**De Computis**

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## ARTÍCULOS DOCTRINALES / *ARTICLES*

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# LUCA PACIOLI, ALGEBRA, AND DOUBLE ENTRY

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**Abstract:** This paper proposes that Pacioli, an accomplished mathematician, recognised that the Venetian method of bookkeeping had an algebraic underpinning, although, when writing his treatise, he converted this algebra into rules. The algebra that he may have used is deduced from a combination of his own words - derived from the translation by Geijsbeek, checked against that of Von Gebsattel and other translations - with quotations from the literature. Perhaps this is the 'buried treasure waiting to be rediscovered' referred to by Macve in 'Pacioli's Legacy'(Macve, 1996, 22). Chapter 23 in the treatise is central to this paper. Here Pacioli advises a merchant on how to run a store, using an approach that is now known as 'the entity theory'. Pacioli also advised the merchants to exclude information about their private possessions from the public domain and this paper describes how this could be achieved. The paper ends with a presentation in the Appendix of the evolution of double entry expressed algebraically in tabular form, along with an illustration of a matrix method approach to recording transactions.

**Keywords:** *Double-Entry, Pacioli, Algebra. Assets, Expenses, Profit, Summa, Treatise.*

## LUCA PACIOLI, ÁLGEBRA Y DOBLE ENTRADA

**Resumen:** Este artículo propone que Pacioli, un matemático consumado, reconoció que el método veneciano de contabilidad tenía un fundamento algebraico, aunque, al escribir su tratado, convirtió este álgebra en reglas. El álgebra que pudo haber usado se deduce de una combinación de sus propias palabras, derivadas de la traducción de Geijsbeek, cotejada con la de Von Gebsattel y otras traducciones, con citas de la literatura. Quizás este sea el "tesoro enterrado esperando ser redescubierto" al que se refiere Macve en "El legado de Pacioli" (Macve, 1996, 22). El capítulo 23 del tratado es fundamental para este artículo. Aquí Pacioli asesora a un comerciante sobre cómo administrar una tienda, utilizando un enfoque que ahora se conoce como "la teoría de la entidad". Pacioli también recomendó a los comerciantes que excluyeran la información sobre sus posesiones privadas del dominio público y este documento describe cómo se podría lograr. El artículo finaliza con una presentación en el Apéndice de la evolución de la doble entrada expresada algebraicamente en forma tabular, junto con una ilustración de un enfoque de método matricial para registrar transacciones.

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## 1. Introduction

Pacioli was only a youth when he went to Venice to further his education. In order to finance himself he became an assistant/apprentice to a leather merchant, Antonio Rompiasi. He worked for Rompaisi for six years, during which time he was provided with many opportunities; he learned bookkeeping, and he represented the merchant on many journeys and sea voyages around the Mediterranean. As well as having to keep accounting records of his activities during these trips, he would have become familiar with the varying customs, weights and measures, currencies, taxes, and fees for various services that any merchant needed to know; he would have become as efficient as any merchant, with his interests being in all things commercial, well before he became a Friar, and later a Professor of Mathematics (Taylor, 1942: 55; and Lane, 1977: 183; as quoted by Sangster, 2021: 137-138). It is probable that he would have seen merchants' businesses fail as well as succeed, so it is not surprising that he began his treatise with these two assertions: *"There are three things needed by anyone who wishes to carry on business …the most important of these is 'cash'. The second is to be a good bookkeeper and a ready mathematician. The third is to arrange all transactions in a systematic way".* and *"I wish to give them* (the merchants) *enough rules to enable them to keep all their books and accounts in an orderly way ……without making the entries systematically it would be impossible for them to conduct their business, for they would have no rest and their minds would be troubled"* (Geijsbeek, 1914: 33).

Von Gebsattel's translation of this paragraph is even more to the point:

*"However, as is well known, there are principally three things necessary for those who wish to trade with due diligence, of which the most important is money and every other asset …. the second thing which one looks for in a merchant is that he be a good reckoner and a quick calculator …. The third and final necessary thing is that all transactions are recorded in good order"* and *"this is most useful to them who would otherwise find it impossible to run their businesses because without due order in the keeping of accounts…their minds would always be greatly troubled"* (Von Gebsattel 1994: 41,42).

## 2. Rules

Rules depend on a logical system, and Pacioli was obviously satisfied that this was what he was dealing with when he wrote his treatise. Lee, (1975) said that:

*"to Pacioli accounting was a set of arithmetical techniques to assist the businessman to conduct his affairs in an orderly, purposeful, and well-informed manner… there was no theory … substituting knowledge of principles for the hard grind of learning detailed rules by rote… was a thing of the future"* (Lee, 1975: 11).

So, given his practical approach, it is not surprising that Pacioli did not even attempt to explain the principles underlying these rules. Even today most sources for teaching double entry still require students to learn rules, with no explanation as to why they work. For example:

*The debit side of an account … receives or gains. The credit side…gives or loses. If this seems a little strange … it does require practice."* (Harrison, 2017: 2)

This explanation has changed little since it was used by Stevin in 1604, when he was explaining double entry to his Prince. Stevin told the Prince that debits were to his advantage, and credits to his disadvantage.

The Prince then asked him: *"Then why is my capital account in credit, surely this account should be to my advantage?"*

Stevin replied that:

*"the capital account had to be the reverse of all other accounts so that debits and credits were not really an exception to the rules that I told to you earlier".* (Stevin, 1604: Forgotten Books 2012:15-16).

One can imagine the confusion in the Prince's mind at this explanation.

Kirkegaard referred to:

*"rules that have to be learned, and conventions that have to be followed"* (Kirkegaard, 1997: 56-57).

Lerner & Gokarn stated that:

*"the way debits and credits affect the categories of assets, expenses, revenues, liabilities, and capital in different ways, is a 'convention'"* (Lerner & Gokarn, 2004: 7).

