The occurrence of prime numbers revisited

Ernesto Tapia Moore
ernesto.tapia-moore@kedgebs.com
e.tapiamoore@odipme.org
Kedge Business School

José Tapia Yañez
j.tapiayanez@odipme.org
DMV
Universidad de Chile (Retired)

ABSTRACT

Based on an arithmetical and autocatalytic approach, the authors propose a solution for the occurrence of prime numbers. Exact arithmetical calculations are provided for: the closest prime to any given positive integer (or any number of bigger or smaller primes from that integer); the quantity of prime (and composite) numbers between 1 and any positive integer; the quantity of prime (and composite) numbers between any two positive integers.

KEYWORDS: Occurrence of prime numbers, occurrence of composite numbers, autocatalysis, activator-inhibitor systems.

INTRODUCTION

The occurrence of prime numbers has been described as "a mystery into which the mind will never penetrate" (Euler in Blinder, 2008, p. 20). Alan Turing was one of the main contributors to the promotion of this concept, to the point that most business transactions executed via electronic media are encoded through the use of prime numbers, mostly based on their seemingly unpredictable occurrence.

Research on the occurrence of prime numbers follows the lines of three different streams: Prescriptive (probabilistic and non-probabilistic), descriptive, and applicative. The probabilistic stream is mainly fueled by business cryptology, especially electronic transactions requiring risk exposure predictions targeting the probability of decrypting a prime-encoded sequence during the time it takes to complete the transaction (Kadane & O'Hagan, 1995; Kelly & Pilling, 2001; Holdom, 2009). Non-probabilistic research, has not been able to solve issues related to integers 2 and 3, although providing slow solutions for the occurrence of primes ≥ 5 (Soundararajan, 2006; Arpe & La Chioma, 2008; Park & Youn, 2012). Descriptive research deals mainly with the prime number phenomenon itself (Holt, 2007; Cusick, Yuan, & Stanica, 2008; Selvam, 2008; Cristea & Darafsheh, 2010). Applicative research mainly provides solutions for cryptologic applications (Luo, 1989; Quesada, Pritchard, & James Iii, 1992; Arnault, 1995; Ker-Chang Chang & Hwang, 2000; Jones & Moore, 2002).

Our work, although non-probabilistic and descriptive, does not fit in any of the above streams, as we have sought to reconsider the problem from an arithmetical point of view, wherein a multiplication is a simplified sum of the same integer. We first explored the partition of natural numbers finding many of the classic patterns described in extant literature, noting that none of these provided a satisfying solution beyond their announced limits. The simple evidence provided in the form of the partition, however, primed us to pursue in the direction of Turing's work, provided one can focus the problem as a dynamic system where only first-time and unique occurrences are allowed, all other occurrences being inhibited by the first. We have found that the occurrence of prime numbers can be solved by a process similar to Alan Turing's two-component reaction-diffusion system (1952). We have observed that the occurrence of composite and prime numbers is predictable yet irregular. Furthermore, we have found that the process behind the occurrence of prime numbers also follows a regular pattern.

In its simplest form, our work is a form of autocatalytic sieving. Our work, however, does solve the problem of calculating the closest prime to any given positive integer; the problem of enumerating the primes located close to any given positive integer as well as those located in between any two positive integers. In doing so, the time lag associated to calculating the prime elements of prime-based encryption keys used for electronic transactions is reduced to that of solving a simple three variable containing one unknown equation. This is not new: It is believed that the complexity of the occurrence of primes is such that the time required to identify which prime is used for the encryption key takes longer than the transaction itself. This belief may need to be revised.

AUTOCATALYSIS

Reaction-diffusion systems, also known as autocatalysis, are part of the activator-inhibitor systems where the "activator generates more of itself by autocatalysis, and also activates the inhibitor. The inhibitor disrupts the autocatalytic formation of the activator. Meanwhile, the two substances diffuse through the system at different rates, with the inhibitor migrating faster" (Ball, 1999, p. 80).

The two-component reaction-diffusion system specific to generating prime numbers follows the structure of a subcritical Turing bifurcation, in two interdependent stages, pertinent to two positive integer categories.

In the first stage, the cardinal ("$|c|$") period ($|z|; z \in Z^+$) generates a first set of activators, which beyond a certain point will generate their inhibitors in a linear monotonic fashion at the ordinal level. The remaining activators found in the area of first-stage inhibition, will generate their own inhibitors in a rhythmic alternating-period manner.

