A CONCISE HISTORY OF ANALYTICAL ACCOUNTING: EXAMINING THE USE OF MATHEMATICAL NOTIONS IN OUR DISCIPLINE[1]

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RESUMEN

Este trabajo ofrece una sucinta revisión de los métodos de matemática analítica empleados en teneduría de libros y contabilidad durante los últimos cinco milenios. Sin embargo, el énfasis se centra en la segunda mitad del siglo XX, en el cual el uso de ideas matemáticas, de conceptos formales y de técnicas es cuando verdaderamente floreció. Aparte de un uso creciente del interés compuesto y de los cálculos del valor actual, las últimas se refinaron grandemente con la introducción de variables probabilísticas, de modelos estocásticos, de errores en términos estadísticos (como se encuentran, por ejemplo, en la versión ampliada y perfeccionada de la *clean surplus theory*). Pero este período (de los últimos 50 años más o menos) comenzó propiamente con modelos contables deterministas (que han probado su valor práctico últimamente en el desarrollo de hojas de cálculo electrónicas y sistemas presupuestarios). Hubo asimismo mucha experimentación con álgebra lineal y no lineal (incluyendo el álgebra matricial) y otras técnicas de investigación operativa, así como en métodos de muestreo estadístico contable que han probado su utilidad particularmente en la auditoría. Con todo, el logro intelectual culminante fue el lento pero impactante desarrollo de las posibilidades de información (*information perspective*) ofrecidas por la contabilidad (incluyendo información económica, la perfeccionada versión de la *clean surplus theory* y la teoría matemática de la agencia. Estos esfuerzos han sido resumidos recientemente en la obra en dos volúmenes de Christensen y Feltham (2003, 2005).

ABSTRACT

The paper offers a succinct survey of analytical-mathematical methods as employed in bookkeeping and accounting during some five millennia. However, the emphasis is on the second half of the 20th century where the

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The paper offers merely a *concise survey* that includes only the most important pertinent publications. It also interprets “analytical accounting” more in the modern sense of relating it to *mathematics* (hence the Subtitle) rather than in the broader sense of logical reasoning in general—though occasionally such extension could not be avoided. However, this interpretation is much broader than that in Wagenhofer (2004) where it is restricted mainly to what in our paper is discussed in Section 4 (information perspective). Furthermore, the expression “analytical accounting” in English must not be confused with the French *comptabilité analytique*, the Italian *contabilitá analitica*, or the Spanish *contabilidad analítica*, all of which used to refer to cost accounting.

use of mathematical ideas, formal concepts and techniques truly flourished. Apart from an increasing use of compound interest and present value calculations, the latter have been greatly refined by introducing probabilistic variables and stochastic models, statistical error terms, etc. (e.g., as encountered in a refinement and extension of the clean surplus theory). But this period (of the last 50 years or so) began rather with deterministic accounting models (that ultimately proved of practical value in the development of computerized spreadsheets and budgeting systems). There also was much experimentation with linear and non-linear algebra (including matrix algebra) and other operation research techniques, as well as with statistical sampling methods for accounting that proved particularly useful in auditing. Yet the crowning intellectual achievement was the slow but impressive development of the "information perspective" of accounting (including information economics, the refined clean surplus theory and mathematical agency theory). These efforts were recently summarized in the two-volume work by Christensen and Feltham (2003, 2005).

PALABRAS CLAVE:
Historia de la contabilidad analítica, uso de nociones matemáticas, álgebra matricial, information perspectiva, clean surplus theory, teoría matemática de la agencia
KEY WORDS:
History of analytical accounting, use of mathematical notions, matrix algebra, information perspective, clean surplus theory, mathematical agency theory

1. Introduction

The relation between mathematics and bookkeeping or accounting is ancient;[2] after all, accounting uses numbers to represent aspects of reality, and it was no lesser than Pythagoras who told us that the universe is written in numbers. So it is hardly surprising that the first published treatise on "double-entry" ("Particularis de computis et scripturis") appeared in a book on mathematics. I refer, of course to Luca Pacioli's (1494) famous *Summa de Arithmetica, geometria, proportioni et proportionalita* (Part I, Section 9, Treatise 11).[3] But this dependence

[2] The following quote from Moonitz and Jordan (1963, p. 5) not only offers a distinction between bookkeeping and accounting, but also hints at the intimate relation of *analytical* procedures to accounting: "Historically, double-entry bookkeeping has developed as a method by which the economic activities of privately owned and operated business concerns are recorded. Accounting, by contrast, may be viewed as the group of *analytical* techniques by which the financial aspects of economic activities are examined, evaluated and cast into the mould of double-entry bookkeeping. These *analytical* techniques consist, in the main, of a body of postulates and procedures, whether logical or conventional in nature, and provide a powerful tool by which an orderly, systematic representation of the complex activities of economic entities may be obtained [my *italics*]." In this paper, however, we often tend to associate analytical methods with mathematical techniques--though, precisely speaking, analytical methods transgress such a narrow definition.

[3] Pacioli's work must not be confused with the "first treatise on bookkeeping". Apart from Leonardo da Pisa's (i.e. Fibonacci's) *Liber abaci* (1202) and probably some preceding Arab texts (cf. Antoni, 1996), a much earlier manuscript on accounting (containing such sophisticated ideas as *purchasing power fluctuations*) can be found in several sections of Kautilia's *Arthasastra* (ca. 300 B.C.) from India—see, Mattessich (2000, pp. 131-149).

was by no means a one-way street. Already centuries before Pacioli, accounting contributed essentially to mathematics itself (see Colebrooke 1817/1973, as well as Mattessich 1998, and 2000, pp. 153-145) by giving Indian mathematicians (like Brahamagupta of the 7th century and Bha skara of the 12th century) the idea and justification of using not only *positive* but also *negative* numbers. They considered debt claims (or debits) as positive items and liabilities (or credits) as negative claims, and thus were the *first to conceive negative numerals as legitimate notions of mathematics* (hundreds of years before the Europeans recognized them as such). And millennia before that, the Sumerians established a "token system" of accounting that not only proved to be the precursor of *writing* but also of abstract *counting* (cf. Mattessich 2000, pp. 45-69).