## 3. The Mathematical Aspect of Double Entry

In his introduction to Geijsbeek's translation of Pacioli's Treatise, Lawrence commented:

*It is a significant fact that the rules and principles elucidated by Pacioli are contained in a book given over to mathematics. One cannot help but believe that the derivation of double- entry bookkeeping is an explanation of the algebraic equation used with such skill by the ancient Greek mathematicians, applied practically to the scientific recording of business transactions for, just as in algebra, the equation, once established, cannot be changed but by the addition of positive or negative quantities. This book will give an added assurance that the apparently empirical rules of commerce are based upon an ancient scientific and mathematical foundation"* (Geijsbeek,1914:1).

However, Durham stated that:

*"The accounting part of the Summa soon became separated as a separate book"*

because

*"this separation of the Treatise from the Summa derives from the apparent irrelevance of most of the remainder of the Summa"* (Durham,1992: 50).

This a rather strange assertion since Pacioli, in chapter one, said:

*"If one has not understood the first part well, (of the Summa) it will be useless for him to read the following"* (Geijsbeek,1914: 33), (Von Gebsattel,1994: 42).

which implies a mathematical approach to the treatise was needed.

Heeffer commented upon the curious coincidence of algebra and double entry bookkeeping, giving several examples of writers who had written books on algebra and on bookkeeping, but stated that: *"there is no direct relationship between algebra and bookkeeping"* (Heeffer, 2010: 116).

Sangster, when writing about the need for accounting students to understand how double entry works, said:

*"The concept of double entry is no more than an example of an application of algebra…double entries can sometimes be complex, requiring sound knowledge of the principles in order to make the appropriate record"* (Sangster, 2010: 26).

Peters & Emery stated that Pacioli rejected negative numbers (Peters & Emery, 1978: 424 - 426), a conclusion with which Scorgie disagreed, proving instead that Pacioli was well aware of them, as he was of subtraction (Scorgie, 1989: 316 - 318).

Mattessich also commented on the early association of positive numbers with Assets, and negative ones with liabilities. He noted that Pacioli did not record either of the equations:

$A = L + OE$
*Or*
$A - L = OE$
*(A=Assets, L = Liabilities, OE= Owners' Equity)*

but he did not consider why Pacioli might have deliberately chosen not to do so (Mattessich, 1998: 130).

Ellerman examined double entry, and described it as being a specific example in group theory, calling it the 'Pacioli group':

*"There is a precise mathematical system underlying the system of double-entry bookkeeping … if the mathematical formulation of any field should be well understood, one would think it might be accounting. Remarkably, however, the mathematical formulation of double entry accounting – algebraic operations on ordered pairs of numbers – is largely unknown in mathematics as well as in accounting"* (Ellerman, 2014: 483).

Pacioli would not have had any knowledge of 'Group Theory' because it was not recognized as a specific type of algebra until the late eighteenth century (Malet, 1984: 52-88).

## 4. Pacioli, and the Entity and Proprietary Theories

Mattessich discussed the basic mathematical equation:

*"If one author pleaded for the equation 'Assets = Liabilities+ Owners' Equity', while his opponent argued in favour of the mathematically equivalent relation of 'Assets – Liabilities = Owners' Equity', intending to emphasise a different kind of classification, (then) an extramathematical element was introduced that a mathematician may find trivial (as for him the two equations are equivalent)"* (Mattessich, 2005: 47-51).

However, Roberts, when summarising a passage from Gilman, explained that the two forms of the equation referred to above, were not alternatives because they implied different accounting theories:

**The Entity Theory:**

*"The equation expressed as*
*'Assets = Liabilities plus Capital'*
*is describing the entity theory; the corporation has all the assets and … claims against the assets are represented by the liabilities and the owners' equity …*

**The Proprietary Theory:**

*whereas, if expressed as*
*'Assets - Liabilities = Equity',*
*it is describing the proprietary theory; the corporation is owned by the stockholders and their claims are residual in nature … and, although many authors use either formula (indiscriminately), they never make a distinction between the two … they should, because if liabilities and capital are considered to be completely different then the transposition to 'Assets - Liabilities = Capital' could be made … (but) if 'liabilities plus capital' is considered to be a single unit of a similar nature then they could not be separated"* (Gilman, 1930: 39, 41, 45; in Roberts, 1955: 174-175).

Gilman, supporting the entity theory, said:

*"actual bookkeeping does treat the proprietor just as though he were a creditor … the elementary plus and minus entries which constitute the bare mechanics of double entry are similar with respect to all sources of capital"* (Gilman, 1930: 90).

Sprague referred to a:

*"supposed entity – the business"* in which all assets are regarded as *"owing to the business, whilst the business is regarded as owing all the liabilities, <u>including proprietary claims</u>"*

which supports the entity theory, but he then qualified this statement, pointing out that only the proprietor's account increased or decreased if the business made a profit or a loss, the other liabilities of the business remaining the same: a fundamental difference, which implies the proprietor theory. He goes on to say that the business is owned by the owners, rather than having the business owing to the owners. On balance it seems more likely that he believed in the proprietary theory (Sprague, 1907: 57).

Stephens was another person who described the proprietor's account (capital) as being a creditor of the business, therefore supporting the entity theory (Stephens, 1735, In Littleton et al. 307-308).

Quoting from Gilman again, Roberts said that:

*"entity theory is often developed in accounting literature by referring to the Roman Empire, where there were slaves acting in business ventures for their masters … if the slave is considered*

*as a person independent of his master's actions, the 'entity theory' of enterprise results ... however, if a slave has no right to own property in his own name ... the 'proprietary theory' results"* (Roberts, 1955: 3).

So, was Pacioli following what is now called the 'entity theory', or was he following the 'proprietary' one? In chapter 23, Pacioli tells the merchant to regard the store as being a:

*"person, separate from its owner"* (Geijsbeek, 1914: 61),(Von Gebsattel, 1994: 73),

which implies he was following the entity theory, but when describing how to enter the inventory into a ledger, Pacioli said:

*"Capital of myself, so and so, is creditor ... capital belonging to me"* (Geijsbeek, 1914: 49),(Von Gebsattel, 1994: 58).

