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Forecast Intervals for US/EURO Foreign Exchange Rate

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ABSTRACT

The main goal of this research is to construct and assess forecast intervals for monthly US/EURO foreign exchange rate. The point forecasts used to build the intervals are based on a vector autoregression (VAR model) and on a Bayesian VAR model for data starting with the first month of 1999. The forecast intervals are based on the prediction error of the previous month. All the interval predictions based on VAR model included the actual values from 2014. The probability that the intervals based on BVAR model include the registered values of exchange rate is less than 0.8, according to likelihood ratio and chi-square tests.

Keywords: Forecast intervals; exchange rate; VAR model; Bayesian VAR model.
JEL classification: C51; C53.
MSC2010: 00A71; 97M10; 62M10; 62P20.

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Intervalos de pronóstico para los tipos de cambio US/EURO

RESUMEN

El objetivo principal de esta investigación es construir y evaluar intervalos mensuales previstos para los tipos de cambio US/EURO. Las previsiones puntuales usadas para construir los intervalos se basan en un modelo de vectores autorregresivos (VAR) y en un modelo VAR bayesiano para los datos a partir del primer mes de 1999. El pronóstico se basa en el error de intervalos de predicción del mes anterior. Todas las predicciones de intervalos basadas en el modelo VAR incluyen los valores reales de 2014. La probabilidad de que los intervalos basados en el modelo BVAR incluyan los valores registrados de los tipos de cambio es inferior a 0,8, según las pruebas de coeficiente de chi-cuadrado y de verosimilitud.

Palabras claves: intervalos de pronóstico; tipos de cambio; modelo VAR; modelo VAR bayesiano.
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1. Introduction

The treasury secretary of USA considered that a stronger Dollar is and will be always a positive aspect for US. However, there was a short period when monthly Euro was stronger than US dollar. The prediction of exchange rate is fundamental for the construction of macroeconomic policies and for making-decision process. The exchange rate predicting is essential to gauge the future evolution of export performance and price competitiveness. Most of the studies use point predictions, but fewer analyses are dedicated to forecast intervals.

The objective of this study is the construction of forecast intervals for real exchange rate, where the predictions are based on different types of econometric models (VAR and BVAR models). These forecast intervals are assessed by using likelihood ratio and chi-square tests.

After the literature review, a presentation of methods to assess the forecast intervals is made. Then, the forecasts based on the two types of models and the associated intervals. Moreover, the evaluation of prediction intervals is made and some conclusions are drawn.

2. Literature review

The real exchange rate is modeled using many types of models of fundamentals from productivity measures, commodity prices, openness, interest rate differentials to capital flows and fiscal balance. Another approach considers the exchange rate as asset price, taking into account not only the current fundamentals but also the fundamentals' expectations at a future moment. The price-asset approach allows the check of efficiency assumption in the exchange foreign markets.

Meese and Rogoff (1983) showed the superiority of random walk forecasts in terms of accuracy compared to structural models predictions. However, Wright (2008) argued the superiority of predictions based on Bayesian Model Averaging technique (BMA technique) compared to naïve forecasts. Kilian and Taylor (2003) are those who showed that the exchange rates the horizons for predictions based on economic model that includes a possible nonlinear exchange rate vary from 2 years to 3 years.

Wing and Wu (2009) attacked the Meese-Rogoff puzzle from the perspective of the interval predicting. The authors started from a group of Taylor rule models for which robust semi-parametric interval forecasting was applied. They discovered that Taylor rule models conduct to tighter forecast intervals compared to random walk. Molodtsova and Papell (2009) showed a significant predictability of exchange rate with Taylor-rule fundamentals.

Fattouh *et al.* (2008) proved, using empirical data, that forecasts based on a linear error correction model including fundamentals outperformed the predictions based on Markov-switching error correction model.