But for centuries the major mathematical aspects of accounting were limited to *arithmetic*. It was mainly in the 19th century that *algebra* was employed to express more general accounting relations. This constituted a revolution that may be difficult to comprehend by modern accountants. The differences between these algebraic relations may seem to us so simplistic that they bordered on the trivial. Yet, Accountants of the 19th century hardly thought so. The evidence lies in the many "theories of accounts classes" and the controversies that followed in their wake. Many an author tried to prove that his classification scheme was better than the others or even the only correct one (for details see Mattessich 2003, particularly pp. 129-131). Usually these schemes were presented in simple algebraic equations, each one a variation or re-arrangement of the equilibrium equation (and from a mathematical point of view usually *equivalent* to the others), expressing that the value of all assets (or debits) equals the value of all equities (credits). Let me give you an illustration: If one author pleaded for the equation "Assets = Liabilities + Owner's Equity", while his opponents argued in favour of the *mathematically* equivalent relation of "Assets – Liabilities = Owner's Equity", intending to emphasize a different kind of classification, an *extra-mathematical* element was introduced that a mathematician may find trivial (as for him the two equations are *equivalent*).

Occasionally, however, more sophisticated ideas entered commercial and accounting thought. One of the first of those notions was the one of compound interest. According to Chatfield (1996a, p. 145) it was already known to the Babylonians between 1600 B.C and 1800 B.C., and was later found by Leonardo da Pisa (1202) as well as by Pacioli (1494). Other related notions were the discounting of debts and the "present value method". According to Schneider (1981, p. 76) this approach was established by Leibniz in 1682 as a by-product of juridical reflections on the premature repayment of debts (cf. collected works of Leibniz, ed. by Gerhard, 1962/1863)--let us call this "PV Phase I". But it appears that it was Stevin (1582)—also deemed to be the *inventor* of the "income statement"–who was "the first to apply the net present value approach to financial investments" (cf. Chatfield 1996b, p. 208) as well as the annuity method (cf. Volmer 1996, p. 565).[4] Seicht (1970, pp. 341-348, 511-547) points out that the application of the present value approach in accounting (and the *kapitaltheoretische Bilanz*) played an important

[4] According to Parker (1959: 35), it was Jean (Jan) Trenchant (1558) who first used "interest tables" and "discussed geometric progressions and compound interest.

role in the German Railway Statutes of 1863 and subsequent legislations. At any rate, the entire complex of present value calculations proved to be not only crucial for finance and investment theory but, ultimately, held its sway over accountants and accounting scholars.

In the nineteenth century the present value and compound interest calculations were important in the insurance business. In the early 20th century, Ciompa (1910), Kempin (1910), Zappa (1920-29), Heina (1925) and Rieger (1928) also seem to have favoured some form of the present value approach to accounting. And the major extensions of the present value method in the 20th century were numerous. They reached from the formulations of Canning (1929) and particularly of Preinreich (see next Sub-Section) to numerous uses in financial and managerial accounting (from tax and depreciation formulas to compound interest for debts and investments as well as goodwill). These efforts were finally crowned by the sophisticated revisions of the clean surplus theory, as executed by Ohlson (1995) and Feltham and Ohlson (1995, 1996) with the incorporation of stochastic variables (for details see Sub-Section 4.3).

Other original mathematical contributions to the 19th century *theory* of accounting were those by August de Morgan and Giovanni Rossi. De Morgan (1846) was the first to introduce an accounting matrix, and Rossi (1889) offered in his book on “the double-entry chessboard” dozens of examples in which accounting matrices played a decisive role, though neither he nor de Morgan used matrix algebra (conceived by Cayley in 1858—see Cayley, 1989-1897). But Rossi did suggest that the accounting matrix (*lo scacchiere a schede*, as he called it) could be converted into a sophisticated computing device. This might be regarded as anticipating the computerized spreadsheet--though Rossi, like Charles Babbage [1791-1871] the inventor of the first ‘digital’ computer, had a mechanical instead of an electronic device in mind. At any rate, if Stevin and Leibniz were the starting points of present value accounting, Rossi pointed at the future of computerized accounting spreadsheets, both of which developed into important branches of mathematical accounting during the 20th century.

2. Mathematical refinements in the first half of the 20th century

Beyond the classificational-algebraic schemes of the 19th and early 20th century, the impact of inflation and current value accounting (after World War I), particularly in Austria, Germany and France (though also in America by Sweeney) helped to improve algebraic thinking in accounting—though systematic mathematical models were rarely used.

Surprisingly, during the first half of the 20th century the present value (discount) method was by no means readily accepted for accounting purposes. Even such an “academic giant” as Schmalenbach rejected it for statement presentation; he admitted “present values” only for the evaluation of firms in special cases (e.g., assessments, liquidations), but only outside the balance sheet. However, some voices of the time were in favour of employing present values in accounting (we might call this “PV Phase II”). One of the first of those voices was that of Canning (1929) who—stimulated by Irving Fisher’s (1906) use of accounting statements as illustrations for his economic theory—recommended present values for accounting purposes. In

Italy it was Zappa (1937) who saw the problem of income measurement as inseparable from the problem of measuring changes in the present value of capital. But a more sophisticated and mathematical use of the present value approach was made by Preinreich (e.g., 1933, 1936a, 1936b, 1937a, 1937b, 1938, 1939, 1996) in a series of highly original publications that were not only neglected at the time, but literally fell into oblivion until they were slowly rediscovered towards the end of the century (see Brief and Owen 1973; Peasnell 1982, Brief 1996; Brief and Peasnell 1996). Indeed, Gabriel Preinreich may be considered the founding father of modern analytical accounting (including the first phase of the clean surplus theory: CST Phase I). His models were designed for depreciation, amortization and investment purposes, based on compound interest, annuity and present value calculations. They analysed such phenomena as "good will" and the "clean surplus".