'Capital as creditor' implies the entity theory whereas 'capital belonging to me' implies the proprietary theory. However, his admonition to use separate sets of books for special cases, plus his concern for protecting a merchant's privacy, suggests that he was indeed using the entity theory for the inventory as well:

- *"According to the circumstances ... you might open other accounts. And this will be sufficient for you (*Geijsbeek, 1914: 63), (Von Gebsattel 1994: 76 (This sentence was not translated by Von Gebsattel))

- "*These accounts must all be entered in all three books separately from your own… keep this cash account separate from your private one…you should have a separate set of books in the same order… I will not give you any further instruction, because what I have said so far will be sufficient for you"* (Geijsbeek, 1914: 59),(Von Gebsattel 1994: 68-69).

- *"Many are accustomed to enter their inventory in this book (the business memorandum) but it is not wise to let people see and know what you possess. It is not wise to enter all your personal property and real property in this book. This book is kept on account of the 'volume of 'business'"* (Geijsbeek, 1914: 39), (Von Gebsattel, 1994: 48).
- *"This inventory was not contained in the memorandum book….and how to make these entries in the journal and the ledger ...I will leave to your ability on which I count very much"* (Geijsbeek, 1914: 51), (Von Gebsattel,1994:60)

Pacioli consistently emphasised that the merchant must keep orderly records. It seems inconceivable then that he would have suggested just having a list of his private possessions, adding to it, and crossing out items as things changed. In fact, he did not; he told the merchant to enter them in a private journal:

*"In this journal, which is your private book, you may fully state all that you own in personal or real property, always making reference to the inventory papers ... which are kept in some (locked?) box"* (Geijsbeek, 1914: 43), (Von Gebsattel, 1994: 52)

If a private journal, then surely a private ledger too? Is he suggesting treating the inventory in the same way as he treated the store – separate from the owner, with its own set of books, and owing all it holds to him?

## 5. Capital

The term 'Capital' has been much discussed in both the accounting and economics literatures. Particularly relevant to this paper are these quotations:
- by Bernardino of Siena, 1380-1444:

*"Capital is not simply money; it is money capable by 'labour and industry' of producing a profit. Capital thus becomes productive money"* (Wood, 2002: Chapter 7).

- by a Franciscan friar, Richard of Middleton, 1249 - 1308:

*"Money of itself is a sterile thing, for it can bring forth no fruit except by the labour and solicitude of the user"* (Wood, 2002: Chapter 7).

- by Rambaud:

*"Capital, as understood by Luca Pacioli …….is already conceptually conceived in a form that includes profit"* (Rambaud, 2019: 20).

Cronhelm, in a comment that seems to amplify Richard of Middleton's statement, said:

*"cash is converted into goods and goods are reconverted into cash"* (Cronhelm, 1818. In Littleton et al. 311).

Also, when adopting a mathematical approach, he used the entity theory:

*"the business classes the proprietor's account as being amongst its other creditors"* (Cronhelm, 1818. In Littleton et al. 310).

Assessing Pacioli's definition of 'Capital':
*"You should charge it (the store) with all the different things that you put into it …. you must imagine that this store is like a person who should be your debtor for all the things you give it or spend for it and on the contrary you shall credit it with all that you take out of it"* (Geijsbeek, 1914, 61),(Von Gebsattel, 1994: 72)

This implies that there would be an account called 'Store' (or 'shop') in the merchant's own ledger, into which he would enter all his transactions with the store, and that the store would have its own set of

books with an account 'Capital' representing the merchant's claim upon it. So Pacioli was following the entity theory, with Capital being regarded as a liability from the business's point of view (Geijsbeek, 1914: 61-62) (Von Gebsattel 1994: 72-73)

Jackson pointed out that:

> *"the complete separation of the owner from the business, the fundamental equation of ownership, and the conception of transactions affecting this equation by increasing or decreasing assets, liabilities or capital…. almost completely disappeared from view in Great Britain"* (Jackson, 1956, In Littleton et al. 312).

## 6. The derivation of Double Entry in equation form; implied by Pacioli's words

In his paper 'Pacioli's Legacy' Macve examines Pacioli's treatise, focussing upon its incompleteness as viewed by a modern accountant. In it he states that Pacioli was not a merchant, and that his knowledge of commerce was gained through acquaintances (Macve1996: 10), This is incorrect however, since Pacioli spent six years working for a merchant, trading on his behalf (Sangster, 2021:137). Although Macve refers to 'Pacioli's legacy', telling us that it is like 'buried treasure, waiting to be rediscovered', he does not tell us what he is expecting to see when it is found (Macve 1996: 22). Perhaps this paper will unearth it.

Pacioli did not claim to have invented double-entry bookkeeping. He made it very clear that he was writing about the Venetian method, expressed as a series of rules for merchants to follow to be in control of their businesses. His background as a humanist, and as a cleric, motivated him to help them, as did his experience as a trader with the Rompaisi family. He even addressed his work to the Duke of Urbino in order to give it, and himself, legitimacy in the eyes of potential readers (Geisjbeek, 1914: 33), (Von Gebsattel, 1994:41)

Pacioli did not record any algebra in his treatise, hence one must read 'between the lines', as it were, to uncover the set of equations that the author considers were in his mind when he was writing it. These equations throw light on how the system evolved from single column, to double column, and eventually to double entry format.

Williams discussed the use of the accounting equation and attributed positive numbers with assets, and negative ones with liabilities, but he took this development no further. He discussed alternative theories concerning the evolution of double entry, and suggested that:

> *"Having grasped the equilibrium nature of the final double-entry product, it would then be a matter of decomposing the whole framework into a series of logical component parts, that is, the journal entries, a trial balance, and a ledger account, complete with calculations of profit."* (Williams, 1978: 37).

Sprague said:

> *"As a branch of mathematical and classificatory science, the principles of accountancy may be determined by a priori reasoning, and do not depend upon the customs and traditions which surround the art. I have endeavoured to set forth these principles …adhering to the fundamental equations and their sub-equations"* (Sprague, 1907: 31).

