

Estimación de volatilidad constante por métodos clásicos y bayesianos en un mercado financiero: Una aplicación a las preferenciales de Bancolombia

Constant volatility estimation by classical and bayesian methods in a financial market: An application to Bancolombia's preferential prices

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RESUMEN

En este trabajo se proponen métodos, desde un enfoque clásico y bayesiano, para estimar la volatilidad constante de un activo cuando no es conveniente ajustar modelos de volatilidad heteroscedástica o estocástica en relación con la serie muestral del activo donde no se observa un aumento elevado de la volatilidad. Para probar cuál de los métodos propuestos se ajusta mejor a la variabilidad de la información y de los pronósticos, se utiliza el modelo estocástico propuesto por Paul Samuelson para estimar, con mínimo error, los precios de cierre de las acciones preferenciales de Bancolombia en un periodo muestral donde no se observa saltos significativos en la evolución de sus precios. Desde el enfoque bayesiano, se asumen a priori las distribuciones gamma inversa, estándar de Levy y de volatilidad de Jeffreys, con la estimación de los hiperparámetros propuestos por los autores. La metodología propuesta se contrasta con la estimación clásica de la volatilidad y el método bootstrap. A partir de los precios de cierre de la acción preferencial de Bancolombia durante un periodo de tiempo donde no existe significancia en el aumento o disminución en su volatilidad de manera temporal, la técnica bayesiana con distribución a priori Gamma Inversa captura la mayor información sobre la muestra de sus retornos, mientras que la estimación clásica de volatilidad pronostica el activo, dentro y fuera de muestra, con menor error. Sin embargo, los pronósticos del activo utilizando técnicas bayesianas o clásicas no muestran un impacto significativo.

PALABRAS CLAVE

Ecuación diferencial estocástica, lema de Ito, rendimientos, proceso estocástico, autocorrelación serial.

ABSTRACT

In this paper we propose methods, from a classical and Bayesian approach, to estimate the constant volatility of an asset when it is not appropriate to fit heteroscedastic or stochastic volatility models relative to the sample series of the asset where no large increase in volatility is observed. To test which of the proposed methods best adapts to the variability of information and forecasts, the stochastic model proposed by Paul Samuelson is used to estimate, with minimum error, the closing prices of Bancolombia's preference shares in a sample period in which no significant jumps in the evolution of their prices are observed. From the Bayesian approach, the inverse gamma, standard Levy and Jeffreys volatility distributions are assumed a priori, with the estimation of the hyperparameters proposed by the authors. The proposed methodology is compared with classical volatility estimation and the bootstrap method. From the closing prices of Bancolombia's preference shares over a period where there is no significant increase or decrease in volatility over time, the Bayesian technique with gamma inverse prior captures the most information about sample returns, while the classical volatility estimation forecasts the asset, in and out of sample, with less error. However, forecasting the asset using either Bayesian or classical techniques does not show a significant impact.

KEYWORDS

Stochastic differential equation, Ito's lemma, returns, stochastic process, serial autocorrelation.

Clasificación JEL: C11, C12, C20, C51, G1.

MSC2010: 39A50, 91B30, 91B62, 91B84.

1. INTRODUCTION

The dynamics of the various economic factors affecting stock markets are of natural interest, given the role of these markets in stabilizing the financial sector to promote a country's economic growth, whose behavior is uncertain and highly dependent on fluctuations in economic fundamentals (Bhowmik, 2013). In this context, the volatility of returns on financial assets is one of the most important uncertainty variables and therefore needs to be understood by market participants. Not only does it affect the efficient allocation of funds, but it can also hamper economic development as it plays a key role in the valuation of derivatives, the calculation of risk measures and the hedging of positions. In general, volatility is an estimate of the uncertainty of a financial asset and is quantified using statistical techniques. Chen (1997) and Chen, Du, Li & Ouyang (2013) stated that volatility should be calculated using the standard deviation of the series of returns in a financial series, since returns represent the fluctuations of the asset without scale and are much more manageable.

However, the assumption of constant volatility in traditional models such as Black & Scholes (1973) has been widely criticized in financial literature for its inability to capture phenomena observed in markets such as volatility smiles and smirks, volatility clustering and price jumps (Ho, Perraudin, & Sørensen, 1996; Eraker, Johannes, & Polson, 2003; Christoffersen, Heston, & Jacobs, 2013). Hull & White (1987) were pioneers in pointing out that volatility is not constant but varies over time and is subject to random shocks. This finding has led to the development of heteroskedastic and stochastic volatility models, where volatility is modelled as a random process with innovations independent of those of the return series (Awartani, & Corradi, 2005; Andritzky, Bannister, & Tamirisa, 2007; Álvarez Franco, Restrepo, & Pérez, 2007; Yang, Zhou, & Wang, 2009).

When the sample financial series exhibits significant and recurrent fluctuations over the period under study, the squared return series exhibits a significant serial autocorrelation structure, that is, a time dependence among its observations (Goudarzi, & Ramanarayanan, 2010; de Jesús Gutiérrez, 2015; Mamtha, & Srinivasan, 2016; Rossetti, Nagano, & Meirelles, 2017; Hendrych, & Cipra,

2018). In such cases, the volatility of the asset should be considered as time-dependent, and its estimation is carried out using heteroscedastic volatility models, such as those belonging to the GARCH family, or stochastic volatility models (Heston, & Nandi, 2000; Khan, Khan, & Khan, 2016; García, & Esquivel, 2018ab; García, & Esquivel, 2019; Byun, Hyun, & Sung, 2021). Parameter estimation for these models is performed using maximum likelihood or Bayesian methods, considering distributions such as normal or generalized GED for the perturbations in this class of models (Villagran, & Huerta, 2006; Espinosa Acuña, & Vaca González, 2017).

However, if there is no serial autocorrelation in the squared sample return series, that is, if the sample financial series does not exhibit significant or recurring fluctuations over the period under consideration, it is not statistically meaningful to fit stochastic or heteroscedastic volatility models (Goudarzi, & Ramanarayanan, 2010; de Jesús Gutiérrez, 2015; Mamtha, & Srinivasan, 2016; Khan, Khan, & Khan, 2016; Rossetti, Nagano, & Meirelles, 2017; García, & Esquivel, 2018ab; Hendrych, & Cipra, 2018; García, & Esquivel, 2019; Byun, Hyun, & Sung, 2021). In such cases, volatility is assumed to be constant. However, even in this scenario, the estimation of constant volatility can be problematic, as the calculation of the standard deviation of returns can generate wide confidence intervals, making the precision of the sample estimate difficult. As the financial series in this study are not a function of intraday data or one-day high/low and opening and closing prices, HAR volatility models are not considered (Qiu, Zhang, Xie, & Zhao, 2019; Clements, & Preve, 2021).

To address this problem, some researchers have proposed alternative methods. For example, Karolyi (2019) uses prior information extracted from cross-sectional patterns of return volatility for groups of stocks sorted by size, financial leverage or trading volume, together with sample information, to derive the posterior variance density. This approach has shown higher predictive accuracy in option pricing than classical methods such as implied volatility or standard historical volatility. Similarly, Ho, Lee & Marsden (2011) use a Bayesian approach to estimate the constant volatility of stock prices by solving the heavy-tailed problem using the t-Student distribution. The authors show that volatility estimation using Bayesian methods is more accurate than classical methods.