Apart from the present value approach, there were geometrical as well as algebraic formulations of different versions of the *break-even chart*, though most of these contributions came from engineers (just as the major contributions to the present value approach derived from economists). The first version of the break-even chart was presented by the German-American engineer Henry Hess [1864-1922] in Hess (1903). Almost at the same time, and independently from Hess, the Scotch chartered accountant Sir John Mann [1863-1955] presented in Mann (1903-07, and particularly in 1904) the same idea. Subsequently, and apparently also independently from their predecessors or even each other, the notion of a break-even chart was introduced by the American engineers C. E. Knoeppel (1920, 1933), linking this graph to *flexible budgeting*, Walter Rautenstrauch [1880-1951] in Rautenstrauch (1922), christening this graph as "break-even chart", and John H. Williams (1922), using not only geometric but also algebraic means to explain various relations of break-even and budgeting analysis (cf. Parker, 1969: 70-71). Such an algebraic analysis can also be found in Rautenstrauch (1930, 1932) and Arthur J. Minor (1930).

3. Analytical Accounting in the Second Half of the 20th Century

The second half of the 20th century introduced a great variety of mathematical approaches and techniques to accounting. First, I shall continue (in 3.1) with the further development and employment of present values in accounting (PV Phase III). Second, there was a widespread use of accounting matrices (that led to computerized spreadsheets) and the application of matrix algebra to accounting theory and practice, particularly in cost and managerial accounting (see 3.2 and 3.5). Third, attempts to axiomatize accounting used set-theoretical and similar mathematical devices to attain rigorous formulations of accounting principles (see 3.3). Fourth, statistical methods (particularly sampling techniques) were introduced to accounting and auditing, and proved (particularly in the latter area) spectacularly successful. Also techniques of statistical hypothesis testing were widely applied in empirical and behavioural accounting theory and, last but not least, operations research models, particularly such techniques as goal-programming, exercised some influence in cost accounting (see 3.4). Related to this are some improvements of the break-event analysis (e.g., Jaedicke, 1961; Charnes et al., 1963).

Out of the concern with accounting matrices arose the idea of building models for computerized budgeting and electronic spreadsheets (see 3.5). Finally, set-theoretical as well as stochastic models were constructed to apply information economics (in the narrow sense), mathematical agency theory, refinements of the clean surplus theory, and similar subjects related to accounting (see 3.6). The last area may, from an intellectual point of view, have been the most exciting achievement. It is for this reason that I have devoted to its further development an entire section (4). Finally, Section 5 is devoted to the conclusion with some speculation about future analytical accounting developments.

3.1 More widespread enthusiasm for present values

During the second half of the century the influence of economics and the emergence of "finance" (as a subject independent of accounting) gave a decisive boost to the further exploration of the present value approach for accounting (including statement presentation). In Continental Europe, Honko (1959) from Finland, Hansen (1962) from Denmark, and Käfer (1962) from Switzerland were among the first who continued to explore this line of research. Later, followed relevant research by Horst Albach (1965) and others in Germany, and Gerhard Seicht (1970) in Austria.

The *synthetic balance sheet theory of* Albach (1965) and the *capital-theoretic balance sheet theory* of Seicht (1970a, pp. 558-619) are both based on the present value approach. Albach's (1965) theory is a *synthesis* of two methods: (1) the valuation of individual *assets* (regarded as past expenditures) *at acquisition costs*; and (2) the valuation of owners equity regarded as *future (expected) receipts* (based on the firm's budget) and discounted at the internal profit rate (*interner Zinsfuß*). In this way the balance sheet became a control instrument for determining the rate at which the firm's budget was on course.

Seicht (1970) was opposed to this approach. His *capital-theoretic* balance sheet theory was claimed to be even more strongly oriented towards the future as he abandoned the acquisition cost basis for the purpose of *asset* valuation. In his notion the firm's assets were stated at the discounted future receipts. Correspondingly, the firm's debts were based on the discounted future expenditures, and the owners equity (but now based on *acquisition costs*) became equated to the discounted future capital re-payments (for details, refer to the original). Here too the discount rate was the internal interest or profit rate (instead of the opportunity cost rate or riskless interest rate used in the more recent American literature). It is interesting to note that in this approach "the sum total of all periodic profits always amounted to the total profit of all periods" (Seicht, 1970, p. 560, translated). This would make Seicht a defender of the clean surplus theory.

In the English accounting literature, the American paper by Alexander (1948) became widely known for its promotion of the present value approach, and so did the paper by Corbin (1962). Furthermore, textbooks, like Moonitz and Jordan (1963) incorporated discussions of discount and present value methods (e.g., for valuation, depreciation, and other purposes). The famous book by Edwards and Bell (1961)--better known for its excellent presentation of the *nominal* and *real* versions of the current (i.e., replacement) value method--also took the present value approach into consideration.[5] Gordon (1960) and Peasnell (1982) took up Preinreich's (1996—posthumous collection) ideas and developed them into the theory of clean surplus (CST Phase II). The major breakthrough in this area, however, came with a series of publications by Ohlson, Feltham and others--to be dealt with separately as CST Phase III (see Sub-Section 4.1).

3.2 Accounting matrices and matrix algebra

As we have seen in Section 1, accounting matrices were already known in the 19th century, but matrix algebra seems to have been used first in macro-accounting by Leontief (1951) and later by Fuerst (1955), then in cost accounting by two Germans, Pichler (1953) and Wenke (1956, 1959), and in general accounting theory by Mattessich (1957, 1964a).