Today, algebraic equations are written in symbol form, but in medieval times they were often expressed verbally. Shakespeare used them in the Merchant of Venice when introducing subtle jokes, this format being well understood by his audiences of lawyers, and other persons of that standing.

One example, concerning the moral worth of Antonio, was reduced to equations by Bady (1985) as:

Bassanio = Antonio + Portia.

When it was revealed that Portia and Bassanio were the same person, the equation became:

Portia = Antonio + Portia,

which reduced to:

Portia - Portia = Antonio = 0,

demonstrating that Antonio had no moral worth at all. (Bady, 1985: 12, cited by Sangster, 2010: 34).

The closest Pacioli came to referring to anything like an equation, was when he wrote:

*"you will have to make two in the ledger….one in the debit and one in the credit… the debitor entry must be at the left and the creditor one on the right"* (Geijsbeek, 1914: 47), (Von Gebsattel, 1994: 56)

Pacioli's first reference to a business was:

*"We will take the case of one who is just starting in business"* (Geijsbeek, 1914, 33), (Von Gebsattel, 1994: 42)

Using the entity method, (therefore including capital in liabilities), the basic equation is:

Assets = Liabilities                                                                 (1)

Then:

*"The purpose of every merchant is to make a lawful and reasonable profit so as to keep up his business"* (Geijsbeek, 1914: 33) (Von Gebsattel, 1994: 42).

Rambaud (2019: 20) referred to 'profit' as being included in capital, therefore any profit the store makes will be owed to the merchant, so, from the store's point of view, profit is a liability.
Equation (1) becomes:
Assets = Liabilities + Profit                                                     (2)
Pacioli's description of how to calculate profit involved several steps: Each goods account was credited with revenue, debited with purchase costs, and with directly attributable costs (a gross profit approach). Only when all of their goods had been sold was the balance on those goods accounts

transferred to the profit and loss account. All other costs were charged to their own expense accounts, and then to the profit and loss account. (Geijsbeek, 1914: 59 - 67), (Von Gebsattel, 1994: 70-80). In this respect, Lee (1975), commented that a merchant had four financial objectives:

*"to maximise sales, to maximise gross profit thereon, to remain solvent, and to live according to his station. Gross profit was significant, net profit was not; it was merely a residue, certain to accrue so long as business was good, and a sharp eye kept on overheads"* (Lee, 1975: 10 - 11).

So, in aggregate, profit can be expressed as:

Profit = Revenue – Expenses       (3)

Substituting (3) into (2), expands the equation to:

Assets = Liabilities + Revenue – Expenses      (4)

Adding expenses to both sides of this equation gives:

Assets + Expenses = Liabilities + Revenue      (5)

Deane et al. also expanded the equation in the form shown as (5) above, but they took its evolution no further. (Deane et al., 2016:12)

Stanfield observed that:

*"Only time separates assets and expenses: assets become expenses as they are used up in trading; they are expenses waiting to happen"* (Stanfield, 2002: 87),

implying that both will be treated in the same way in the double entry system.

Sprague (1907) made a similar point, saying:

"*Assets … are a storage of services to be received*" (Sprague 1907: 46).

These assertions are now supported by this algebraic explanation as well; assets and expenses are treated in the same way in the double-entry system because they are both on the same side of the equation.

Cronhelm (1818) subtracted the liability items from both sides of the equation:

*"a + b + c ...- l - m - n...= ±s"*

(s stood for capital), with assets being plus items and liabilities minus ones, the sum of them all being capital 's'. Then he subtracted 's' from both sides to give:

*"a + b + c ...- l -m - n... ∓s = 0"* (Cronhelm, 1818. In Littleton et al. 310).

Applying this approach to equation (5) gives:

Assets + Expenses - Liabilities – Revenues (= 0)     (6)

Capital (including profit) is included in liabilities in this equation. This is a 'single columns double entry system', as described by Ellerman (Ellerman, 2014: 487-488), each line of which balances to zero because a plus figure in one of the columns is balanced by a corresponding minus one in another. No signs are needed on the column headings because plus and minus signs are attached to the numbers that are entered into them. So, equation (6) becomes:

Assets   Expenses   Liabilities   Revenues      (7)

From a merchant's point of view, equation (7) has a major disadvantage in that the columns will have both plus and minus numbers in them, one below another, making it difficult to calculate inflows and outflows. This is a matter of particular concern for the cash column, because keeping control of cash is essential. Pacioli emphasised:

*"if you are in business and do not know all about it, your money will go like flies – that is, you will lose it."* (Geijsbeek, 1914, 63), (Von Gebsattel, 1994: 73).

Stephens suggested that having to add positive and negative numbers that are in a single column was '*tedious*' and suggested that this was the reason for *'having two columns'*, one for plus and one for minus items so that:

*"they might at pleasure be compared by one subtraction, after their sums were severally added up"* (Stephens, 1735: In Littleton et al. 308).

Although subtraction was not what Pacioli had in mind when he explained how to find a balancing figure through addition instead:

"*As I have often told you, add the difference to the column, either debit or credit, which shows a smaller total*" (Geijsbeek,1914: 73), (Von Gebsattel, 1994: 84).

Hence, we arrive at equation (8):

Assets  Expenses  Liabilities  Revenues $\qquad\qquad\qquad$ (8)
  +  -    +  -      +  -      +  -

The next development in the evolution of the equation was to eliminate the positive and negative signs; they would not have been understood by the general public in the fifteenth century, because, as Heeffer said:

*"the use of negative quantities* (was) *a concept which was gradually introduced only from the sixteenth century onwards"* (Heeffer, 2010: 123).

Sangster described one way that this elimination was achieved:

*"the verbs 'dare' and 'avere' (to give and to have) became technical words, finally replaced by 'debit and 'credit'"* (Sangster, 2016: 275-276).