In this context, our study proposes methods for estimating volatility when the sample return series follows a normal distribution and does not exhibit significant lags in squared returns. These estimates are made using classical and Bayesian methods, assuming as a priori distributions the inverse gamma, standard Lévy and Jeffrey distributions. These distributions are chosen to respect the fundamental properties of asset volatility, that is, non-negative and leptokurtic. In addition, the estimation method that best adapts to the variability of the information and predicts, with the minimum error, the daily closing prices of Bancolombia's preferred shares between 02/01/2019 and 30/12/2019 (G. Bancolombia, 1875), which has no significant time jumps in its price evolution, is tested.

Our approach contributes to the literature by addressing the limitations of classical constant volatility estimation methods and by proposing Bayesian methods that improve the precision of the estimates. Moreover, the study is in line with recent literature highlighting the importance of independent volatility shocks and the flexibility of the correlation between returns and volatility (Eraker, Johannes & Polson, 2003; Christoffersen, Heston & Jacobs, 2013; Byun, Hyun, & Sung, 2021). By combining classical and Bayesian techniques, our work provides a flexible platform for estimating volatility in scenarios where heteroskedastic or stochastic models are not applicable in the sample time series.

2. RESEARCH METHODOLOGY

According to numerous studies (Goudarzi, & Ramanarayanan, 2010; de Jesús Gutiérrez, 2015; Mamtha, & Srinivasan, 2016; Khan, Khan, & Khan, 2016; Rossetti, Nagano, & Meirelles, 2017; García, & Esquivel, 2018a; García, & Esquivel, 2018b; Hendrych, & Cipra, 2018; García, & Esquivel, 2019; Byun, Hyun, & Sung, 2021), the following steps are suggested to determine whether volatility should be considered constant in a sample series of asset prices:

- Conversion of the sample time series of the asset into a series of returns (Tsay, 2014),
- Use the Dickey-Fuller test to determine that the series of returns follows a stationary process (Tsay, 2014),
- Use the Ljung-Box test to verify if the series of squared returns has significant lags in its autocorrelations (Tsay, 2014). If there are no significant lags, volatility is considered constant.

For the case where the return series follows a normal distribution with zero population means, this paper proposes alternatives using classical and Bayesian methods to estimate the constant volatility of an asset over a given period. If we are interested in estimating asset prices for a short period of time, the next section presents a difference equation for estimating returns, which is a discrete solution of a stochastic differential equation describing the asset returns and whose parameters are given by the sample mean and variance of the returns, together with a random behavior given by the interaction of volatility with fluctuations determined by a normal distribution.

3. MODEL DESCRIPTION

The following is a mathematical model that can be used to estimate the returns of an asset, the parameters of which are given by the sample mean and variance of the return series. Similarly, conditions are given to determine whether the population means, or variance should be considered constant.

3.1. Model for estimating sample returns

Consider a historical record of the monthly prices of an asset $\{P_t\}_{t=0}^n$, whose monthly asset returns $\{R_t\}_{t=1}^n$ are of the form

3.1

$$R_t = \frac{P_t - P_{t-1}}{P_{t-1}},$$

with monthly mean estimation

3.2

$$\hat{\mu} = \frac{1}{n} \sum_{t=1}^n R_t,$$

and monthly variance estimation

$$\hat{\sigma}^2 = \frac{1}{n} \sum_{t=1}^n (R_t - \hat{\mu})^2,$$

where the standard deviation σ is known as the volatility of the monthly return.

If returns $\{R_t\}_{t=1}^n$ are standardized,

$$\tilde{R}_t = \frac{R_t - \hat{\mu}}{\hat{\sigma}},$$

and the histogram of $\{\tilde{R}_t\}_{t=1}^n$ coincides with the density function of a random variable $\epsilon_t \sim N(0,1)$, then the monthly returns $\{R_t\}_{t=1}^n$ follow a normal distribution, with a monthly mean $\hat{\mu}$ and a monthly variance $\hat{\sigma}^2$. In this case

3.3

$$\frac{P_{t-1} - P_t}{P_{t-1}} = \hat{\mu} \times \text{month} + \sqrt{\hat{\sigma}^2 \times \text{month}} \epsilon_t.$$

If Δt expresses a month and generally denotes the unit of time separating observations $P_0, P_1, \dots, P_n$, by (3.3) we have that:

3.4

$$\frac{P_{t-1} - P_t}{P_{t-1}} = \hat{\mu} \Delta t + \hat{\sigma} \sqrt{\Delta t} \epsilon_t,$$

equivalently,

$$\frac{\Delta P_t}{P_{t-1}} = \hat{\mu} \Delta t + \hat{\sigma} \Delta W_t,$$

where $\Delta W_t \sim N(0, \Delta t)$, with $E[\Delta W_t] = 0$, and $\text{Var}[\Delta W_t] = \Delta t$.

If Δt becomes smaller and smaller, for example, from months to weeks, from weeks to days, from days to hours, from hours to seconds, from seconds to moments, then the model (3.4) is equivalent to:

3.5

$$\frac{dP_t}{P_t} = \hat{\mu} dt + \hat{\sigma} dW_t,$$

where $dW_t \sim N(0, dt)$ are the daily fluctuations in returns. The stochastic model (3.5) was introduced by Paul Samuelson (Venegas-Martínez, 2008).

In practice, we assume that μ and σ are known and that the only uncertainty in the model (3.5) is the random noise $dW(t)$. Therefore, the simulation of the model (3.5) can be used to evaluate the stock price.

Using Ito's Lemma (Venegas-Martínez, 2008), the model (3.5) is equivalent:

$$3.6 \quad d\ln(P_t) = \left(\hat{\mu} - \frac{\hat{\sigma}^2}{2} \right) dt + \hat{\sigma} dW_t.$$

If $d\ln(P_t) = \ln(P_t) - \ln(P_{t-1})$ and S , then the discrete version of the model (3.6) is

$$3.7 \quad P_t = P_{t-1} \exp \left(\hat{\mu} - \frac{1}{2} \hat{\sigma}^2 + \hat{\sigma} \epsilon_t \right),$$

where $\epsilon_t \sim N(0,1)$, and P_0 is the initial price of the asset.

To predict asset prices using the model (3.7), we need to see that the parameters $\hat{\mu}$ and $\hat{\sigma}$ do not depend on time or position, that is, they are considered as constants. Therefore, we must check whether the series $\{R_t\}_{t=1}^n$ behaves as a stationary process and does not show significant autocorrelations between two periods t and s of the series R_t and R_t^2 , respectively (Tsay, 2014).

According to Tsay (2014), to determine whether a series $\{R_t\}_{t=1}^n$ is stationary, we must be checked that

$$R_t = R_{t-1} + \epsilon_t,$$

with $\epsilon_t \sim N(0,1)$. For this purpose, the following model is considered

$$3.8 \quad R_t = \phi_1 R_{t-1} + \epsilon_t,$$

and the Dickey-Fuller test is proposed, which contrasts

$$3.9 \quad \begin{aligned} H_0: \quad \phi_1 &= 1 \\ H_a: \quad \phi_1 &< 1, \end{aligned}$$

with test statistician

To construct an estimator of ϕ_1 , note that the model (3.8) is equivalent to model

$$X_{t-1} = \delta R_{t-1} + \epsilon_t,$$

where $X_{t-1} = R_t - R_{t-1}$ and $\delta = \phi_1 - 1$ is estimated by a simple linear regression without intercept.