There were large preoccupations in literature for constructing theoretical models like Band-Threshold Autoregressive (Band-TAR models) and exponential smooth-transition autoregressive models (ESTAR models) that include transaction costs. A discrete regime switching is the main characteristic of Band-TAR models, while the ESTAR model supposes smooth transition between regimes.

Aye *et al.* (2013) compared the performance of real exchange rate predictions based on linear and, respectively non-linear models in South Africa. The authors presented all the varieties of forecasts: point predictions, interval forecasts and density forecasts for linear autoregressive models, ESTAR and TAR models. The differences in performance of forecasts for these types of models are non-significant for the real exchange rate in South Africa (South Africa Rand/US dollar and South Africa Rand/British pound).

Lam *et al.* (2008) compared the performance of predictions based on Bayesian model averaging technique, Purchasing Power Parity model, Uncovered Interest Rate Parity model and combined forecasts based on these models. The reference prediction is the naïve one and the historical average return. The combined predictions gave better results in terms of accuracy than the expectations based on a single model of those mentioned above.

Mtonga (2006) explained the necessity of predicting the real exchange rate for reducing the uncertainties in policy and decision making. The deviations of the exchange rate from the long-run equilibrium are more important for the policy making.

The forecast interval is built starting from the point forecasts and prediction error and a probability is attached in accordance to the hypothesis regarding the errors distribution. In general, we assume that the random shocks follow a normal distribution $e_t \to N(0,\sigma_\varepsilon^2)$, which implies a normally distributed probability density $x_{t+h} \to N(\hat{x}_{t+h},\sigma_h^2)$.

Olave and Miguel (2012) proposed a bootstrap method in order to construct forecast intervals starting from ARCH models of exchange rate. The main advantage of this approach is the fact that there is no assumption regarding the conditional repartition of errors.

Boero and Marrocu (2004) compared the performance of predictions based on GARCH and SETAR models for exchange rate, using as reference model a random walk. The assessment criteria refer to point and interval forecasts.

For a fixed probability (nominal coverage), the empirical coverage of forecast intervals based on an economic model is a probability that the registered exchange rates be placed inside the intervals. The difference between the upper and the lower limit of an interval is called length of the interval, being a measure of tightness. Most of evaluation methods suppose the comparison of empirical coverage across models. The best model is the one with the most accurate coverage. If we have equal coverage accuracy, we have to check if the intervals have the same length. Tighter the forecast interval is, better the forecasting model is. Wing and Wu (2009) showed that in the case of equal coverage accuracy the economic models provide tighter prediction intervals compared to random walk. The authors proposed some loss criteria to assess the quality of intervals and some statistics presented by Giacomini and White (2006).

The forecast intervals are assessed by making the comparison with the out-of-sample interval predictions from the random walk. The likelihood ratio tests for conditional, unconditional and independent coverage were used by Wallis (2003) and Aye *et al.* (2013) to compare the predictions based on linear random walk models to those based on non-linear random walk models.

Empirical forecast intervals were built by Lee and Scholtes (2014) that used the repartition of the previous prediction errors. The construction of the prediction interval is based on the distribution quantiles for the prediction errors and the point forecasts based on a certain model.

For high-frequency data, Cai and Zhang (2016) analyzed the exchange rate movement predictability. The authors used an autoregressive conditional multinomial–autoregressive conditional duration model and they found a high predictability in data. By filtering the data, the forecasts performance improved.

A Bayesian VAR and a VAR model with Dornbusch prior were used by Ca'Zorzi *et al.* (2015) to predict the exchange rate. The standard VAR predictions and the naïve forecasts were outperformed by the predictions based on the VAR model with Dornbusch prior.

3. The assessment of forecast intervals

For assessing the density or forecast intervals more tests are used. Wallis (2003) used inferences on p-values rather than the asymptotic repartition of chi-square tests. This author developed the conditional, unconditional and independent coverage proposed by Christoffersen (1998). Good interval forecasts should be independently distributed, having an acceptable coverage.