Another possibility can be deduced by considering the store, which owed everything it held to the merchant, or to other persons who provided them. These persons trusted the store to eventually repay them, or to return the items. The Latin term for 'owe' is 'debet', and for 'trust' is 'credet', terms which became 'debit' (DR) and 'credit' (CR), replacing the plus and minus signs at the heads of the columns. Hence, we arrive at equation (9), which has been expanded to allow for Pacioli's statement that:

*"you can make your purchase in cash, on time, by exchanging something, or by draft. I need not explain the ways of selling"* (Geijsbeek, 1914: 43), (Von Gebsattel, 1994: 51)

So new columns are needed for debtors, creditors, and cash, plus a temporary one in which to calculate the profit or loss before transferring it to the capital account. The names 'Assets' and 'Liabilities' in equation (9), should now be understood to refer to 'Other Assets' and to 'Other Liabilities'.

Assets Debtors Cash Expenses Liabilities Creditors Revenues P & L $\quad$ (9)
 DR CR  DR CR  DR CR DR  CR   DR  CR    DR  CR   DR  CR   DR CR

Ellerman stated that:

*"the choice is between unsigned (single sided) numbers in double sided accounts (i.e. DEB) or…. signed numbers (double sided) in single sided accounts"* (Ellerman 2014: 487-488).

Equations (8) and (9) are the 'unsigned numbers in double sided accounts' referred to by Ellerman; we have arrived at traditional double entry. All that is needed now is to turn the columns into pages, in as many ledgers as one wishes.

Why, when, and how these developments took place has been a matter of conjecture for many researchers. de Roover thought that:

*"Double entry did not grow out of any pre-established theory but was developed step by step by a process of trial and error"* (de Roover,1956:115).

Most researchers, when exploring how double entry developed, looked for evidence in historical records. Sangster took a different stance in that he discussed its development in two main ways. The first focussed on the needs of bankers and merchants to have a secure way of recording transactions, one that could be checked for accuracy by anyone. The second reviewed the system's development from single to dual entries, and finally to double entry, each step making the system more efficient, and free from error:

*"In 1211, the form of entry had reached dual entry, a starting point towards double entry. It was neither entirely single entry nor double entry (Usher 1934). How and why did the transition beyond that point to double entry occur?....... the local bankers in Florence needed to devise an appropriate system to maintain a historical record of their transactions to a standard that would enable checks to be undertaken by external auditors……. That this externally driven motivation to devise a structured system of bookkeeping is indisputable……in all likelihood, it led to double entry bookkeeping first emerging in the local banks of Florence….. Bankers were not alone in maintaining records for debtors and creditors: "the statutes of many of the artisan guilds…. required that their members keep an accounting record of their affairs" (Goldthwaite 2009: 354) ….. so merchants embraced a bookkeeping method developed by bankers"* (Sangster, 2016: 6)

| EVOLUTION OF SYSTEMS | SINGLE ENTRY | DUAL ENTRY | DOUBLE ENTRY |
|---|---|---|---|
| ENTRIES MADE IN | 1 ACCOUNT | 2 ACCOUNTS | 2 ACCOUNTS |
| EACH ENTRY INCLUDES THE PAGE NUMBER WHERE THE CONTRA ENTRY CAN BE FOUND | NO | NO | YES |

*"Thus, the development from single column to dual columns, and eventually to double entry took place"* (Sangster, 2016: 6).

De Morgan suggested an alternative way to record transactions; he used a matrix into which only one figure had to be entered, not two, as in normal double entry (De Morgan, 1846, In Littleton et al. 298). Kemeny et al. commented that:

*"the important point about double entry is <u>not</u> that each transaction is <u>recorded</u> twice, but rather that each transaction is <u>classified</u> twice – once as a debit and once as a credit"* (Kemeny et al. 1962: 347).

When using the same matrix as De Morgan, they commented that, although it would be ideally suited for computers, it was too unwieldy a method for manually recording transactions, (because of the need to have multiple entries into the same cells) a process for which the double entry system was better suited. (Kemeny et al. 1962: 350).

An example of this matrix is shown in the appendix.

Although this system of equations reflects the underlying logic of double entry, Pacioli only tangentially referred to the first equation when he required the sum of all the debits to equal the sum of all the credits, (Geijsbeek, 1914: 73), (Von Gebsattel, 1994, 86).

The reason for him not including any other reference to algebra is probably the same as that given by Cripps when discussing the way in which Pacioli ensured the treatise would be comprehensible to merchants:

*"Pacioli chose to write…'in the vernacular'…. (because it was) … the language of the market"* (Cripps, 2017)

After his experiences as a trader and later as a Friar, Pacioli would have known how best to communicate with the merchants. McCarthy et al. said that:

*"Pacioli had a range of choices …. concerning the language and style of writing to use … he had many years' experience of teaching in the vernacular… (*and*) as a Franciscan preacher he would have been used to addressing a common crowd … he would have known precisely how to pitch his text for his audience"* (McCarthy, 2008:199).

He would have known too, that had he included equations in his treatise they would have confused many of the merchants, and this was the last thing he would have wanted to happen; the very reason he wrote the treatise in the first place, was to help them.

Ellerman expressed an opinion supporting the idea that Pacioli would have made such a decision:

*"it may well be that Pacioli, as a mathematician, implicitly assumed an underlying equation…. but did not mention it explicitly in his treatise intended for practical business purpose"* (Ellerman, 2014: 499, note 3)

The evolution of these equations is illustrated in the appendix, using tables and figures. The scenario is taken from Shakespeare's play, 'The Merchant of Venice'.

## 7. The Necessary and sufficient conditions

Vaughn, in his book 'Living Philosophy' said:

*"According to Plato, there are three necessary and sufficient conditions for one to have knowledge; (1) the proposition must be believed; (2) the proposition must be true ;(3) the proposition must be supported by good reasons, which is to say, you must be justified in believing it"* (Vaughn, 2014).

Being a mathematician, Pacioli would have been aware of these propositions, and so he included a version of them in his treatise, the sufficient condition first, and then the necessary one. This is not the order in which a mathematician would expect them to be presented, but Pacioli did not write his

treatise for academics; he wrote it to help merchants, who would not even have known those technical terms existed.