An estimator of δ by the least squares method is

$$\hat{\delta} = \frac{\sum_{t=1}^n X_{t-1} R_{t-1}}{\sum_{t=1}^n R_{t-1}^2},$$

with $R_0=0$, and

$$\text{Var}[\hat{\delta}] = \frac{\sigma_\epsilon^2}{\sum_{t=1}^n R_{t-1}^2},$$

were

$$\hat{\sigma}_\epsilon^2 = \frac{1}{n-1} \sum_{t=1}^n (X_{t-1} - \hat{\delta} R_{t-1})^2 = \frac{1}{n-1} \sum_{t=1}^n (R_t - \hat{\phi}_1 R_{t-1})^2.$$

Since

$$\text{Var}[\hat{\phi}_1] = \text{Var}[\hat{\delta}],$$

then an estimator of $\hat{\phi}_1$ is given by

$$\hat{\phi}_1 = \frac{\sum_{t=1}^n R_t R_{t-1}}{\sum_{t=1}^n R_{t-1}^2},$$

and

$$\text{DF} = \frac{\sum_{t=1}^n R_{t-1} (R_t - R_{t-1})}{\hat{\sigma}_\epsilon \sqrt{\sum_{t=1}^n R_{t-1}^2}}.$$

3.2. Constant mean and constant volatility conditions

If $\{R_t\}_{t=1}^n$ is stationary, then the central idea behind fitting an ARMA model to account for μ as a non-constant parameter in $\{R_t\}_{t=1}^n$ is to test whether there are significant autocorrelations between two variables R_t and R_{t-k} , separated by k periods, defined by

3.10

$$\rho_k = \frac{\text{cov}[R_t, R_{t-k}]}{\sqrt{\text{Var}[R_{t-k}]} \sqrt{\text{Var}[R_t]}} = \frac{\text{cov}[R_t, R_{t-k}]}{\text{Var}[R_t]}.$$

For this purpose, the Ljung-Box test is used, which tests the following hypothesis statement:

3.11

$$\begin{aligned} H_0 &: \rho_1 = \rho_2 = \dots = \rho_k = 0 \\ H_a &: \rho_i \neq 0 \text{ for some } i = 1, 2, \dots, k \end{aligned}$$

where k is the number of lags to test, with statistic

$$LB = n(n+2) \sum_{i=1}^k \left(\frac{\hat{\rho}_i^2}{n-i} \right) \sim \chi^2(k),$$

and

$$\hat{\rho}_j = \frac{\sum_{t=j+1}^n (R_t - \hat{\mu})(R_{t-j} - \hat{\mu})}{\sum_{t=1}^n (R_t - \hat{\mu})^2}.$$

If $\{R_t\}_{t=1}^n$ has significant autocorrelations, then $\hat{\mu}$ is not considered a constant. Therefore, the mean of $\{R_t\}_{t=1}^n$ is fitted by an ARMA model such that

$$\hat{\mu}_t = E[R_t | R_{t-1}, \dots, R_1].$$

In addition, if there are significant autocorrelations in the series $(R_t - \hat{\mu})^2$, that is, $\hat{\sigma}_t^2 = \text{Var}[R_t | R_{t-1}, \dots, R_1] = E[(R_t - \mu_t)^2 | R_{t-1}, \dots, R_1]$, then $\{R_t\}_{t=1}^n$ is fitted by an GARCH family model, so $\hat{\sigma}$ is not considered a constant.

However, if $\{R_t\}_{t=1}^n$ follows a stationary process and has no significant lags, then $\hat{\mu}$ can be estimated by equation (3.2). In addition, if there is not enough evidence to reject the hypothesis for the series of returns, which contrasts

3.12

$$\begin{aligned} H_0 &: \mu = 0 \\ H_a &: \mu \neq 0, \end{aligned}$$

then we could consider $\mu=0$.

Similarly, if the series $\{R_t^2\}_{t=1}^n$ has no significant lags by means of the Ljung-Box test (3.11), then $\hat{\sigma}$ is considered as constant. Therefore, we must choose the best estimator of σ .

4. CONSTANT VOLATILITY ESTIMATION

We will show two techniques for estimating the constant volatility of an asset from the series of returns explained in Section 3.

4.1. Classical approach

To perform the classical estimation procedure, it is necessary to define the probability distribution associated with the behavior of the returns R_t (3.1). If $R_t \sim N(0, \phi)$, with $\phi = \sigma^2$, then the density function is given by

$$f(R_i|\phi) = \frac{1}{\sqrt{2\pi\phi}} \exp\left(-\frac{R_i^2}{2\phi}\right) \quad \phi > 0$$

To obtain an estimate of ϕ , the maximum likelihood method and the bootstrap technique are used as tools. The first method is to find $\hat{\phi}_{MV}$ such that it maximises the likelihood function

4.1

$$\begin{aligned} L(R_1, \dots, R_n|\phi) &= \prod_{i=1}^n f(R_i|\phi) \\ &= (2\pi\phi)^{-\frac{n}{2}} \exp\left(-\frac{1}{2\phi} \sum_{i=1}^n R_i^2\right). \end{aligned}$$

Indeed, since the equation (4.1) is equivalent to,

$$\ln L(R_1, \dots, R_n|\phi) = -\frac{n}{2} \ln(2\pi\phi) - \frac{1}{2\phi} \sum_{i=1}^n R_i^2,$$

then

$$\hat{\phi}_{MV} = \frac{1}{n} \sum_{i=1}^n R_i^2,$$

with confidence interval at $100(1-\alpha)\%$ given by:

$$\phi_{MV} \in \left(\frac{(n-1)\hat{\phi}_{MV}}{\chi_{1-\frac{\alpha}{2},(n-1)}^2}, \frac{(n-1)\hat{\phi}_{MV}}{\chi_{\frac{\alpha}{2},(n-1)}^2} \right).$$

On the other hand, if we consider bootstrap samples $R^b = (R_1^b, R_2^b, \dots, R_n^b)$, $b = 1, \dots, B$, as random re-sampling of $(R_1, R_2, \dots, R_n)$, then by using an estimator $\hat{\theta}$, for example $\hat{\theta} = \phi_{MV}$ to each bootstrap sample R_b , that is,

$$\hat{\theta}^*(b) = \hat{\theta}(R^b), \quad b = 1, 2, \dots, B$$

an estimator is proposed for ϕ given by

$$\hat{\phi}_B = \frac{1}{B} \sum_{b=1}^B \hat{\theta}^*(b),$$

with quasi-variance

$$\text{Var}(\hat{\phi}_B) = \frac{\sum_{b=1}^B (\hat{\theta}^*(b) - \hat{\theta}^*(\cdot))^2}{B-1},$$

and confidence interval at a $100(1-\alpha)\%$ for ϕ_B given by

$$\left(\hat{\phi}_B - z_{\alpha/2} \sqrt{\text{Var}(\hat{\phi}_B)}, \hat{\phi}_B + z_{\alpha/2} \sqrt{\text{Var}(\hat{\phi}_B)} \right).$$

In particular, the above method is known as the bootstrap method (Efron, 1992).