I. Likelihood ratio (LR) tests for forecast intervals

We consider time series for the forecast intervals and we fix the probability π for the value to be inside the interval. We registered time series for the results registered in reality and we fix as objective the evaluation of *ex-ante* probability correction. If n_1 results are inside the forecast intervals and n_2 outside it, the coverage probability is: p= n_1/n. The distribution is binomial, under the null hypothesis and the likelihood is $L(\pi)=(1-\pi)^{n_2}\,\pi^{n_1}$; while under the alternative hypothesis, it is $L(p)=(1-p)^{n_2}\,p^{n_1}$. Christoffersen (1998) established the statistic of likelihood ratio as $LR_{UC}=2(n_2\log\frac{1-p}{1-\pi}+n_1\log\frac{p}{\pi})\underset{H_0}{\longrightarrow}\chi_1^2$.

This is a test of unconditional coverage that is actually unsuitable for time series. Therefore, Christoffersen (1998) proposed another test that combines the test of unconditional coverage with the independence one.

The independence test is based on matrix of transition frequencies $[n_{ij}]$, which is the number of observations that are in state *i* at moment *t*-1 and in state *j* at moment *t*. The maximal likelihood estimations of transition probabilities are computed as a ratio between frequencies in a cell and the total number of frequencies of a line. For a forecast interval, two cases are possible: The values are inside or outside the interval, being denoted with 1 and 0. The transition matrix of estimated probabilities is:

$$P = \begin{pmatrix} 1-p_{01} & p_{01} \\ 1-p_{11} & p_{11} \end{pmatrix} = \begin{pmatrix} \frac{n_{00}}{n_{0.}} & \frac{n_{01}}{n_{0}\cdot} \\ \frac{n_{10}}{n_{1.}} & \frac{n_{11}}{n_{1.}} \end{pmatrix}$$

The likelihood for P is $L(P) = (1-p_{01})^{n_{00}} \cdot p_{01}^{n_{01}} \cdot (1-p_{11})^{n_{10}} \cdot p_{11}^{n_{11}}$. The null hypothesis of independence test fixes that the *t*-1 state is independent of *t* state, which is equivalent with $\pi_{01} = \pi_{11}$. The estimator of maximal likelihood of the common probability is $p = \frac{n_{.1}}{n}$. The likelihood under the null hypothesis assessed at p is: $L(p) = (1-p)^{n_{.0}} \cdot p^{n_{1.}}$. The LR test statistic is: $LR_{ind} = -2\log\frac{L(p)}{L(P)} \underset{H_0}{\longrightarrow} \chi_1^2$. The test proposed by Christoffersen (1998) that combine the unconditional coverage test with the independence one has the statistic: $LR_{CC} = -2\log\frac{L(\pi)}{L(P)} \underset{H_0}{\longrightarrow} \chi_2^2$.

If the first observation is ignored, then: $LR_{CC} = LR_{UC} + LR_{ind}$.

II. Chi-square (χ^2) tests for forecast intervals

Stuart *et al.* (1999) demonstrated that likelihood ratio tests are equivalent with Pearson's goodness-of-fit tests. Wallis (2003) used them for the first time for density and forecast intervals. Chi-square test for unconditional coverage uses the statistic $X^2 = \frac{n(p-\pi)^2}{\pi(1-\pi)}$. If we consider the matrix of observed frequencies,

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix},$$

then

$$X^2 = \frac{n(ad-bc)^2}{(a+b)(c+d)(a+c)(b+d)}$$

The conditional coverage test combined with the independence test uses the contingency table of the observed frequencies with expected frequencies under the null hypothesis of independent lines and using the coverage probability π.

The matrix of expected frequencies is

$$\begin{bmatrix} (1-\pi)(a+b) & \pi(a+b) \\ (1-\pi)(c+d) & \pi(c+d) \end{bmatrix}.$$

The proportions column is considered under the test hypothesis; the tests has 2 degrees of freedom. The statistic is computed as a sum of square normal standard statistics of the sample of proportions, a proportion for each row of the table. For low-volume samples the additive relationship of LR statistics cannot be transposed exactly in chi-square test terms.