For the sufficient condition Pacioli stipulated:

*"Get a helper, as you could hardly do this alone. Give him the journal, for greater precaution, and you shall keep the ledger. Then you tell him, beginning with the first entry in the journal, to call the numbers of the pages in your ledger where the entries have been made, first in the debit and then in the credit. If all agrees it will mean that all the accounts are correct, and the entries are entered correctly"* (Geijsbeek, 1914: 69), (Von Gebsattel, 1994: 81-82)

He described the necessary condition as:

*"summarize on a sheet of paper all the debit totals … and place them on the left, then you shall write down all the credit totals and place them on the right……if these grand totals are equal … then you shall conclude that your ledger was very well kept"* (Geijsbeek,1914: 73), (Von Gebsattel,1994: 86)

In chapter 36 Pacioli stated this condition in another way:

*"By a trial balance …we write down all the creditors… and all the debtors… we see whether the total of the debits is equal to that of the credits, and if so, the ledger is in order"* (Geijsbeek, 1914: 77), (Von Gebsattel, 1994: 89).

The merchant is 'listening' to the clerk, and the Latin for 'he listens' is 'audite'. Pacioli was including the practice of auditing in his treatise as well, as further evidenced by his advice to a merchant to have his books checked by an external source (Geijsbeek,1914: 41), (Von Gebsattel,1994: 49)

Kemeny et al. also implied that both these conditions should be included:

*"the fact that the sums of the debits and the credits balance is a necessary, but not a sufficient, condition that the transactions have been properly recorded"* (Kemeny et al., 1962: 347).

## 8. An Assessment of Alternative Translations of the Treatise

Since this paper has only quoted from the translations of Geijsbeek and Von Gebsattel, four other translations have been reviewed to see whether they invalidated any of the quotations or conclusions reached.

Cotrugli's Chapter (Sangster& Rossi, 2018)

Written in 1458, well before Pacioli wrote his treatise, this chapter gives instructions on how to write up the books of account. The concept of 'Capital' is used in conformity with how the term is used in Geijsbeek's translation. No contradictions were found.

Translation by P. Crivelli (1924)

All the quotations from Geijsbeek were checked against this translation, and although the wording differs, the substance was the same.

Translation by R.G. Brown and K. S. Johnston (1963)

This translation, like that of Cripps which follows, is in clear English and easy to read. All the quotations have been checked against it and all would have led to the same conclusions if they had been used. Therefore, the conclusions drawn in this paper are not invalidated by this translation.

Translation by J. Cripps (1994)

No points were found that would invalidate the conclusions of this paper. Chapter 27 stated that all the trading account balances in the ledger were closed to the profit and loss account. This does not agree with Geijsbeek's translation, which implies that only those goods accounts that had sold all their produce and therefore made a gross profit or loss, would be transferred to the profit and loss account.

## 9. Conclusions

The hypothesis of this paper is that Pacioli, a skilled mathematician, would have been aware of the algebraic logic that supported the rules he gave the merchants. There can be no historical evidence to support this proposition; it has been deduced from Pacioli's own words as translated by Geijsbeek, and then checked to other translations. None the less, it is hoped that the arguments put forward will convince the reader that there is a strong possibility that Pacioli may have had such equations in mind when writing his treatise. Perhaps they are 'Pacioli's Legacy'; Macve's 'buried treasure', now unearthed (Macve, 1996: 22).

Pacioli's view of capital follows the entity theory, and his description of how to account for a store is central to the whole of the treatise. His suggestion that the store should be regarded as being a person, separate from him, is applied throughout the treatise. Taking into consideration his recommendation that separate sets of books should be opened when the occasion calls for them, it is considered likely that the merchant would have had a private master set of books, in the ledger of which would be an account for every commercial activity he was involved in - even one for dealing with the inventory of his personal possessions, regarding 'inventory' too as if it were a 'person' owing the possessions to him.

Pacioli wrote the treatise to help merchants run their businesses efficiently. After six years of being amongst merchants, he knew very well how to pitch his treatise to them. That is why he wrote in the vernacular and why he chose to avoid the explicit use of equations; the merchants would not have understood them. Instead, he gave them rules, based on the same logic.

It is hoped that these equations will be of help in the teaching of double entry. Their progression, ending with the replacement of plus and minus signs with debit and credit names, removes the mystery that surrounds those terms. There is no need to learn rules by rote anymore.

The equations also provide answers to student questions such as:

*'Assets are nice to have, expenses are not, so why does the double entry system treat them in the same way?'*

The connection of 'plus with debit' and 'minus with credit' will also make it easier to explain how and why asset, liability, expense, and revenue accounts are affected in different ways by these terms.

Prince Maurits would not have been so confused if Stevin had been able to explain to him that he should think of his business as if it were a person, separate from himself, working hard on his behalf. Then he could have told his Prince that credit increases in the capital account were good; they meant that the business had created profits, and therefore had increased the amount that it owed to him.

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**Annexes**

## APPENDIX - TABLES TO ILLUSTRATE THE EVOLUTION OF DOUBLE ENTRY

In 'The Art of Something – Arithmetic in the Merchant of Venice - David Bady (1997) identified verbal equations, so as this play already has mathematical connections, it's scenario is used to illustrate the evolution of double entry in tabular form.

Antonio, a merchant, invests all his money (100) in a business. The business buys ships (80), hires crews (50), provides them with provisions, (30) and buys goods, on credit (70), with which to trade.

Some weeks later a friend, Bassanio, asks Antonio for a loan (3), but since all of Antonio's money has been invested in his business, he has to borrow this amount from Shylock, a money lender. Shylock insists that if he is not repaid at the agreed time, he will be entitled to a pound of Antonio's flesh.

When the time arrives, Antonio cannot repay because the business's ships have not returned, indeed he is told the ships have been lost at sea and he has no other money. Shylock takes him to Court and demands that the contract be carried out. The terms of the contract so shock the Court that it rules against Shylock, and orders him to cancel Antonio's debt, and to forfeit his whole estate (1,000), one half of which is to be given to Antonio and the other half to the State.