The second-stage inhibition is multi-linear, yet is better understood under planear settings (explaining the use of a cardinal-ordinal notation $|z| \cdot z$, simplified to $|z|z$). The remaining set of non-inhibited second-stage activator positive integers coincides with the set of all prime numbers.

Conceptually, the "root" of the system is $|z|1$, also named the meta-integer. It determines the smallest interval of the scale in which the phenomenon is observed.

Autocatalysis happens in stages. The first stage typically offers a stable zone, followed by a second turbulent or hysteretic one.

First stage autocatalysis

We refer to this stage as the "proto-autocatalysis." It has a period of $|1|z$. It offers a clear boundary between the stable and the hysteretic regions. Furthermore, the configuration of the stable region does not offer any room for the appearance of an inhibitor. The characteristics of this stage are the following:

- The boundary is situated at $|1|2^2$;
- The stable region is $< |1|2^2$ (includes $|2|1$ and $|3|1$);
- The hysteretic region is $\geq |1|2^2$.

The mechanics of this first stage are as follows:

- First $|2|1$ occurs and then "autocatalytically migrates" at $|2|1$ intervals towards $+\infty$. As it proceeds, it inhibits all $|2|z > |2|1$ at $|1|z$ level, effectively inhibiting all even integers;
- Then, $|3|1$ occurs and following the same mechanism inhibits all larger $|3|z$, or multiples of 3 at the $|1|z$ level, amongst the remaining non-inhibited $|1|z$ integers.

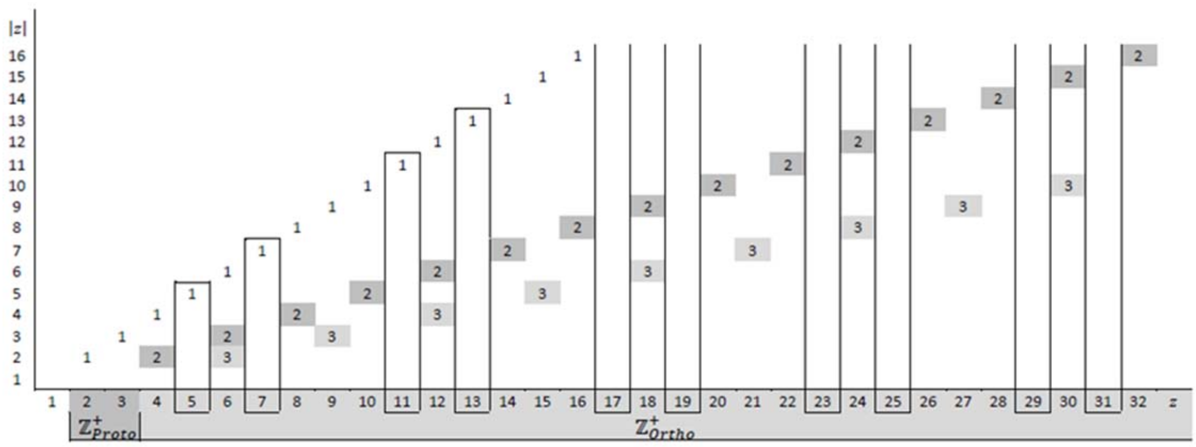
Once the first-stage inhibition has taken place, the remaining integers are all odd, are not multiples of 3, and are all located in the hysteretic region of this first autocatalysis. We refer to these remaining integers as "alphas" (α). Namely: $Z^+ \backslash \{ |2|z \ , |3|z\} = |\alpha|z$.

Alphas present themselves in pairs. We designate the first α of a pair α'; the second α of the same pair is written α". The interval between α' and α'' is 2; so that $\alpha' + 2 = \alpha''$. The interval between two neighboring pairs of alphas is 2^2. That is: the second alpha (α'') of a given pair of alphas is at $|z|2^2$ from the first alpha (α') of the following pair of alphas. This generates the sequence of $|1|\alpha$: 5, 7, 11, 13, 17, 19, 23, 25, 29, 31, 35, 37, …, +∞.

Consequently, the occurrence of alphas in the hysteretic region of the proto-autocatalysis, has an alternating period of $|z|2$ and $|z|2^2$.