Subsequently, a flood of pertinent publications in the application of matrix algebra, linear and non-linear programming and other mathematical techniques appeared (e.g., Rosenblatt, 1957, 1960); and some fifty of those publications have been collected in Williams and Griffin (1967). Among the books in this area the following must be emphasized: Ijiri's (1965a) dissertation on goal-oriented models, and the publications by Williams and Griffin (1964) and Corcoran (1968) both on mathematical applications in accounting. Furthermore, one might mention two other books. First, the Japanese translation of a collection of papers by several North American authors (Corcoran, Goetz, Mattessich, and Richards) edited by Koshimura (1969) under the translated title *Contributions to matrix accounting.* And second, a Brazilian work by Florentino (1965) under the translated title *Mathematical foundations of accounting and its educational application to programming and bookkeeping analysis.* Later books on matrix accounting were those by Shank (1972) and the doctoral thesis by E. C. Smith (1974).

3.3 Attempts to axiomatize accounting

In the 1950s and early 1960s analytical accounting took a new direction. It concerned attempts to axiomatize accounting theory that arose out of the dissatisfaction with the traditional framework of accounting rules and loosely connected principles. This was preceded by Aukrust's (1955) attempt to axiomatize national income accounting. At a time when related disciplines (such as economics, finance, operations research, etc.) reached for more sophisticated mathematical methods and tools, young scholars felt the need for a more analytical and systematic approach in the construction of a conceptual framework of business accounting. This

[5] Edwards and Bell (1961) also helped to promote algebraic thoughts in accounting, although the book did not present those models of inflation and current value accounting in a comprehensive *mathematical* scheme —for such a model or family of models, see Mattessich (1995, pp. 97-124).

direction was actually pursued by two groups that partly competed, partly cooperated with each other. The first "postulational" direction (preceded by one of the chapters in Paton, 1922, and a paper by Sweeney, 1937) hardly qualifies as a "mathematical-analytical" approach as it avoided mathematical concepts. The other "axiomatic" direction followed a more rigorous methodology with clear assumptions, mathematical theorems and corresponding proofs (yet, in comparison to the more recent trend of *stochastic-analytical* accounting, those earlier applications of mathematical concepts were *deterministic* and relatively moderate—though, nevertheless, heavily attacked, at the time, as "too sophisticated").

The first camp (Moonitz, 1961; Sprouse and Moonitz, 1962; Study Group of the University of Illinois, 1964, Givens, 1966) got its impetus from Chambers (1955, 1957, 1966), while the second one (Winborne, 1962, and Ijiri 1965b, 1967, 1971, 1975) may have been stimulated by the experiments in Mattessich (1957, 1964a). Such publications as Zeff (1982), Slaymaker (1996) and others indicate that some of those endeavours may ultimately have influenced the FASB (Financial Accounting Standards Board, 1978 and subsequent pertinent publications) and led to its conceptual framework and similar attempts. However, this conceptual framework was occasionally criticized (e.g., Archer, 1993) and its limited academic success may well be attributed to the fact that the foundations of such conceptual frameworks were not formulated rigorously enough. On the other hand, a mathematical and axiomatic accounting framework was premature at a time when practitioners were not even ready for the much less demanding mathematics of inflation accounting (cf. Mattessich, 1995, p. 97).

On a purely theoretical level, the second (i.e., mathematical) camp influenced a wide range of scholars in North America as well as abroad in examining, in a fairly rigorous way, the foundations of accounting. In Germany, these efforts included a German version (Mattessich 1970) of *Accounting and analytical methods*, as well as axiomatization attempts by Kosiol (1970, 1978) and Schweitzer (1970, 1972), both using the pagatoric (i.e., cash-flow) basis. The approach of these two authors was criticized by Schneider (1973) for paying insufficient attention to purpose-orientation (*Zwecksetzung*) and to the valuation problem. Indeed, in this neglect lies the decisive difference between the postulational or axiomatic attempts of Kosiol and Schweitzer, that is not explicitly purpose-oriented, and that by Mattessich (1964a, 1970) which has such an orientation. In Australia this search for an axiomatic basis was continued by Wells (1971, 1976), and in Japan by Saito (1972, 1973) with a response by Mattessich (1973).

In the late 1970s this trend was resumed in America and Great Britain as, for example, in a doctoral dissertation by Orbach (1978) and a paper by Tippett (1978). In Italy a similar interest was manifested in Onida (1970) and Galassi (1978), and in the Spanish language area pertinent publications were those by García-García (1972), Requena-Rodríguez (1972), and Buenos-Campos (1974).

Axiomatization and related analytical efforts continued even into the 1980s, 1990s and beyond, though by that time this area no longer occupied centre-stage. In America it was Carleson and Lamb (1981) as well as Mattessich (1995: Chapter 5); in England Willett (1985, 1987, 1988), Gutiérrez (1990, 1992), Gutiérrez and Whittington (1997); in Germany Herde (1992), Balzer and Mattessich (1991, 2000); in the Latin-American countries, Ávila et al. (1988) and Sá (1995); in Japan, Harada (1978), Deguchi and Nakano (1986) as well as Koguchi (1990). The latter author found further applications of this methodology in situations of macro- and meso-accounting that refer primarily to more local environmental accounting issues (apart from his translation of Balzer and Mattessich, 1991, into Japanese in 1994).

3.4 Sampling techniques, hypothesis testing, econometrics, and OR

The application of statistical sampling methods to accounting and, particularly, to auditing (audit risk models) has become one of the most successful mathematical tools of the public accounting profession. Sadly enough, its pioneer Laurence Vance (1950) is hardly ever mentioned (he is neither *profiled* in the encyclopeadic work of Chatfield and Vangermeersch, 1996, nor even listed in its lengthy entry on "Statistical Sampling" by Tucker, 1996)—and this despite the fact that some years later he published in co-authorship a more "popular" version as a textbook (see Vance and Neter, 1956). Yet his work seems to have been eclipsed by more sophisticated sampling techniques as presented in such texts as Trueblood and Cyert (1954), Trueblood and Cooper (1955), Cyert and Trueblood (1957), Stringer (1961), Arkin (1984) and others[6]—or in research papers like Ijiri and Kaplan (1971).