Fortunately, the business's ships have not perished; they return with the proceeds of their sales (200), and Antonio withdraws some money (40), from the business, and also cancels Bassanio's debt.

The tables below separate the business transactions from the personal ones. Pacioli advised the merchants to conceal their private possessions and lives from prying eyes, and one way this could have been achieved would be to have a master set of books with accounts within it for their businesses, partnerships, and any other activities that they wished to keep separate. Pacioli said:

*According to the circumstances ... you might open other accounts. And this will be sufficient for you* Geijsbeek, 1914, 63).

   "*These accounts must all be entered in all three books separately from your own... keep this cash account separate from your private one...you should have a separate set of books in the same order.... I will not give you any further instruction, because what I have said so far will be sufficient for you*" (Geijsbeek,1914:59).

*"Many are accustomed to enter their inventory in this book (the business memorandum) but it is not wise to let people see and know what you possess. It is not wise to enter all your personal property and real property in this book. This book is kept on account of the volume of business".* (Geijsbeek,1914:39).

"*This inventory was not contained in the memorandum book....and how to make these entries in the journal and the ledger ...I will leave to your ability on which I count very much*" (Geijsbeek,1914:51).

Table 1 - based on equation (5) Assets + Expenses = Liabilities + Revenues (business transactions only)

| Table 1 | Ships | Goods | Exp. | Cash | = | Cap | Rev | Crs | P&l |
|---|---|---|---|---|---|---|---|---|---|
| Cash invested | | | | 100 | = | 100 | | | |
| Business buys ships | 80 | | | -80 | = | | | | |
| Crs. Supply goods | | 70 | | | = | | | 70 | |
| Wages paid | | | 50 | -50 | = | | | | |
| Provisions | | | 30 | -30 | = | | | | |
| Sales revenue | | | | 200 | = | | 200 | | |
| Transfer revenue | | | | | = | | -200 | | 200 |
| Charge cogs | | -70 | | | = | | | | -70 |
| Transfer expenses | | | -80 | | = | | | | -80 |
| Transfer profit | | | | | = | 50 | | | -50 |
| Pay creditors | | | | -70 | = | | | -70 | |
| Ant. Withdraws | | | | -40 | = | -40 | | | |
| Balance sheet | 80 | 0 | 0 | 30 | = | 110 | 0 | 0 | 0 |

Although this is a double entry system the existence of an 'equals' sign makes it unsuitable for recording transactions.

2 – Based on equation (7) Assets Expenses Liabilities Revenues (business transactions only)

| Table 2 | Ships | Goods | Exp. | Cash | Cap | Rev | Crs | P&l |
|---|---|---|---|---|---|---|---|---|
| Cash invested | | | | 100 | -100 | | | |
| Business buys ships | 80 | | | -80 | | | | |
| Crs. Supply goods | | 70 | | | | | -70 | |
| Wages paid | | | 50 | -50 | | | | |
| Provisions | | | 30 | -30 | | | | |
| Sales revenue | | | | 200 | | -200 | | |
| Transfer revenue | | | | | | 200 | | -200 |
| Charge cogs | | -70 | | | | | | 70 |
| Transfer expenses | | | -80 | | | | | 80 |
| Transfer profit | | | | | -50 | | | 50 |
| Pay creditors | | | | -70 | | | 70 | |
| Ant. Withdraws | | | | -40 | 40 | | | |
| Balance sheet | 80 | 0 | 0 | 30 | -110 | 0 | 0 | 0 |

This is the double entry system described by Ellerman as being 'an unsigned column system with signed numbers' (Ellerman, 2014: 487-488). It would work, but from a merchant's point of view it has a major disadvantage in that each column has plus and minus numbers in them, one below another. This would make it difficult to calculate inflows and outflows, a matter of particular concern for the cash column, because keeping control of cash is essential. So, format 3 came about.

Format 3 Assets    Debtors    Cash    Expenses    Liabilities    Creditors    Revenue

+ -    + -    + -    + -    + -    + -    + -

Where the term 'Assets' represents other assets, and 'Liabilities' represents other liabilities, including capital. This is the two- column system described by Ellerman as 'a signed column system with unsigned numbers' (Ellerman, 2014: 487-488). This format transforms into the debit and credit format by replacing the plus and minus signs, so only that format is shown below as table 3

Table 3: Based on equations (8) converted to (9); The debit, credit table: double entry. (business transactions only)

| | Ships | | Goods | | Expenses | | Cash | | Capital | | Rev | | Crs | | P & l | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | DR | CR | DR | CR | DR | CR | DR | CR | DR | CR | DR | CR | DR | CR | DR | CR |
| Capital | | | | | | | 100 | | | 100 | | | | | | |
| Ship | 80 | | | | | | | 80 | | | | | | | | |
| Wages | | | | | 50 | | | 50 | | | | | | | | |
| Food | | | | | 30 | | | 30 | | | | | | | | |
| Goods | | | 70 | | | | | | | | | | | 70 | | |
| Sold | | | | | | | 200 | | | | | 200 | | | | |
| Transfer | | | | | | | | | | | 200 | | | | | 200 |
| Transfer | | | | | | 80 | | | | | | | | | 80 | |
| Cogs | | | | 70 | | | | | | | | | | | 70 | |
| Transfer | | | | | | | | | | 50 | | | | | 50 | |
| Pay | | | | | | | | 70 | | | | | 70 | | | |
| Withdraw | | | | | | | | 40 | 40 | | | | | | | |
| Totals | | | | | | | | | | | | | | | | |
| Carried fwd. | | 80 | | | | | | 30 | 110 | | | | | | | |
| Totals | 80 | 80 | 70 | 70 | 80 | 80 | 300 | 300 | 150 | 150 | 200 | 200 | 70 | 70 | 200 | 200 |
| Brought dn. | 80 | | | | | | 30 | | | 110 | | | | | | |

Note that the lack of knowledge about negative numbers explains why Pacioli told the merchants how to balance accounts without involving subtraction:

*"As I have often told you, add the difference to the column, either debit or credit, which shows a smaller total.* (Geijsbeek, 1914: 73)

Now all that was necessary was to instruct the clerks to make sure that there was a credit for every debit; there are no plus or minus signs in sight, nor is there an equation. However, the clerks had to learn 'rules', a task that has always presented difficulties; learn the rules and follow them, even poems have been used to instill them, for example:

The owner, or the owing thing,

Or whatsoever comes to thee,

Upon the **left** hand see thou bring

For there the same must placed be.