Furthermore: $|\alpha_1| = |z|5 \, , |\alpha_2| = |z|7 \, , |\alpha_3| = |z|11 \, , |\alpha_4| = |z|13 \, , |\alpha_5| = |z|17 \, , |\alpha_6| = |z|19, \ldots$ allowing for an ordinal sequence of alphas ($\alpha_1 \, , \alpha_2 \, , \alpha_3 \, , \alpha_4 \, , \alpha_5 \, , \alpha_6 \, , \ldots , \alpha_{+\infty}$).

Fig. 1: First-stage *proto*-integer autocatalysis generating pairs of alphas in the *ortho*-integer, or *proto*-hysteretic, region.



Second stage autocatalysis

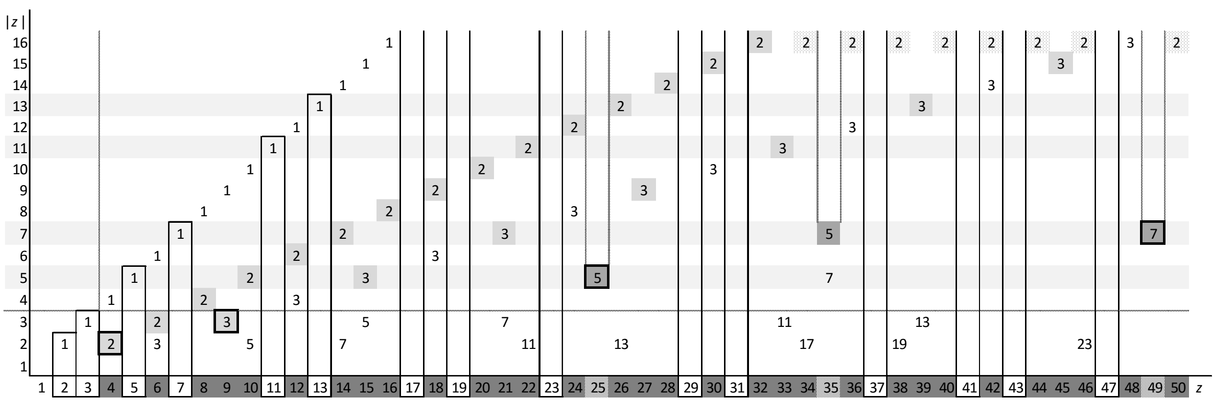
We refer to this stage as the "ortho-autocatalysis." It is located in the proto-hysteretic region. The activators in this stage are alphas generated in the preceding one. Alphas behave as autosolitons in the proto-hysteretic region, coexisting with the $|z|1$ periods of both proto and ortho-autocatalysis.

Alphas generate ortho-inhibitors, referred to hereafter as "betas" (β), where $|\alpha_i|\alpha_j = \beta_m$. Inhibitions occur when $\alpha = \beta$, that is when $\alpha - \beta = 0$.

Alphas possess the same characteristics as the activators of the proto-autocatalysis:

- Their boundary is situated at $|\alpha^2| = |\alpha_i|\alpha_i = \alpha^2 = |\beta_1|$;
- their stable region is $< |\alpha^2|$;
- their hysteretic region is $\geq |\alpha^2|$.

Fig. 2: First and second stage autocatalysis. The second stage inhibitions are dashed-out. Please note the boxed-in *z*² boundaries separating stable and hysteretic regions in the $|z|z$ plane.



Furthermore, betas have the same periodic characteristics as alphas. Likewise, betas occur in pairs, so that $|\alpha|\beta' + |\alpha|2 = |\alpha|\beta''$ and the second beta (β'') of a given pair of alphas is at $|\alpha|2^2$ from the first beta (β') of the following pair of betas.

Like alphas, betas allow for an ordinal sequence ($\beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \beta_6, \dots, \beta_{+\infty}$) where:

$$|\beta_1| = |\alpha_i|\alpha_1 = \alpha^2, |\beta_2| = |\alpha_i|\alpha_2, |\beta_3| = |\alpha_i|\alpha_3, |\beta_4| = |\alpha_i|\alpha_4, |\beta_5| = |\alpha_i|\alpha_5, |\beta_6| = |\alpha_i|\alpha_6$$

Also, when $|\beta_1|$ is generated by an odd alpha (such as $\alpha_1, \alpha_3, \alpha_5, \alpha_{7,}$ ...) the period between $|\alpha_i|\beta_1$ and $|\alpha_i|\beta_2$ is $|\alpha_i|2$, meaning this is the first beta of a pair ($|\alpha_i|\beta'$). When $|\beta_1|$ is generated by an even alpha (such as $\alpha_2, \alpha_4, \alpha_6, \alpha_{8,}$ ...) the period between $|\alpha_j|\beta_1$ and $|\alpha_j|\beta_2$ is $|\alpha_j|2^2$, meaning this is the second beta of a pair ($|\alpha_j|\beta''$).