To those efforts might be added the innumerable *empirical* accounting publications employing hypotheses testing and other statistical techniques that are indispensable for this kind of direction. However, to list here even a few representative examples would be hardly appropriate as it would transgress the confines of a survey on *analytical* accounting. Though it is correct that the foundations of statistical hypotheses testing are still mathematical, hence 'analytical', the application of these tools (whether in accounting or elsewhere) is the contrary, namely 'inductive-empirical'. Something similar may hold for the econometric studies of cost behaviour by managerial economists like Dean (1948, 1951, 1954) or Johnston (1960) that are no less inductive-empirical. And as to the promising achievements of empirical accounting research, the latter is likely to attain its full long-run potential only if based on sound analytical models.

Among the Operations Research techniques applied to accounting, particularly to cost accounting, the best example is the two-volume work on goal-programming by Charnes and Cooper (1957, 1961), and Charnes et al. (1963), though some parts of these works might have to be regarded as management model building in a more general sense.

[6] Books on sampling techniques for auditing and accounting have meanwhile become numerous. A recent listing in the WorldCat (internet subscription for libraries) showed no less than 75 books.

3.5 Mathematical accounting models for computer programmes

Another area where the expression "analytical" may be more controversial than that of "mathematical" is the application of computerized spreadsheets to business accounting. The first suggestion of an electronic spreadsheet seems to have been in my paper on budgeting models and system simulation (Mattessich, 1961). The subsequent elaboration of this idea consisted in presenting as a prototype a mathematical budget model of an entire firm in Mattessich (1964a) as well as a complete computer program in Mattessich (1964b). The latter was supplemented by a computer output of some 48 computerized spreadsheets (with sub-budgets for cash flows, labour costs, material costs, purchases, sales, overhead expenses with proper allocations, as well as a projected income statement and balance sheet). That this was the first electronic spreadsheet (in FORTRAN IV for mainframe computers) and the forerunner of such best-selling spreadsheet programmes (for personal computers) of the 1980s, such as *VisiCalc*, *SuperCalc*, *Lotus 1-2-3*, and *Excel*, is acknowledged by MacHaney's (2000) article in the *Encyclopedia of computers and computer history*, vol. II, as well as in other encyclopedic works (see Murphy, 1996), and documented in detail by Mattessich and Galassi (2000).

Apart from electronic spreadsheets, computers have played a long-standing role in accounting, as manifested in such books as McRae (1964) and Li (1968) as well as the many articles of the journal *Computers in Accounting* (1984-1993). In the 1990s related model building led to attempts of introducing *expert systems* to accounting and finance—see, for example, Thierauf (1990), O'Leary (1992), Brown and Wensley (1995).

3.6 The beginning of information economics in accounting

The veritable pioneer of the application of information economics to accounting is G. A. Feltham (1967) with his Berkeley doctoral dissertation and subsequent works (e.g., Feltham, 1968, 1972). Joel Demski, inspired by Feltham (both then at Stanford University) soon joined forces with him in such significant publications as Feltham and Demski (1970) and Demski and Feltham (1972, 1976, 1978) that earned them jointly the highest research award of the American Accounting Association. Related or alternative information approaches to accounting theory (as well as extensions to the original theory) were presented by Butterworth (1967), Lev (1969), Mock (1969, 1971, 1976), Baiman (1975) or the information inductance approach by Prakash and Rappaport (1977), and many others.

Information economics was originally developed by such economists as Stigler (1961, 1962), J. Marschak and Miyasawa (1968), Marschak and Radner (1972), J. Marschak (1974), and others for the purpose of determining the economic value and consequences of information. Feltham goes even further back in time and points out that:

> The development of models of *rational choice under uncertainty* by such pioneers as von Neumann and Morgenstern…can be viewed as the starting point of

> information economics. They demonstrate that if an individual's choice behaviour satisfies a few rather basic "consistency" axioms, then this behaviour can be represented as the maximization of his expected utility for the consequences of the actions available to him. (Feltham, 1984, pp. 181-182).

Thus the roots of information economics go back to Bayesian decision and game theory as well as the probabilistic version of the *states-acts-outcome* matrix, as originally conceived by Savage (1954). The results of such a matrix can serve as a valuable basis of comparison (i.e., telling us what the outcome would be *without* additional information).[7]

However, in the hands of accountants the "information perspective of accounting" (as nowadays called by Christensen and Demski, 2003) covers a much broader area. In the second half of the 1970s, it was noticed that basic agency theory (Alchian and Demsetz, 1972; Williamson et al., 1975; and above all, Jensen and Meckling 1976)--dealing with the contractual relation between "principal" and "agent", various agency costs, risk-sharing and incentive problems—offered a welcome extension of the information perspective to managerial accounting. This resulted in what one might call "analytical agency (or contract) theory" (for detailed survey articles, see Baiman 1984, 1991). This approach introduced a kind of *rationalistic* version of the *stewardship notion* to our discipline and made it the central paradigm of managerial accounting (cf., Gjesdal, 1981)—and this, despite previous attempts to elevate *investment decision-making* to this pivotal role. And for an attempt to relate agency theory to ethics, see Noreen (1988)—for further details on agency theory, see below, Sub-Section 4.2.

Soon afterwards another mathematical (and stochastic) theory, that of "clean surplus"-- as extended by Ohlson (1987a, 1987b, 1990, 1995), Feltham and Ohlson (1995, 1996) and others – found its way into the now broad research area of the information perspective (see Sub-Section 4.1). Indeed, this field has become the dominant area of analytical accounting research. Even a cursory survey of its present status needs to be accommodated in a separate (forth) section.