4.2. Bayesian approach

From the Bayesian approach (Bernardo, & Smith, 2009; Box, & Tiao, 2011), the parameter $\phi = \sigma^2$ is considered as a random variable modelled by a continuous prior probability distribution $h(\phi)$, whose information can be updated by observations from a sample $(R_1, \dots, R_n)$. Consequently, we obtain a posterior distribution $h(\phi | R_1, \dots, R_n)$ from which we have a complete description of the knowledge about ϕ obtained from the a priori distribution $h(\phi)$ and the sampling information $(R_1, \dots, R_n)$.

From Bayes' theorem we have that,

$$\text{Prob}(\phi | R_1, \dots, R_n) = \frac{\text{Prob}(R_1, \dots, R_n | \phi) \text{Prob}(\phi)}{\text{Prob}(R_1, \dots, R_n)},$$

with

$$\text{Prob}(R_1, \dots, R_n) = \int_0^{\infty} \text{Prob}(R_1, \dots, R_n | \phi) \text{Prob}(\phi) d\phi,$$

and

$$\text{Prob}(\phi) = h(\phi).$$

From the likelihood function $L(R_1, \dots, R_n | \phi)$ (4.1), the population means of returns $\mu=0$ and a prior distribution $h(\phi)$, the posterior distribution of ϕ can be found as follows:

4.2

$$h(\phi | R_1, \dots, R_n) = \frac{L(R_1, \dots, R_n | \phi) h(\phi)}{\int_0^{\infty} L(R_1, \dots, R_n | \phi) h(\phi) d\phi}$$

The following candidate prior distributions for the volatility of an asset are proposed by the authors below:

- Since $\phi > 0$, the a priori inverse Gamma distribution is a candidate for ϕ since it allows for skewness within its structure. The density function of the inverse Gamma distribution, for the variable ϕ is

4.3

$$h_1(\phi) = \frac{\beta^\alpha}{\Gamma(\alpha)} \phi^{-(\alpha+1)} \exp\left(-\frac{\beta}{\phi}\right) \quad \alpha, \beta > 0$$

From (4.1), the posterior distribution is

$$h_1(\phi|R_1, \dots, R_n) = \frac{\beta_1^{\alpha_1}}{\Gamma(\alpha_1)} \phi^{-(\alpha_1+1)} \exp\left(-\frac{\beta_1}{\phi}\right),$$

with $\alpha_1 = \alpha + \frac{n}{2}$ and $\beta_1 = \beta + \sum_{i=1}^n \frac{R_i^2}{2}$. This distribution coincides with the Inverse Gamma distribution (4.3), with parameters (α_1, β_1) .

- One of the main strengths of Jeffreys' method is its invariance under injective transformations, which is based on Fisher's information measure on $\phi > 0$, that is,

4.4

$$I(\phi) = E \left[\left(\frac{\partial \ln L(R_1, \dots, R_n | \phi)}{\partial \phi} \right)^2 | \phi \right] = -E \left[\frac{\partial^2 \ln L(R_1, \dots, R_n | \phi)}{\partial \phi^2} | \phi \right]$$

Thus, the prior distribution proposed by Jeffreys for the uniparametric case,

4.5

$$h_2(\phi) \propto \sqrt{I(\phi)}.$$

From (4.1), (4.4) and (4.5) we have that the a priori distribution is defined as

4.6

$$h_2(\phi) \propto \frac{\sqrt{n}}{\phi},$$

and from (4.2) we have

4.7

$$h_2(\phi|R_1, \dots, R_n) = \frac{\beta_2^{\alpha_2}}{\Gamma(\alpha_2)} \phi^{-(\alpha_2+1)} \exp\left(-\frac{\beta_2}{\phi}\right),$$

with $\alpha_2 = \frac{n}{2}$ and $\beta_2 = \sum_{i=1}^n \frac{R_i^2}{2}$.

It is observed that the posterior distribution (4.7) is equivalent to the Inverse Gamma distribution (4.3) with parameters (α_2, β_2) .

- Given that financial series can exhibit high volatility conditions, the a priori standard Levy distribution $\text{Levy}(0, \tau)$ is considered as a candidate because it exhibits heavy tails. The density function of the Levy distribution is defined as follows

4.8

$$h_3(\phi) = \left(\frac{\tau}{2\pi}\right)^{\frac{1}{2}} \phi^{-\frac{3}{2}} \exp\left(-\frac{\tau}{2\phi}\right) \quad \tau > 0$$

with posterior distribution

$$h_3(\phi|R_1, \dots, R_n) = \frac{\beta_3^{\alpha_3}}{\Gamma(\alpha_3)} \phi^{-(\alpha_3+1)} \exp\left(-\frac{\beta_3}{\phi}\right),$$

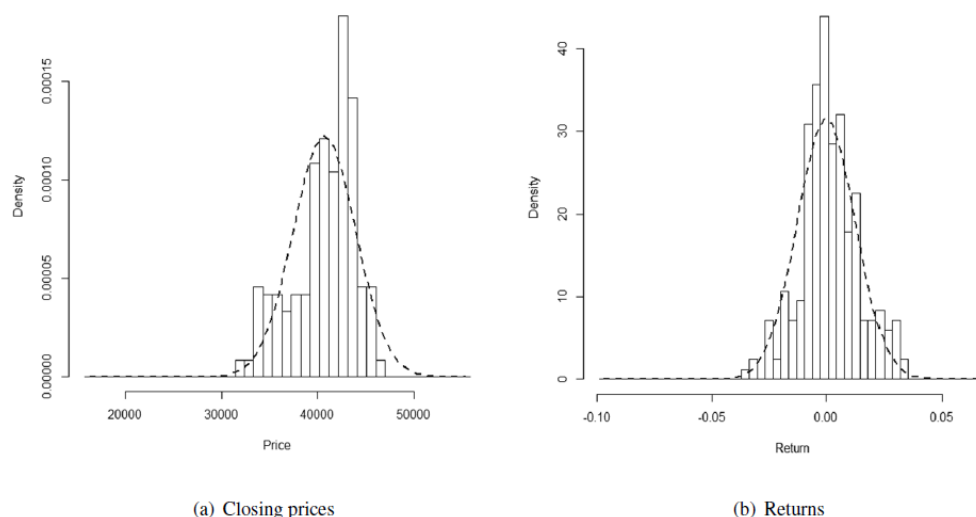
which is an Inverse Gamma probability distribution (4.3), with $\alpha_3 = \frac{n+3}{2}$ and $\beta_3 = \frac{1}{2}(\tau + \sum_{i=1}^n R_i^2)$.

5. PRESENTATION OF THE INFORMATION

The temporal series to be used are the daily returns of the preferred shares of the banking entity Bancolombia (G. Bancolombia, 1875). For the calculation of this return's series (3.1), the daily closing prices of shares in which there is market in the period between 02/Jan/2019 and 30/December/2019 have been used, with a total of 245 observations (Bolsa de valores de Colombia, 2001).

The histogram in Figure 1 shows that the price series data does not follow a normal distribution, but is slightly skewed to the left and leptokurtic, with a skewness coefficient of -0.6363245 and a kurtosis of 2.26349.