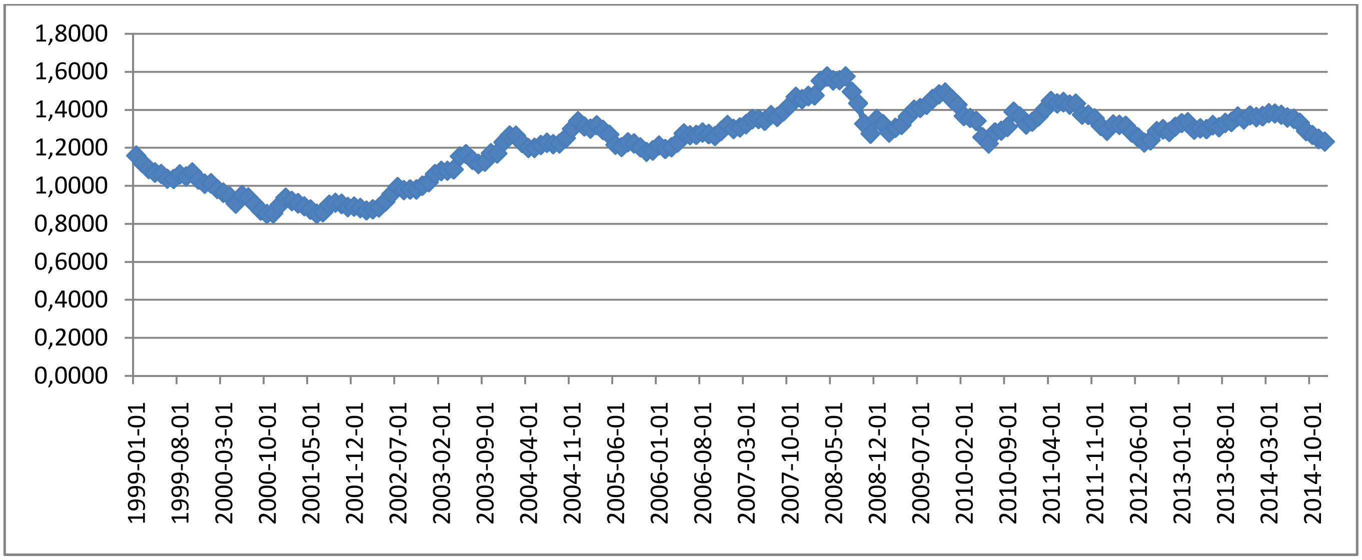
4. The evaluation of exchange rate forecast intervals

The exchange rate forecasts were built using VAR and Bayesian VAR models for monthly US/Euro exchange rate (ER) and consumer price index (CPI) over the period 1999:01-2014:12. The data are provided by Federal Reserve Bank of St. Louise from Federal Reserve Economic Database. The aggregation method for foreign exchange rate is based on the average. The consumer price index is the average monthly change in the goods and services prices, being paid by the consumers between two periods. The data series for CPI is seasonally adjusted and the reference base is the index from 1982-1984.

As we can see in Figure 1, there were some periods with values of exchange rate under 1: 2000:02-2002:10. This might be caused by the demand expansion by lowering the interest rates.

The stationary character of the data series is checked using ADF test. The series in level presents unit roots and the stationary character is ensured by the first differentiation. The variables corresponding to the new data series are denoted by Δcpi and Δer (see Table 1)

Figure 1. The evolution of monthly US/Euro foreign exchange rate (1999:01-2014:12)



Source: Own elaboration.

Table 1. The Augmented Dickey-Fuller (ADF) test for the transformed data series

Variable	Model	ADF statistic	1% critical value	5% critical value	10% critical value
Δcpi_t	Trend+intercept	-6.5162	-4.0101	-3.4348	-3.1411
	Intercept	-6.4737	-3.4667	-2.8771	-2.5750
	None	-4.2398	-2.5766	-1.9414	-1.6166
Δer_t	Trend+intercept	-5.7196	-4.0101	-3.4348	-3.1411
	Intercept	-5.7115	-3.4667	-2.8771	-2.5750
	None	-5.7126	-2.5766	-1.9414	-1.6166

Source: Author's computations.