4. The present status of the information economic perspective

It took some 30 years for this branch to mature to its present status. The reason for this long digestion period lies, partly, in the wearisome process of building the relevant mathematical models, partly, in the meticulous work of the scholars involved in this endeavour. There exist

[7] The most fundamental theorem of information theory was proven by Blackwell (1951, 1953), though "couched in terms of experiments, but various authors have explored its implications for comparing information systems (e.g., Marschak and Miyasawa [1968], Marschak [1974], Marschak and Radner [1972], and McGuire [1972])" (Feltham, 1984, p. 182). Thus the *value* of an *information system* can be derived by comparing the maximized expected utilities of such a system with that of a simple decision system (e.g., based on a probabilistic version of Savage's *states-acts-outcome* matrix). A more pragmatic way to determine this value would be to ask how much people are willing to pay for such an information system.

several papers that offer pertinent surveys (e.g., Feltham, 1984, Mattessich, 2005), but the best and most comprehensive survey are the two volumes of P.O. Christensen and Feltham (2003, 2005). A less comprehensive work, intended as an *introduction* for graduate students to the information perspective, is the book by J.A. Christensen and Demski (2003). Whether these works mark a breakthrough in this field—i.e., extending the interest in this area from a relatively small elite of specialists to a broader or even global interest in analytical accounting—still is to be seen. But the mathematical rigor presented in these publications (particular in the work of the two volumes by Christensen and Feltham with some 250 propositions and their mathematical proofs) indicate that academic accounting may be on a critical threshold. It may be added that in Germany and Austria the books by Ewert and Wagenhofer (1993/2003) and Wagenhofer and Ewert (2003), and in Italy the one by Cilloni (2004) made strides to familiarize accountants with information economics and its relation to our discipline.

The major items of critique advanced against this approach are, first that it is actually not dealing with "accounting" but rather with the "economics of accounting" (as admitted in the title of Christensen and Feltham, 2003, 2005); and second, that it is too esoteric. Indeed, most of its literature as well as the books by Christensen and Demski (2003) and Christensen and Feltham (2003, 2005) operate with a conceptual apparatus very alien to the average accountant and, what may be worse, a neglect of the traditional tools used by accountants. But this hurdle may be overcome to some extent, as proven by two German books on this subject (so far not translated into English). The first is by Ewert and Wagenhofer (1993/2003), dealing mainly with managerial accounting, while the second one, by Wagenhofer and Ewert (2003), concerns financial accounting. Both publications succeeded fairly well in combining major aspects of the information perspective with conventional accounting and, what is most important, without throwing the accustomed vocabulary and concepts overboard.

Another argument, perhaps less valid but interesting, runs as follows: information economics is basically concerned with determining the value of information. That is to say with the question: How much resources ought to be spent in search of information, and when are these costs of searching for information higher than the value of the information itself (cf. Stigler, 1961)? But it does not seem that the information perspective of accounting is applying this maxim of minimizing information costs to its own activity. In other words: Does, perhaps, our building of complex information models cost more than the benefits derived from it? And do we, at all, try to answer such a question? The reason why this argument itself is controversial, is because it refers to *scientific research* where such reasoning can be self-defeating.

4.1 The information perspective vs. traditional accounting

The information perspective as seen by its major pioneers has several features that distinguish it decisively from conventional accounting. Therefore, a rapprochement or reconciliation between these two areas may require some compromise in both quarters. And the first and foremost prerequisite for this, is a clear vision of the major differences between them. We may distinguish two such categories; the first refers to the philosophic and methodological differences, the second to the conceptual or technical ones.

(1) While traditional accounting focuses on *valuation*, viewing it as a pragmatic process that depends on the information objective, the information perspective (particularly as pursued by Christensen and Demski, 2003) seems to reject traditional valuation as the purpose of accounting. It rather sees the function of accounting as *providing information* (thereby emphasizing the *value* or *content* of information and the underlying *uncertainty* that gives information its meaning) as well as revealing where and in which way accounting information (as well as non-accounting information) is produced. This perspective also emphasizes the distinction between *public* and *private* information and the implications of each for financial markets.

Christensen and Feltham (2003, 2005) also stress the distinction between accounting's *decision-facilitating role* (affecting the decision maker's beliefs about the consequences of his actions) vs. its *decision-influencing* role (where anticipated accounting reports concerning a decision maker's actions may influence his action choices). While the former is likely to play a more important role in financial accounting, the latter is usually more relevant in managerial accounting (including agency theory). As to the "philosophic" issue, I believe that at a time when our discipline has greatly expanded in many directions, it is only proper to count the economics of accounting as an integral part of modern accounting.

(2) Whereas the information perspective employs stochastic models, statistical expectations (expressing objective as well as subjective beliefs), formal utility functions, partitions and sub-partitions of the state space, and other sophisticated concepts (of modern economics, probability and decision theory, operations research, game theory, and finance theory), traditional accounting usually disregards most of these developments. A wide variety of model types are being used by the different branches of the information perspective, but as Wagenhofer (2004, p. 9) points out, most "of them belong to the family of games with incomplete information. These games are characterized as having at least two players with different information endowments and potentially different interests. These characteristics seem to fit well with the general setting in which accounting is embedded. The common solution concept for such games is the (Bayesian) Nash equilibrium."[8]

4.2 The modern clean surplus theory

The essence of *clean surplus* is easy to describe. Provided there are no contingent claims (such as convertible debts, executive stock options) involved, this notion refers to an income derived from a comparison of the book value of a firm's owners equity at the beginning of the

[8] A simplistic way to explain the Nash equilibrium is the example of a contractual situation in which one player A knows that player B is cheating, but B knows that A knows it, and so on *ad infinitum*. So each player accommodates himself or herself to this fact, and an equilibrium emerges.