Figure 1. Price and return series PFBCOLOM



When modelling time series, it is important to determine whether the phenomenon under analysis follows a stationary process, that is, with unchanging meaning and variance in the period under study $0 \leq t \leq 244$. To this end, using the Dickey-Fuller test (3.9), it is concluded that, at a significant level of 5%, the price series does not statistically behave with a time-varying mean and variance. In this case, P_t does not behave statistically as a stationary process with a probability of 25.63%. It is therefore of great interest to analyse the returns of the series using the returns of this stock.

Figure 2 shows the differences between the closing price series $\{P_t\}_{t=0}^{244}$ and the return series $\{R_t\}_{t=1}^{244}$, which statistically behaves like a stationary process with a probability <1% given by the Dickey-Fuller test (3.9) at a significance level of 5%.

Figure 2. Price and return series PFBCOLOM

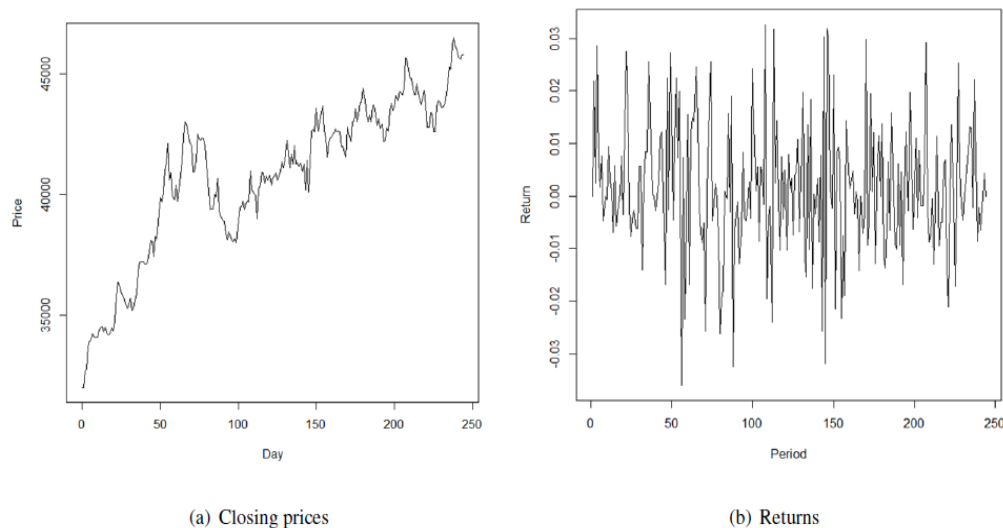


Table 1 shows that the series R_t behaves like a normal distribution, as it is slightly skewed to the left and platykurtic, with a skewness coefficient of 0.076 and a kurtosis of 0.347. However, the Shapiro-Wilk and Jarque-Bera test is used to determine whether the series R_t behaves like a normal distribution.

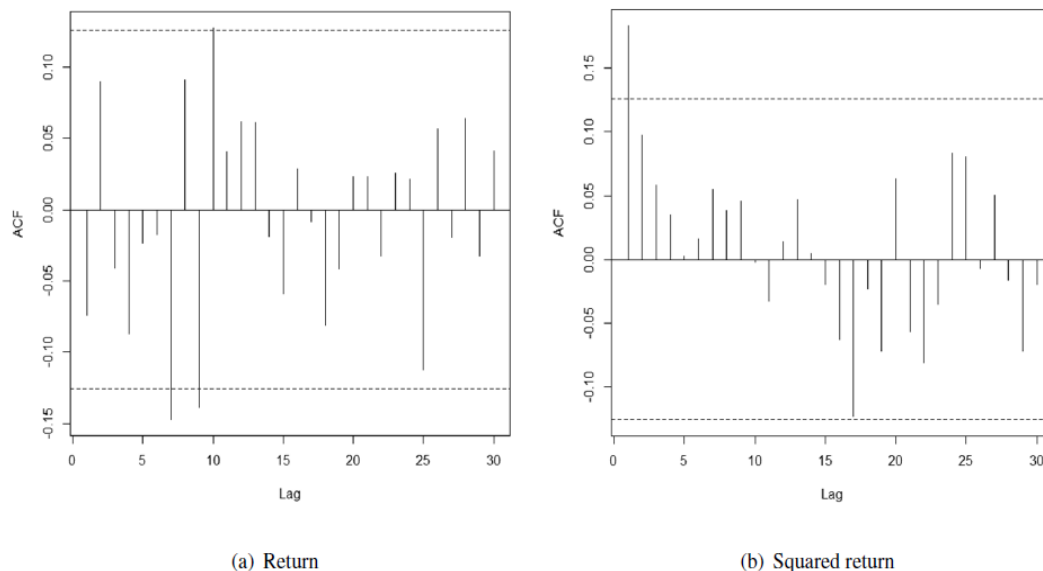
Table 1. Description of PFBCOLOM shares

Measure	Statistician	
	Closing Price	Returns
Media	38767.78	0.001678714
Median	39700	0.0005376344
Variance	7774182	0.0001655227
Standard deviation	2788.222	0.01286556
IQR	4215	0.01316559
Skewness	-0.6363245	-0.07809873
Kurtosis	2.26349	3.361937

The probabilities associated with the Shapiro-Wilk and Jarque-Bera tests are 1.4% and 4.43%, respectively, which is insufficient evidence to determine that the series R_t does not follow a normal distribution, with a significance level of 5%.

Figure 3, the simple autocorrelation function (3.10) of the series R_t and R_t^2 have few significant lags that are outside the 95% confidence intervals, however, the Ljung-Box test (3.11) is applied to check the significance of the autocorrelations up to lag 30.

Figure 3. Correlation of return and squared return series



The probabilities associated with the LB test statistic for the series R_t and R_t^2 are 65.41% and 40.89% respectively, which is insufficient evidence to determine that the series R_t and R_t^2 have significant lags at the 5% significance level, and therefore there is no presence of serial autocorrelation in either series. This suggests that an ARMA model for the mean and GARCH family models for the volatility of the returns should not be fitted. Therefore, μ and σ of the series R_t are considered as constants.

Applying a test t_{n-1} on the return series (3.12), it appears that, at a significant level of 5%, the population mean of R_t can be considered statistically zero with a probability of 5.6%. Next, we proceed to estimate the hyperparameters for each prior distribution to be considered to calculate the parameter σ .

6. HYPERPARAMETER ESTIMATION

Since the hyperparameters α and β of the inverse gamma prior (4.3) and τ of the Levy distribution (4.8) are unknown and no expert judgement is available to determine their values, the following methods are proposed to compute them (Tovar Cuevas, 2015).

We take the daily prices of Bancolombia's preferred shares, obtained from the Bolsa de valores de Colombia (2001), during the period from 22/February/2010 to 28/December/2018, with a total of 2161 observations. From the data corresponding to the returns $\{R_t\}_{t=1}^{2160}$, 72 excluded subsets of 30 elements each are formed, and, for each subset, the sample variance is calculated to obtain a vector of estimates $\hat{\phi} = \{\hat{\phi}_m\}_{m=1}^{72}$.