However, unlike the activators of the first stage, alphas do not share a common stable region. The stable regions of alphas include proto-inhibitions as well as any lesser alpha inhibitions.

For example:

- When $\alpha = 5 = \alpha_1$:

  $|\alpha_1|z$ generates (in $|1|$ values): 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, … Where 10,15, 20, 30, 40, 45, 50, and 60 are the first $|2|z$ and $|3|z$ inhibitions; 25 is the boundary ($|\beta_1|$); 35, 55, and 65, are the first of the subsequent betas generated by $|\alpha_1|\alpha_i$; inhibitions 10, 15, 20 are proto-autocatalytic occurring before the boundary. There are no ortho-autocatalysis inhibitions for $|\alpha_1|$.

- When $\alpha = 7 = \alpha_2$:

  $|\alpha_2|z$ generates (in $|1|$ values): 7, 14, 21, 28, 35, 42, 49, 56, 63, 70, 77, 84, 91, … Where 14, 21, 28, 35, and 42 are the first $|2|z$ and $|3|z$ inhibitions; 49 is the boundary ($|\beta_1|$); 77 and 91 are the following betas generated by $|\alpha_1|\alpha_i$; inhibitions 14, 21, 28, 35, 42 are the proto-autocatalytic inhibitions occurring before the boundary. There is only one ortho-autocatalytic inhibition for $|\alpha_2|$ in its stable region, which is $|\alpha_2|\alpha_1 = |\alpha_1|\beta_2 = |1|35$.

Likewise, the ortho-autocatalytic inhibitions in the stable region of the following greater alphas are:

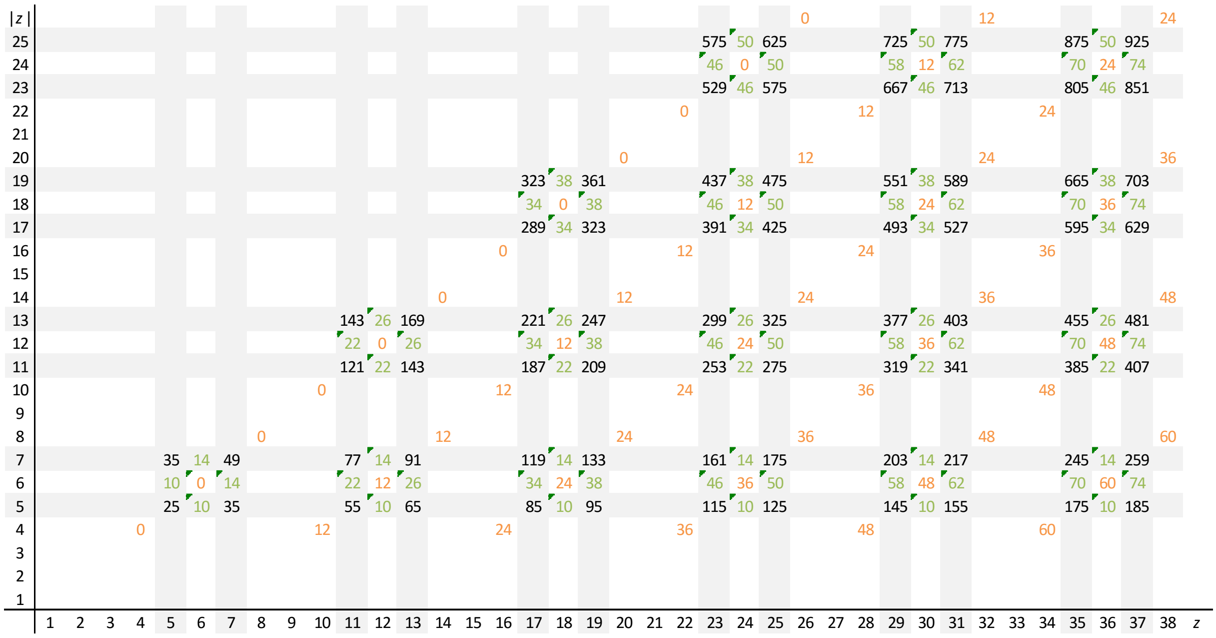
- $|\alpha_3|$ ($|1|11$): $|\alpha_3|\alpha_1 = |\alpha_1|\beta_3 = |1|55$, and $|\alpha_3|\alpha_2 = |\alpha_2|\beta_2 = |1|77$;
- $|\alpha_4|$ ($|1|13$): $|\alpha_4|\alpha_1 = |\alpha_1|\beta_4 = |1|65$, $|\alpha_4|\alpha_2 = |\alpha_2|\beta_3 = |1|91$, and

