 & 0,993548 & 0,968153 \end{pmatrix}$$

$$I2=\begin{pmatrix} 0,09\ /0,09 & 0,11\ /0,1 & 0,115/0,11 \\ 50\ 000/50\ 000 & 52\ 000/45\ 000 & 53\ 000/47000 \\ 15\ 000/15\ 000 & 15\ 300/15\ 000 & 15\ 000/15700 \end{pmatrix} = \begin{pmatrix} 1 & 1,1 & 1,045455 \\ 1 & 1,155556 & 1,12766 \\ 1 & 0,987097 & 0,955414 \end{pmatrix}$$

$$E=\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

This mathematical operation assists to compare changes of risk values assessed in different units of measurement (money units of measurement, per cent, coefficients or pieces).

3) Next, it is needed to take into account the importance of each kind of risk and the time period on the basis of pair comparisons methods. Pair comparisons method of Louis Thurstone was used. On the basis of chosen pair comparisons method weighting coefficients of each kind of risk and each time period were defined. The weighting coefficient of the value of bank interest rate is 0,5, the mean of total revenue over the time period is 0,3 and the value of spoilt production is 0,2. The weighting coefficient of time period one is 0,1, time period two is 0.7 and time period three is 0,2. Then the weights coefficients of each kind of risk and time periods are multiplied and Matrix of Weighting Coefficients is obtained (W - matrix).

$$W = \begin{pmatrix} 0{,}05 & 0{,}35 & 0{,}10 \\ 0{,}03 & 0{,}21 & 0{,}06 \\ 0{,}02 & 0{,}14 & 0{,}04 \end{pmatrix}$$

The matrix describes the importance of each kind of risk according to the time period. The sum of all the elements of weight coefficients matrix is equal to 1.

4) Later each element of Risk Index Matrices (I 1, I 2 matrices) and the Basic Unit Matrix (E matrix) is multiplied by the elements of Weight Coefficients Matrix. Thus, Weighted Base Unit Matrix (E º) and Weighted Risk Index Matrices (I 1 º, I 2 º matrices) are set.

$$E^{o} = \begin{pmatrix} 0{,}05 & 0{,}35 & 0{,}10 \\ 0{,}03 & 0{,}21 & 0{,}06 \\ 0{,}02 & 0{,}14 & 0{,}04 \end{pmatrix}$$

$$I\,1^{o} = \begin{pmatrix} 0{,}05 & 0{,}35 & 0{,}095455 \\ 0{,}03 & 0{,}228667 & 0{,}065106 \\ 0{,}02 & 0{,}139097 & 0{,}038726 \end{pmatrix}$$

$$I\,2^{o} = \begin{pmatrix} 0{,}05 & 0{,}385 & 0{,}104545 \\ 0{,}03 & 0{,}242667 & 0{,}067660 \\ 0{,}02 & 0{,}138194 & 0{,}038217 \end{pmatrix}$$

5) Next, the Integral Sum of Differential Weighted Indexes of Risks (ISDWIR) between Weighted Base Unit Matrix (E º) and each Weighted Risk Index Matrix (I 1 º, I 2 º matrices) is calculated in accordance with formula 4.

ISDWIR 1 = $(0{,}05 - 0{,}05 + 0{,}35 - 0{,}35 + 0{,}095455 - 0{,}10) * (-1) + (0{,}03 - 0{,}03 + 0{,}228667 - 0{,}21 + 0{,}065106 - 0{,}06) * 1 + (0{,}02 - 0{,}02 + 0{,}139097 - 0{,}14 + 0{,}038726 - 0{,}04) * (-1) = 0{,}030496$

ISDWIR 2 = $(0{,}05 - 0{,}05 + 0{,}385 - 0{,}35 + 0{,}104545 - 0{,}10) * (-1) + (0{,}03 - 0{,}03 + 0{,}242667 - 0{,}21 + 0{,}06766 - 0{,}06) * 1 + (0{,}02 - 0{,}02 + 0{,}138194 - 0{,}14 + 0{,}038217 - 0{,}04) * (-1) = 0{,}004371$

6) The values of the Integral Sums of Differential Weighted Indexes of Risks for each variant of risk management decision are compared. The company should choose decision one, because it has greater value of ISDWIR than decision two and it is 0,030496.