Based on the above, the first and second moments of the inverse gamma distribution (4.3) are calculated, that is,

6.1

$$E[\hat{\phi}] = \frac{\beta}{\alpha-1} \quad \alpha > 1$$

$$E[\hat{\phi}^2] = \frac{\beta^2}{(\alpha-1)(\alpha-2)} \quad \alpha > 2$$

and determined the sample variance of the vector of estimates $\hat{\phi}$ as a function of moments, to form a system of linear equations with two unknowns given by:

6.2

$$E[\hat{\phi}] = \frac{\beta}{\alpha-1}, \quad \alpha > 1$$

$$Var[\hat{\phi}] = E[\hat{\phi}^2] - E[\hat{\phi}]^2 = \frac{\beta^2}{(\alpha-1)(\alpha-2)}. \quad \alpha > 2$$

The solutions of the system (6.2) are given by

$$\alpha = \frac{E[\hat{\phi}]^2 + 2Var[\hat{\phi}]}{Var[\hat{\phi}]},$$

$$\beta = E[\hat{\phi}] \left(\frac{E[\hat{\phi}]^2 + 2Var[\hat{\phi}]}{Var[\hat{\phi}]} - 1 \right),$$

which defines a first case for the hyperparameters of the Inverse Gamma prior distribution (4.3).

A second case consists in calculating the historical volatility for each 20 days of the above series of returns, that is,

$$\hat{\phi}_m = \frac{1}{20} \sum_{i=m-19}^m |R_i - N_m|^2, \text{ with } N_m = \frac{1}{20} \sum_{i=m-19}^m R_i^2$$

to obtain a vector of estimates $\{\hat{\phi}_m\}_{m=20}^{2160}$ and estimate α and β as posed in system (6.2).

A third case consists in taking a sample of the returns corresponding to the period 02/January/2018 and 28/December/2018, that is, $\{R_t\}_{t=1}^{242}$ and the mean and variance are calculated to replace them in the system (6.2) to obtain a third Inverse Gamma prior distribution (4.3).

To obtain prior information about the parameter τ of the Standard Levy distribution (4.8), we divide the set $\hat{\phi}$, generated in the first case, into 8 exhaustive excluding subsets with 9 elements each, that is, $\{\phi_{i_m}\}_{m=1}^9$, $i, i=1, \dots, 8$ and, for each of the sets, we estimate $\hat{\tau}$ by the maximum likelihood method, that is,

$$\hat{\tau}_i = \frac{10}{\sum_{m=1}^9 \phi_{i_m}^{-1}}, \quad i = 1, \dots, 8,$$

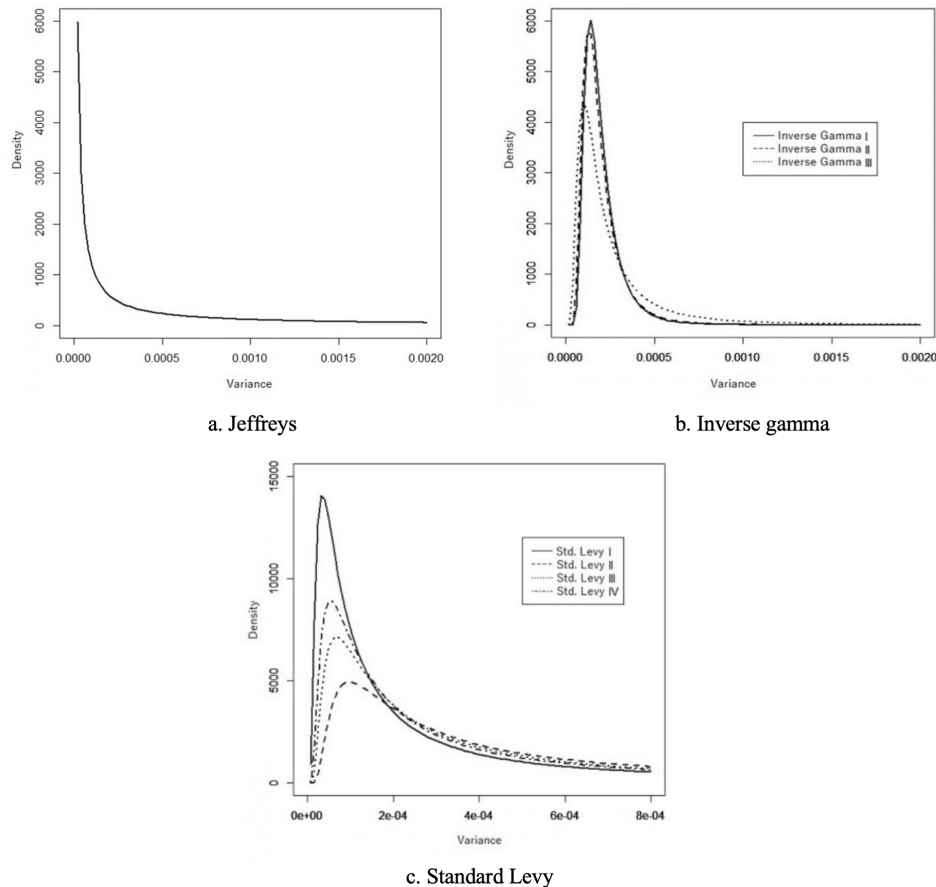
and the minimum, maximum, mean, and median values of these estimates are taken to establish values of the hyperparameters of the Standard Levy prior distribution.

Table 2 shows the estimates for the hyperparameters of the Inverse Gamma (4.3) and standard Levy prior distributions (4.8). In the first case of the Inverse Gamma distribution, the parameter α is the largest compared to the other Inverse Gamma prior distributions, which causes the distribution to be more leptokurtic, that is, it has a smaller variance in its structure compared to the others as observed in Figure 4(b). In the Standard Levy distribution, as the hyperparameter becomes smaller, the distribution tends to become leptokurtic, as observed in Figure 4(c).

Table 2 Estimates of hyperparameters for each prior distribution

Distribution $h(\phi)$	Hyperparameter
Inverse Gamma I	$(\alpha, \beta) = (5.255163, 0.000859)$
Inverse Gamma II	$(\alpha, \beta) = (4.481333, 0.000704)$
Inverse Gamma III	$(\alpha, \beta) = (2.000404, 0.000306)$
Standard Levy I	$\tau = 0.000103$
Standard Levy II	$\tau = 0.000295$
Standard Levy III	$\tau = 0.000203$
Standard Levy VI	$\tau = 0.000163$

Figure 4. Prior Distributions



7. RESULTS

Using the mean as a measure that summarizes the σ^2 information for each of the posterior distributions, Table 3 shows the estimates of the volatilities by the proposed methods, where the estimation was done by the bootstrap method with $B=200$ and the initial estimator generated by the maximum likelihood method $\hat{\phi}_{MV}$.

Table 3 shows that the $\hat{\sigma}$ are quite similar for the different methods. However, the 95% confidence regions for the Bayesian models tend to have less variability than those observed by the 95% confidence intervals given by the classical methods, which allows a better precision in the volatility estimation.