Most of the lag length criteria indicated that the suitable lag equals 2 (see Table 2).

Table 2. Lag length criteria for VAR model

Lag	LogL	LR	FPE	AIC	SC	HQ
0	189.8130	NA	0.000381	-2.196644	-2.159899	-2.181734
1	219.9155	59.14865	0.000281	-2.501935	-2.391701*	-2.457207*
2	225.6149	11.06554*	0.000275*	-2.521811*	-2.338088	-2.447264
3	227.0109	2.677775	0.000284	-2.491356	-2.234143	-2.386990
4	228.1586	2.174527	0.000294	-2.457995	-2.127294	-2.323811
5	232.1550	7.478671	0.000294	-2.457953	-2.053763	-2.293950
6	233.5947	2.660437	0.000303	-2.428008	-1.950328	-2.234186
7	238.7297	9.369111	0.000299	-2.441283	-1.890113	-2.217642
8	240.7302	3.603297	0.000306	-2.417897	-1.793239	-2.164437

Source: Author's computations.

The corresponding equations of the VAR model are represented below:

$$\Delta er = 0.3190269538 * \Delta er(-1) - 0.06897735018 * \Delta er(-2) - 0.003635677141 * \Delta cpi(-1) - 0.002265028041 * \Delta cpi(-2) + 0.00357020649;$$

$$\Delta cpi = 3.427668687 * \Delta er(-1) + 1.876892616 * \Delta er(-2) + 0.4382151242 * \Delta cpi(-1) - 0.2301331038 * \Delta cpi(-2) + 0.3058719426.$$

Table 3. VAR Residual Portmanteau test errors' autocorrelation

Lags	Q-Stat	Prob.	Adj Q-Stat	Prob.	df
1	0.073084	NA*	0.073499	NA*	NA*
2	0.967128	NA*	0.977761	NA*	NA*
3	3.356869	0.5000	3.408704	0.4919	4
4	5.282142	0.7270	5.378492	0.7165	8
5	12.64331	0.3955	12.95365	0.3724	12
6	14.12884	0.5891	14.49130	0.5622	16
7	24.67919	0.2140	25.47607	0.1838	20
8	25.84405	0.3611	26.69608	0.3188	24
9	28.17443	0.4552	29.15130	0.4049	28
10	29.28258	0.6048	30.32581	0.5514	32
11	35.70315	0.4826	37.17184	0.4148	36
12	42.93435	0.3466	44.92894	0.2730	40

Source: Author's computations.

In the first period (see Table 4), the variation in exchange rate is not due to consumer price index changes. In the second period, 0.41% of the variance in exchange rate is due to CPI changes. Starting from the third period, more than 1.2% of the variation in US/EUR foreign exchange rate is explained by CPI.