_____

They unto whom thou dost owe,

Upon the **right** let them be set,

Or whatsoever doth from thee go

To place them there - do not forget

(dafforne)

Although table 3a does represent double entry, table 2 better represents the system used by computerized accounting systems, which, naturally, are not concerned about having positive and negative numbers in the same column.

The personal accounts of Antonio and of Shylock are given below:

Table 4 Antonio's personal accounts

| Assets - liabilities = 0 | Business | Jewels | Linen | Cash | Bassanio | Capital | Shylok |
|---|---|---|---|---|---|---|---|
| Jewels | | 200 | | | | -200 | |
| Linen | | | 60 | | | -60 | |
| Personal wealth | | | | 100 | | -100 | |
| Invests in business | 100 | | | -100 | | | |
| Shylock lends Antonio | | | | 3 | | | -3 |
| Antonio lends Bassanio | | | | -3 | 3 | | |
| Loan from Shylock is cancelled | | | | | | -3 | 3 |
| Business makes profit | 50 | | | | | -50 | |
| Withdraws from business | -40 | | | 40 | | | |
| Shylock pays | | | | 500 | | -500 | |
| Antonio cancels Bassanio's debt | | | | | -3 | 3 | |
| Antonio's revised wealth | 110 | 200 | 60 | 540 | 0 | -910 | 0 |

Antonio receives half of Shylock's wealth. The State gets the other half.

Table 5 Shylock's personal accounts

| Assets - liabilities = 0 | Cash | Antonio | Capital |
|---|---|---|---|
| Personal wealth | 1003 | | -1003 |
| Shylock lends Antonio | -3 | 3 | |
| Loan cancelled | | -3 | 3 |
| Shylock pays | -1000 | | 1000 |
| Revised wealth | 0 | 0 | 0 |

Shylock's personal wealth has been converted to cash and given away. He has lost everything.

## Matrix presentation

A square matrix can be used to present the mathematical structure of double entry, it was first used by De Morgan in 1869 and developed by Kemeny et al. (Ellerman, 2014, 490).

Such a table is shown below, using the Merchant of Venice figures.

| | Table 6 | Dr | Dr | Dr | Dr | Dr | Dr | Dr | Dr | Dr | Dr | Dr |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Cash | Ships | Goods | Wages | Provns. | Rev. | Exp | P&L | Cap | W'Draw | Totals |
| CR | Cash | | 80 | 70 | 50 | 30 | | | | | 40 | 270 |
| CR | Ships | | | | | | | | | | | 0 |
| CR | Goods | | | | | | | 70 | | | | 70 |
| CR | Wages | | | | | | | 50 | | | | 50 |
| CR | Provns. | | | | | | | 30 | | | | 30 |
| CR | Rev. | 200 | | | | | | | | | | 200 |
| CR | Exp. | | | | | | | | 150 | | | 150 |
| CR | P&L | | | | | | 200 | | | | | 200 |
| CR | Cap. | 100 | | | | | | | 50 | | | 150 |
| CR | W'Draw | | | | | | | | | 40 | | 40 |
| CR | Totals | 300 | 80 | 70 | 50 | 30 | 200 | 150 | 200 | 40 | 40 | 1,160 |

Data is then extracted and presented in Table 7 to give a closing balance sheet.

| | TABLE 7 | | | | | | | | | | B |
|---|---|---|---|---|---|---|---|---|---|---|---|
| DR | Cash | - | CR | Cash | 300 | - | 270 | = | **30** | A |
| DR | Ships | - | CR | Ships | 80 | - | 0 | = | **80** | L |
| DR | Goods | - | CR | Goods | 70 | - | 70 | = | **0** | A |
| DR | Wages | - | CR | Wages | 50 | - | 50 | = | **0** | N |
| DR | Provns. | - | CR | Provns. | 30 | - | 30 | = | **0** | C |
| DR | Rev. | - | CR | Rev. | 200 | - | 200 | = | **0** | E |
| DR | Exp. | - | CR | Exp. | 150 | - | 150 | = | **0** | S |
| DR | P&L | - | CR | P&L | 200 | - | 200 | = | **0** | H |
| DR | Cap. | - | CR | Cap. | 40 | - | 150 | = | **-110** | E |
| DR | W'Draw | - | CR | W'Draw | 40 | - | 40 | = | **0** | E |
| DR | Totals | - | CR | Totals | 1,160 | - | 1,160 | = | **0** | T |

Manual use of this system would be difficult, because, as the transactions multiplied, many of the cells would contain multiple numbers in them. Kemeny et al. point out though, that this matrix could form the basis for a computerised system. (Kemeny et al., 1962)

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Biography:

DEREK STONE BA FCA FRSA

Derek obtained an Honours Degree in Economics and Mathematics at Nottingham University. He became a Chartered Accountant, and then joined a Management Consulting Company before becoming the first appointed Lecturer in Accounting at the newly formed Manchester Business School. Then followed three years at Stanford Business School, as a Foundation for Management Education Fellow, studying and researching in the Organisational Behaviour Area. On returning to England, he joined the London Business School where he lectured in both Financial and Management Accounting, the latter with a behavioural slant, until his retirement.

You can learn more about him and his work by visiting either of his two websites: understandaccounting.com or understandmathematics.com His book 'Understand Accounting' is available on Amazon in its third edition. At present he is writing two books on Mathematics, one for students, and one for people who feel that they 'missed out' on mathematics whilst at school.