Table 3. Estimation of σ^2 from the different methods proposed

Method	Type	σ^2	Interval estimator
Classic	Maximum Likelihood	0.000158796	(0.0001339515,0.0001912957)
	Bootstrap	0001586208	(0.0001260133,0.0001912284)
	Jeffreys	0.0001618451	(0.0001515057,0.0001934298)
Bayesian	Inverse Gamma I	0.000161998	(0.0001518425,0.0001929266)
	Inverse Gamma II	0.0001616447	(0.0001515535,0.0001924145)
	Inverse Gamma III	0.0001617566	(0.0001515217,0.0001932139)
	Standard Levy I	0.0001602782	(0.0001501442,0.0001913579)
	Standard Levy II	0.0001610221	(0.0001507602,0.0001920557)
	Standard Levy III	0.0001607126	(0.0001505169,0.0001918419)
	Standard Levy VI	0.0001605483	(0.0001503757,0.0001914762)

Since each of the priori distributions has the same posteriori distribution, with different hyperparameters, it is necessary to determine which a posteriori distribution is the most appropriate for the estimation of the volatility parameter. From a Bayesian point of view, the Deviance information criterion (DIC) is used, which is based on the posterior distribution, since the log-likelihood of the model is a measure of model fit to the data, and the deviance is a measure of discrepancy between the model and the data.

The DIC criterion is calculated using the expression

$$DIC = P_d + P_h,$$

Where

$$\begin{aligned} P_d &= P_h - E[-2\ln L(R_1, \dots, R_n | \text{Var}[h(\phi | R_1, \dots, R_n)])] \\ P_h &= E[h(\phi | R_1, \dots, R_n)]. \end{aligned}$$

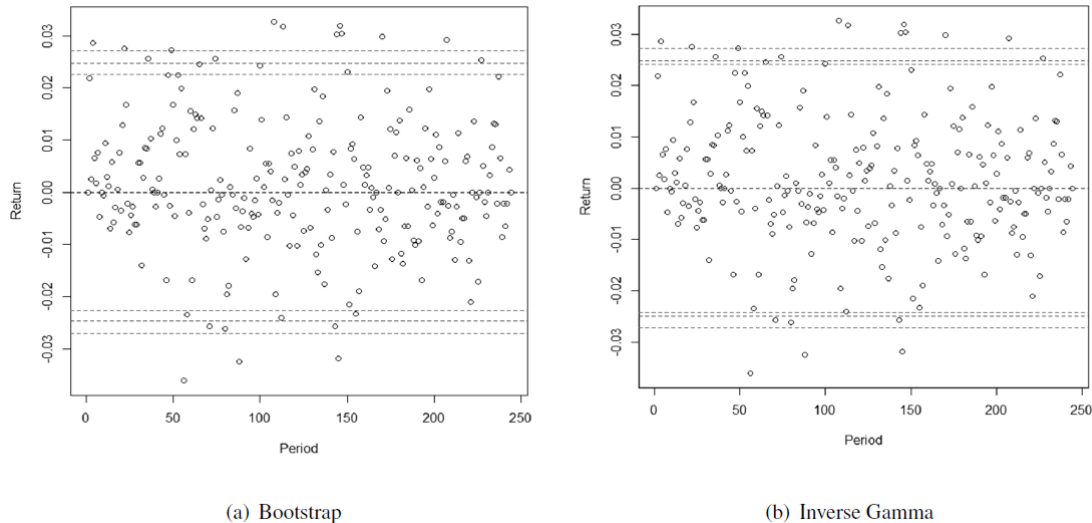
Since that the bootstrap method has lower variability in the confidence interval than the maximum likelihood method, and the Inverse Gamma III prior distribution has a lower DIC, as observed in Table 4, Figure 5 shows the confidence bands for the volatility of the returns, at a 95% confidence level of both the point estimates and the extremes of the confidence bands for both methods. The two selected estimation methods do not capture the totality of returns. However,

when analyzing the extremes of the intervals of the σ estimates, the Bayesian method can capture more information than the classical method. Therefore, the Bayesian method, in addition to showing less variability in its 95% confidence intervals, manages to capture more information about returns than the classical method.

Table 4. DIC criteria

Previa	Posterior	DIC
Jeffreys	Inverse Gamma	12532.52
Inverse Gamma I		12519.46
Inverse Gamma II	Inverse Gamma	12564.64
Inverse Gamma III		12517.85
Standard Levy I		12765.72
Standard Levy I	Inverse Gamma	12651.82
Standard Levy II		12723.19
Standard Levy III		12724.53

Figure 5. 95% confidence bands for return volatility



On the other hand, if the initial asset price is $\tilde{P}_0 = 31400$, corresponding to 28/December/2018, the model (3.7) with $\hat{\mu} = 0.001549458$ is used to estimate the closing prices and compare its results with the real prices within the sample, that is, from 02/January/2019 to 30/December/2019, where

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Christian Cortés-García; Alvaro Cangrejo-Esquivel

$$\begin{aligned} \text{MAE} &= \frac{1}{n+1} \sum_{t=0}^n |P_t - \hat{P}_t|, \\ \text{MAPE} &= \frac{1}{n+1} \sum_{t=0}^n \left| \frac{P_t - \hat{P}_t}{P_t} \right|, \\ \text{AMAPE} &= \frac{1}{n+1} \sum_{t=0}^n \left| \frac{P_t - \hat{P}_t}{P_t + \hat{P}_t} \right|, \end{aligned}$$

are the errors of these forecasts, with actual closing prices P_t and forecast closing prices $\hat{P}_t$.

Considering 1000 numerical solutions, with fixed random values $\epsilon_t \sim N(0,1)$ for each solution, we can be seen from Table 5 that the σ estimator using the bootstrap technique has the lowest means of MAE, MAPE and AMAPE, with respect to the Maximum Likelihood method and the Bayesian methods, which have lower DIC in each of their prior distributions, confirming that this method predicts Bancolombia's stock prices with minimum error.

Table 5. In-sample forecast errors by classical and Bayesian methods

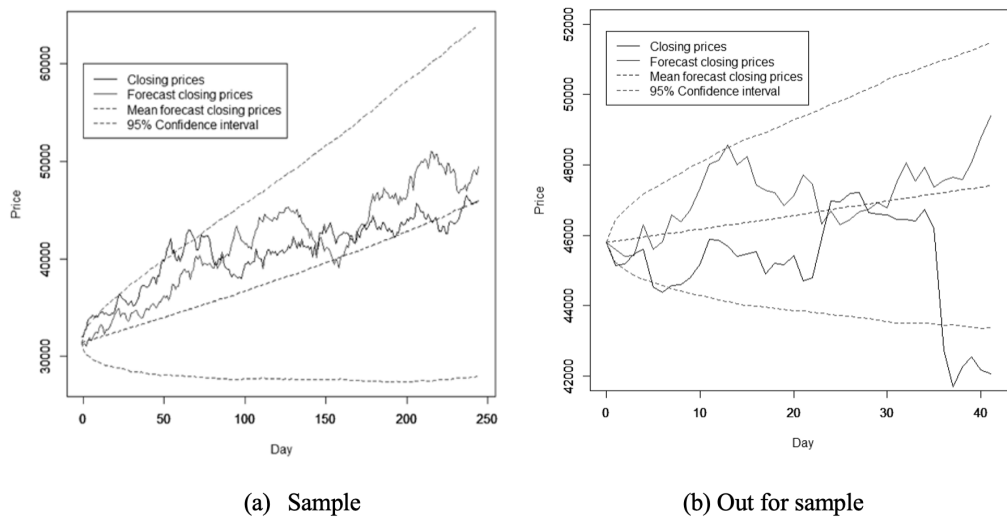
Method	Type	MAE	MAPE	AMAPE
Classic	Maximum Likelihood	5159.698	0.1247814	0.06561538
	Bootstrap	5157.789	0.1247363	0.06559164
Bayesian	Jeffreys	5192.784	0.1255642	0.06602711
	Inverse Gamma III	5191.827	0.1255416	0.0660152
	Standard Levy II	5183.877	0.1253535	0.06591627

However, if we compare the residuals $e_t = P_t - \hat{P}_t$ for each of the 1000 asset price predictions made by each Bayesian volatility estimation method with lower DIC and the bootstrap method relative to the actual prices between the period 02/January/2019 to 30/December/2019, then from the mean of the 1000 Diebold-Mariano (DM) tests (Diebold, & Mariano, 1995; Harvey, Leybourne, & Newbold, 1997), we conclude that the two forecasts have the same accuracy as observed in Table 6.