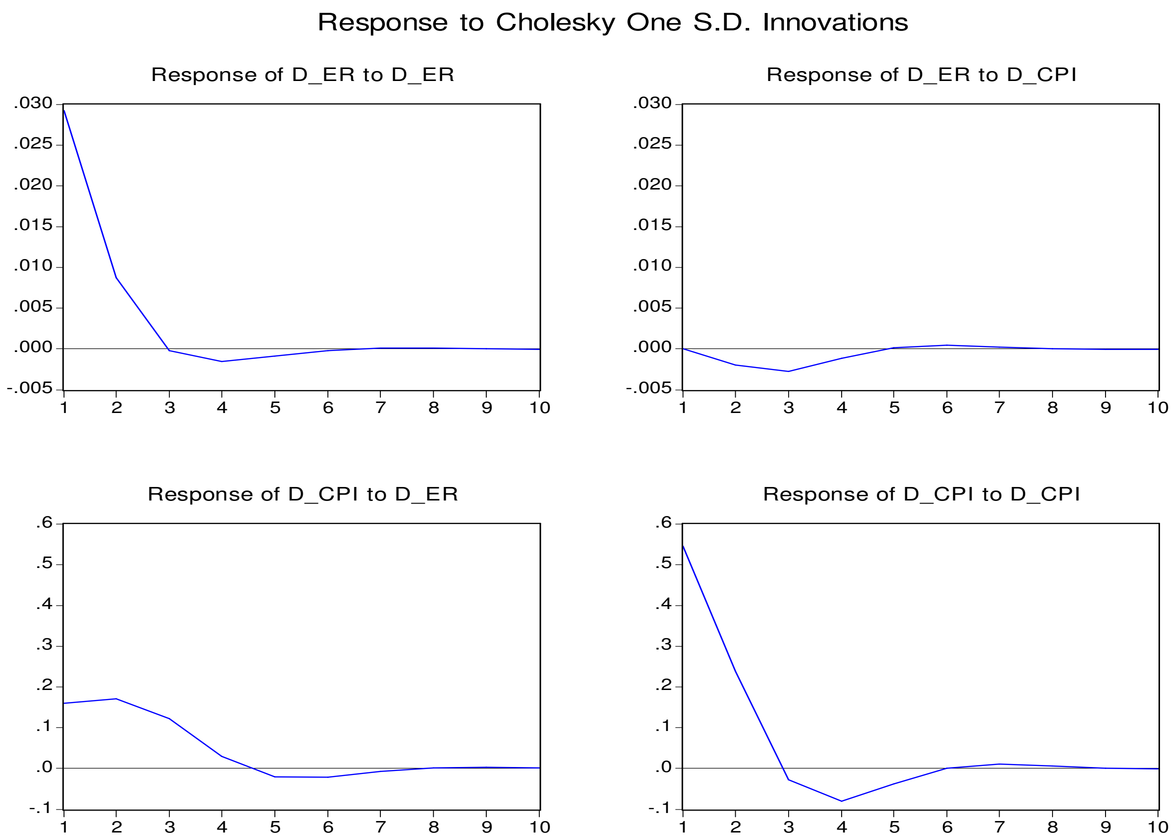
Table 4. Variance decomposition of Δer

Period	S.E.	Δer	Δcpi
1	0.029250	100.0000	0.000000
2	0.030594	99.58078	0.419218
3	0.030717	98.79185	1.208151
4	0.030776	98.65042	1.349579
5	0.030788	98.64855	1.351445
6	0.030792	98.62717	1.372829
7	0.030793	98.62223	1.377769
8	0.030793	98.62225	1.377746
9	0.030793	98.62185	1.378146
10	0.030793	98.62172	1.378283

Source: Author's computations.

In the second, the third and the fourth period (see Figure 2), a shock in the consumer price index determined a negative response of exchange rate.

Figure 2. Impulse-response functions



Source: Own elaboration.

A Bayesian VAR model of order 2 is also built. The means of posterior coefficients are used as coefficient to make forecasts for the months of 2014. The VAR model is also used to make forecasts for 2014.

The BVAR model has the following form:

$$Yi = Xi * Phi\_i + ui , \qquad \text{where } ui \sim N(0,s^2);$$
$$Phi\_i \mid tau\_i \sim tau\_i * N(0,V1) + (1\text{-}tau\_i) * N(0, V2) , V1 > V2$$

tau_i = 1 suggests that a variable was chosen;

tau_i = 0 shows that Phi_i can be excluded.

The estimation algorithm is a Gibbs sampler with hierarchical priors:

First stage: s^2 ~ IG(a,b), Beta_i | tau_i ~ tau_i * N(0,V1) + (1-tau_i) * N(0, V2).
Second stage: tau_i | pi ~ Bernoulli (pi).
Third stage: pi ~ Beta (a', b').