accounting period with the firm's value at the end of this period (eliminating new investments or withdrawals by the owners). It results from the "all-inclusive" income statement where the sum-total of all the annual earnings, from the beginning of a firm (e.g., starting with cash) to its liquidation, equals the sum-total of all annual incomes of the firm's lifetime. This can be expressed in the following equation (though this and the second equation are merely a highly simplified version of the "modern clean surplus theory" as presented by Ohlson, 1995; and Feltham and Ohlson, 1995):

1. The *Clean Surplus Relation* (CSR):
Book value (at *t*) = book value (*t*-1) + income (period ending at *t*) – dividends (at *t*)

In contrast to this, the *conventional* current operating income (or its statements) do *not* provide such a "clean" surplus (or "income"), because the sum of all their annual income figures do not necessarily add up to the firm's life-time earnings. Those "dirty" income statements *exclude many kinds of extraordinary items* (be they capital and non-operating gains or losses, or residuals from past periods, etc.), relegating them to separate surplus statements. But this does not lessen the respectability of such dirty income statements. Indeed, they permit *income smoothing* over several accounting periods (avoiding erratically fluctuating annual income figures) that reveal the long-term profit trend. Apart from resurrecting several pertinent insights of Preinreich,[9] Ohlson argued that only a simple notion of "pure" earnings is economically relevant and sufficient to determine a security's pay-off (i.e., its price plus dividend).

A second notion plays a decisive role in launching this theory. It is that of *residual income* (also called "excess income" or "`abnormal earnings"), defined (under CSR) as "the net income minus a capital charge, which equals the riskless interest rate *i* (i.e., start-of-period interest rate times start-of-period book value) according to Christensen and Feltham (2003, p. 281).

2. *The Residual Income Relation* (RIR):
Market value (at *t*) = Book value (at *t*) + All future "residual income" discounted at rate *i*.

Hence residual income can be regarded as the difference between the firm's income and the cost of capital. This theory, together with further extensions made in Ohlson (1995) and Feltham and Ohlson (1995, 1996), has been called (by Bernard, 1995) a "paradigm-shift" away from the market based research of Ball and Brown (1968) as well as Beaver (1968)—yet, not

[9] E.g., such a statement as: "The yield of common stocks is not dependent upon the corporate dividend policy so long as that policy conforms to the principle of good management" (Preinreich 1936a, p. 137). This is an anticipation of a well-known insight credited to the Nobel laureates Modigliani and Miller (1958) and Miller and Modigliani (1961), known as "Modigliani-Miller dividends irrelevancy". Or "A fundamental truth about accounting is that, given perfect and unlimited foresight, no matter at what value an asset is placed on the books and no matter in what haphazard way it is amortized over its unexpired life, the discounted excess profits plus the recorded value will always give the true fair market value, even though both the investment and the excess profits are measured incorrectly. This statement is a simple theorem of arithmetic." (Preinreich (1937b, p. 220).

only towards a more analytical approach but, perhaps surprisingly, also one more in conformity with *accounting tradition*. As hinted at in Section 1, this theory can be regarded as an elegant extension (in several directions) of the present value model in determining the value of a firm and its shares.

However, the original clean surplus relation (as shown above) still has an important limitation. Its income or dividends referred to future expectations and not to the "past" figures of accounting statements. To overcome this impediment, the first extension introduced by Ohlson (1995) was to relate past figures to future expectations. He created a "linear information dynamics" by adding to the right hand side of the residual income relation RIR (equation 2) a set of *stochastic variables* and *error terms* that imposed a series of restrictions (e.g., risk neutrality of investors) and linearity assumptions about the *probabilistic time-series behaviour* of *abnormal earnings* and of *information about other than abnormal earnings* (e.g., innovations). This "other information" could reflect, for example, the reports of security analysts as well as some unpredictable *random disturbances*. Such a modification attempts to project accounting figures of the past into the future. To what extent these or similar regressions will prove satisfactory has, of course, to be tested empirically.

A second extension might be seen in "objectifying" the original *subjective* present value approach of Fisher and Canning in as far as the RIR equates the *present value* (of all expected residual incomes + present book value that *already* is "discounted") with the *market value* (that is considered *objective*).

Third, Ohlson and Feltham (1995) introduced into this theory a distinction between *financial* and *operational* activities of a firm (here not shown). The latter activities are considered an important residual, after borrowing and lending data are separated--though remaining tied to the first by cash flows--although this distinction between financial and operational activity is admittedly arbitrary.

Fourth, a more important distinction (introduced also by Feltham and Ohlson, 1995, 1996) is that between *unbiased* accounting (based on *economic* valuation) and *conservative* accounting (based on *traditional* accounting valuation). Under unbiased accounting (on average) the market value will equal the book value; whereas under conservative accounting the market value (on average) exceeds the book value. Both of these distinctions constitute significant steps in *bridging* the gap between the information perspective and traditional accounting.[10]

There are, of course, many other analytical as well as empirical attempts to develop sufficiently accurate means of predicting future earnings (for details, see Lee, 1999). After all, such predictions are the central object of any modern value theory of the firm. But the extended clean surplus theory of Feltham-Ohlson has, so far, been the most popular among the many attempts. Even commercial variations of this model have been marketed under such trademarks

[10]Cf. Bernard (1995, p. 733): "These studies return to issues so basic as to render them direct descendants of work done no later than the 1960s....Ohlson (1995) and Feltham and Ohlson (1995) represent the base of a branch of capital market research might have followed, but did not."

and brand names as: Stern Stewards EVA, Hot Value Associate's CFROI, McKinsey Economic Profit Model, etc.

4.4 Mathematical agency theory

As pointed out above, agency theory is now a well-established part of the information perspective (as information plays a decisive role in entering into a specific agency contract as well as in monitoring the agent's activity). Some experts may regard the original part of the information perspective (plus some parts of the extended clean surplus theory) as closer to financial accounting, while the agency part corresponds more to managerial accounting. This seems to be reinforced by the division of the work by Christensen and Feltham (2003, 2005) where the first volume is devoted to "information in markets", and the second to "performance evaluation".