Table 6. Comparison between the accuracy of in-sample asset price forecasting using the bootstrap method and Bayesian estimation

Distribution $h(\phi)$	Mean p-valor Diebold-Mariano test
Jeffreys	0.3943
Inverse Gamma III	0.3875
Standard Levy II	0.3842

By performing 5000 numerical solutions of the model (3.7), for $\hat{\sigma}^2$ as estimated by the bootstrap method, and by computing the computational mean and variance for each partition $t \in [0,243]$, Figure 6(a) shows the relationship between Bancolombia's actual closing prices, during the period 02/January/2019 to 30/December/2019, and one of the predictions along with the 95% confidence interval. Although an adequate fit was not achieved, due to noise generated by model (3.7), such estimates can be considered by investors for decision making.

Figure 6. Forecasts closing price by Bootstrap method

Finally, considering as initial asset price $P_0=45800$, corresponding to 30/December/2019, we contrast the classical and Bayesian volatility estimation methods to estimate the closing prices by model (3.7) and compare their results with prices between 02/January/2020 to 28/February/2020, for a total of 41 data.

When considering 1000 numerical solutions, with fixed random values $\epsilon_t \sim N(0,1)$ for each solution, and as shown in Table 7, it is evident that the bootstrap method, besides forecasting with lowest means error the in-sample asset prices, also forecasts the out-of-sample asset prices with lowest means of MAE, MAPE and AMAPE errors. However, as shown in Table 8, the mean Diebold-Mariano (DM) test, which compares the 1000 forecasts of each Bayesian method with respect to the bootstrap method, confirms that both classical and Bayesian estimation methods predict out-of-sample asset prices similarly. Figure 6(b) compares the actual asset prices to those predicted by the out-of-sample bootstrap method, which 95% confidence interval captures much of the information from the actual prices.

Table 7. Out-of-sample forecast errors by classical and Bayesian methods

Method	Type	MAE	MAPE	AMAPE
Classic	Maximum Likelihood	2811.727	0.05746561	0.03001113
	Bootstrap	2810.969	0.05745062	0.03000334
Bayesian	Jeffreys	2824.885	0.05772597	0.03014635
	Inverse Gamma I	2825.543	0.057739	0.03015312
	Inverse Gamma II	2824.022	0.0577089	0.03013749
	Inverse Gamma III	2824.504	0.05771843	0.03014244
	Standard Levy I	2818.133	0.05759236	0.03007696
	Standard Levy II	2821.341	0.05765584	0.03010993
	Standard Levy III	2820.007	0.05762944	0.03009622
	Standard Levy IV	2819.298	0.05761542	0.03008894

Table 8. Comparison between the accuracy of out-of-sample asset price forecasting using the bootstrap method and Bayesian estimation

Distribution $h(\phi)$	Mean p-valor Diebold-Mariano test
Jeffreys	0.2972
Inverse Gamma I	0.2822
Inverse Gamma II	0.2964
Inverse Gamma III	0.2737
Standard Levy I	0.2844
Standard Levy II	0.2785
Standard Levy III	0.2972
Standard Levy IV	0.2923

8. CONCLUSIONS

Since there is not much research on the estimation of constant volatility, apart from the classical method and through resampling methods, it is interesting to propose alternatives to be contrasted with the classical estimates. One of these alternatives is to propose estimates using Bayesian methods, since from a priori distribution of the possible behavior of the volatility, a posteriori distribution is determined, which is generated by the information of the sample returns, and which can be used to estimate the volatility.

For each posteriori distribution, the Bayes estimator was evaluated with respect to the quadratic loss function, that is, the expected value of the distribution is taken as a measure that summarizes the volatility information (Tovar, 2015), although other possible statistics to estimate the volatility could be accepted. At the sensitivity level, such methods, both classical and Bayesian, could be contrasted by the smaller range of their confidence intervals or regions, as there would be less variability in the sample estimate of volatility.

Confidence intervals for the classical approach (CI) and credible regions by estimating the IQR of the posterior distribution are used in the Bayesian approach (CR) to assess the quality of volatility estimates, although CI and CR are concepts with completely different interpretations. The CI sets two values for the estimator within which it is expected to find the random quantity of interest a certain percentage of the time, whereas the CR should generate a proportion of values of the unknown quantity equal to a specified probability. Although conceptual confidence intervals and credible regions are different, for practical reasons their lengths and the average coverage obtained after many iterations of the estimation procedure can be used as indicators of the performance of the estimates obtained.

On the other hand, the choice of the priori distribution for volatility is made based on experience or on the suggestion of the researcher or an expert. In this case, given that the Jeffrey, Gamma – Inverse and Standard Levy distributions have been chosen as prior distributions to describe volatility, other possible distributions similar in nature to those chosen in this work could be proposed as future work, that is, they should not contain negative values in their structure. In addition, due to the lack of information from an expert, an empirical Bayesian method (Tovar-Cuevas, 2015) was proposed for the selection of the hyperparameters of the priori distributions.

In the case of constant volatility to describe the risk of an asset, the normal distribution has been considered as a candidate to describe the returns and thus construct the likelihood function to

calculate the a posteriori distribution. However, other possible distributions could be considered as the series generally behave as leptokurtic or with heavy tails. As possible future work, the Laplace or GED generalized distribution could be considered to describe the return series.

Therefore, since the Colombian Stock Exchange only presents price records on business days, equation (3.1) is used to estimate simple returns. On the other hand, if asset prices are recorded daily, the logarithm of simple returns could be considered as an estimator of returns, since the origin of this formula is related to the continuous interest rate, which is used in many financial fields as the interest rate that operates different instruments, in this case stock returns, although from a mathematical point of view both estimates are equivalent.

From an economic point of view, the difference between constant volatility and time volatility applied to the return series is that the series does not show any significant variability, i.e. the increase in the price of the asset does not show any drastic fluctuations because external economic factors have not caused a significant increase or decrease in the price of the asset, as in the case of the price series of Bancolombia during the period implemented, so that there are no significant autocorrelations in the series of returns squared of the preference shares of Bancolombia, it does not make sense to fit a volatility model, so it should be considered as constant and either classical or Bayesian estimation techniques should be applied.

In the comparison of Bayesian methods for estimating the volatility of the series of returns on Bancolombia's preferred shares, when the inverse gamma prior distribution is taken, it presents a lower value in the DIC, indicating that the distribution has a better fit than the others. However, the classical method was able to make predictions with minimal error compared to the Bayesian methods. On the other hand, regardless of the hyperparameters in the different procedures, it has been demonstrated that the results do not present significant differences and that the proposed methodology can be applied in different fields of study where the variable of interest presents positive asymmetric behavior.

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