Table 5. Estimation of Bayesian VAR model

Constant	Posterior Phi1 coefficients		Posterior Phi2 coefficients		Posterior covariance matrix of the VAR system	
0.6894	1.35	4.83	-0.35	-4.35	0.32	0
0.0104	0	1.3	0	-0.33	0	0.01

Source: author's computations

Starting from the point forecasts based on VAR and BVAR models, some forecast intervals were built by considering the forecast error from previous period.

The prediction intervals are built as follows:

$$(er_t(k) - z_{\alpha/2} \cdot e(k-1), er_t(k) + z_{\alpha/2} \cdot e(k-1)), \quad k = 1,..., K .$$

where:

$er_t(k)$ is the point forecast of exchange rate for period (t+k); and

$z_{\alpha/2}$ is the quantile α/2 of standard normal distribution.

Table 6. The point forecasts and prediction intervals for US/Euro foreign exchange rate

Month	Point forecasts based on VAR model	Point forecasts based on BVAR model	Forecast intervals based on VAR model (intervals' limits)		Forecast intervals based on BVAR model (intervals' limits)		Actual values
2014:01	1.3798	1.3702	**1.3446**	**1.4149**	**1.3350**	**1.4054**	1.3618
2014:02	1.3815	1.3778	**1.3521**	**1.4108**	**1.3485**	**1.4071**	1.3665
2014:03	1.3820	1.3806	**1.3803**	**1.3836**	**1.3790**	**1.3822**	1.3828
2014:04	1.3830	1.389	**1.3791**	**1.3868**	**1.3692**	**1.3851**	1.3810
2014:05	1.3845	1.39	**1.3637**	**1.4053**	**1.3379**	**1.3692**	1.3739
2014:06	1.3862	1.3903	**1.3338**	**1.4386**	1.3228	1.3379	1.3595
2014:07	1.3880	1.3907	**1.3200**	**1.4559**	1.2763	1.3228	1.3533
2014:08	1.3896	1.3902	**1.2757**	**1.5035**	1.1899	1.2763	1.3315
2014:09	1.3912	1.3905	**1.1907**	**1.5918**	1.1494	1.1899	1.2889
2014:10	1.3929	1.3947	**1.1475**	**1.6382**	1.1097	1.1494	1.2677
2014:11	1.3945	1.3982	**1.1060**	**1.6830**	1.0826	1.1097	1.2473
2014:12	1.3961	1.4025	**1.0762**	**1.7161**	0.0000	1.0826	1.2329

Source: author's computations.

All the forecast intervals based on VAR models includes the actual values, but only 5 out of 12 intervals based on BVAR model contain the registered values of exchange rate (see Table 6). Starting from the half of 2014, the exchange rate underwent a fast decrease that was not anticipated by the models. Moreover, the predictions based on BVAR model anticipated a higher increase than VAR model forecasts, fact that explains the differences between forecast intervals.

The forecast intervals based on the point predictions of VAR and Bayesian VAR models are assessed using likelihood ratio test and chi-square test. Considering an *ex-ante* probability of 0.8, we assess the hypothesis that the empirical probability is 0.8.

Table 7. The evaluation of prediction intervals for US/Euro foreign exchange rate

LR_{CC} statistic		Chi-square statistic	
VAR model predictions	BVAR model predictions	VAR model predictions	BVAR model predictions
4.4629	3.7523	3	25.5208

Source: author's computations.

For a level of significance of 5%, both tests suggest that the probability that forecast intervals include the actual values is higher than 0.8 for the forecasts based on VAR models, but this probability is lower than 0.8 for intervals based on BVAR predictions.

5. Conclusions

In this study, point forecasts are built for US/EURO foreign exchange rate for months in 2014. The forecasting method is represented by econometric models like: VAR model and Bayesian VAR model. The results indicated that all the prediction intervals based on VAR model included all the actual values unlike the intervals based on BVAR model.

In a future research, other types of forecast intervals should be developed and other econometric models should be chosen.

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