Agency theory is probably that part of the information perspective most familiar to most accounting academics. Thus I merely recapitulate its major activities and notions. Agency theory attempts to determine (depending on individual circumstances) the appropriate type of contract between principal and agent (usually between owner and manager). Thereby the following notions play an important role: first best vs. second best contracts, performance and reporting incentives through bonus plans, stock options, etc., (requiring incentive compatibility between both parties), performance evaluation, and asymmetry of information (often the agent has more information than the principal). Further key notions are: the danger of deliberately supplying misinformation, moral hazard, risk preference (e.g., *risk taking* by the principal vs. *risk aversion* of agent), observability vs. unobservability or indirect observability of the agent's performance, monitoring the agent's activity through surprise inspections or auditing activities, agency costs (due to either employing monitoring devices or, alternatively, income loss by the principal due to insufficient monitoring). Finally, the distinction between private vs. public information plays an important part, as does the variety of information sources, the mixing and separation of different information sources, earnings management (using inter-temporal accruals or income smoothing for shifting profits from one period to another by the agent), and so on.

Depending on the particular organizational structure, these numerous notions offer a rich pallet for designing mathematical models supposed to reflect the first-best or second-best management contracts. The alternative combinations are large, and so are the potential model types--for a survey of this literature, see Christensen and Feltham, 2005).

5. Summary and Conclusion

The influence of mathematics and analytical thinking on accounting has come a long way—and I do not refer so much to the past five thousand years, rather to the last 50 years or so. Of course, originally it may have begun with the token-envelope accounting of the Sumerians with simple *clay tokens* (representing individual assets) and their *impressions on clay envelopes* (as an inseparable representation of the corresponding equity) or with later classification schemes. In classical and medieval times (not to forget India and the Arab world) as well as during the Renaissance it was arithmetic that dominated the scene. From the 16th century onwards valuation issues based on discounted debts, annuity methods and present values occasionally touched our discipline (Stevin, Leibniz). Finally, in the 19th century algebra played an increasing role in the teaching of bookkeeping and in conveying different classification schemes. During this period, accountants became more interested in compound interest and present value calculations. The notion of fluctuating prices and price levels (particularly during rampant inflations) also led to algebraic thinking in accounting. All this opened the door to a more general and abstract vision of our discipline during the first half of the 20th century--with scholars like Canning promoting present values, and Preinreich pioneering pertinent model building in accounting and finance.

But only the second half of the 20th century opened the floodgates of our discipline to the use of higher mathematics on a broad scale. Accounting academics of this period were not merely more receptive towards employing the present value method for statement presentation, they also experimented with a wide variety of mathematical techniques and concepts. Above all, this period began with attempts to axiomatize accounting in order to replace the loose set of accounting rules or principles with a more rigorous foundation. The use of accounting matrices and models led to increasing interest in matrix algebra as well as linear and non-linear algebra in general. The consequence of this type of *deterministic* model building led inevitably to more comprehensive accounting models that proved crucial for the development of computerized spreadsheets and electronic budgeting. Furthermore, operations research techniques were applied to cost and managerial accounting, and statistical sampling methods became a highly successful tool for accounting and, particularly, for auditing practice. Other statistical techniques, such as hypothesis testing played an indispensable role in modern empirical accounting research—that, however, already transgresses into the area of non-analytical accounting.

Since the late 1960s information economics played a steadily increasing role in analytical accounting research. This application was introduced by a series of doctoral dissertations at the University of California at Berkeley (among which that by Feltham proved to be the most significant launching pad). Originally the determination of the *value and content* of information (or of an entire information system) formed core and pivot. As time went by, and non-mathematical agency theory came about, agency theory was “mathematized” and incorporated into the information perspective. Finally, the clean surplus theory (particularly as revised by Ohlson and Feltham since the mid-1990s) became an integral part of the information perspective of accounting. At the beginning of the 21st century the advent of such graduate texts as J. A. Christensen and Demski (2003) and P. O. Chrsistensen and Feltham (2003, 2005) in North America, as well as the German texts by Ewert and Wagenhofer (1993/2003) and

Wagenhofer and Ewert (2003) have summarized the major tenets of this sub-field, and made it, hopefully, more palatable to a larger audience of academic accountants.

Although the information perspective constitutes a formidable intellectual achievement of analytical accounting, it is not without its critiques. As it is more "economics of accounting" (than accounting in the traditional sense), and as it requires considerable prerequisites of mathematics, finance and economics, it is inaccessible not only to the majority of practitioners but also to many accounting academics. Surprisingly, the two German texts (see paragraph above) have been more successful in integrating major aspects of the information perspective with traditional accounting than were the major American texts. The latter continued to disregard conventional accounting terminology and made little effort to explain their insights in a language accessible to most accountants--though it has to be emphasized that this sub-discipline aims foremost towards a general and theoretical understanding than to practical know-how.

Attempts have been made (and are continued to be made) to factually verify some of the analytical propositions of this sub-area. But it may be too early to assess the empirical relevance of many of them. It still has to be seen to what extent this achievement will influence or even determine the future of accounting as an applied and normative discipline. However, to close on a positive note, let me quote a summary of the major aspects of analytical accounting as presented by Wagenhofer:

> *The major advantage of analytical research lies in its precision and the rigor of its analysis, its explicit assumptions, and the logic behind its results. The results can be intersubjectively confirmed. This methodology is particularly useful for gaining insights into situations that are characterized by the strategic interactions of different decision-makers with information asymmetry and potentially diverging interests. The resulting equilibria often include prima facie counterintuitive results, and uncover important conditions that must be present for certain results to hold. The best justification for the use of a model is if the result is surprising relative to prior knowledge*. (Wagenhofer, 2004, p. 26).

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