  $|\alpha_4|\alpha_3 = |\alpha_3|\beta_2 = |1|143$;
- $|\alpha_5|$ ($|1|17$): $|\alpha_5|\alpha_1 = |\alpha_1|\beta_5 = |1|85$, $|\alpha_5|\alpha_2 = |\alpha_2|\beta_4 = |1|119$,
- $|\alpha_5|\alpha_3 = |\alpha_3|\beta_3 = |1|187$, and $|\alpha_5|\alpha_4 = |\alpha_4|\beta_2 = |1|221$;

And so on.

Betas positioned in a $|\alpha|\alpha$ plane form commutative diagrams, where the $|\alpha|2$ period betas generate $|2^2 \cdot 3|z$ (or $|1|12 \cdot z$) as a commutative constant, and $|\alpha|2^2$ period betas generate $|2^3 \cdot 3|z$ (or $|1|24 \cdot z$) commutative constant values, both following the z diagonals in the said plane.

Fig. 3: Beta values (in black) and their commutative diagram values.



The remaining uninhibited alphas ($\alpha\backslash\beta$) are not composite integers ($k;\ k_i \in K$), therefore uninhibited alphas are all prime integers ($\pi;\ \pi_i \in \Pi$). Inhibitors are all composite integers.

THEOREM

Synthesizing the developments of both sections above, we state:

$$Z^+ \equiv K \cup \Pi;\ K \equiv \{\ |2|z\,, |3|z\,, |\beta|z\ \} \ \therefore\ Z^+ \backslash \{\ |2|z\,, |3|z\,, |\beta|z\ \} \equiv \Pi$$

Given $Z^+ \backslash \{\ |2|z\ \ , |3|z\} = |\alpha|z$ and $|\beta|z \in |\alpha|z$, our theorem is therefore:

$$\forall\ |\alpha|z \setminus |\beta|z \equiv \Pi$$

Consequently, our two-stage diffusion-reaction system yields prime and composite numbers in a predictable and exact manner.

PROOF

We first solve the question of knowing if a positive integer $z \geq 4$ is prime ($\pi\ ;\ \pi \in \Pi$) or not. For z to be prime, it must satisfy two conditions:

- Pass $z = \alpha$ (alpha test), because alphas contain all prime integers and some composite;
- Fail $z = \beta$ (beta test), because betas are all composite integers.

Previous screening of numbers whose unit digit is even or 5 will avoid unnecessary calculations, as these are all multiples of 2 and 5. This leaves numbers ending in $1, 3, 7, 9$.

Alpha test

Alphas are all neither even nor multiples of 3 ($\alpha \notin \{|2|z, |3|z\}$). Assuming z is odd, if the result of $\{z/3\} \in Q$, then $z \notin |3|z$ and is therefore an alpha. In other words, having excluded all even integers, the alpha test is simply:

$$z/3 \stackrel{?}{=} q \; ; \; q \in Q \tag{1}$$

The result is interpreted as follows:

- $\{z/3 \in Q\} \Rightarrow z = \alpha \, ; \alpha \in A \ni z$
- $\{z/3 \notin Q\} \Rightarrow z \neq \alpha \, ; z \in |3|z$

If $z \in A$, we may then proceed on to the beta test because z could be prime, provided $z \geq |1|25$.

If $z \in A$ and if $z < |1|25$, the test would end here since the small value of z implies it cannot enter the beta test, because the smallest value of β is $|1|25$. Consequently, $\forall \, \{\alpha < 25\} \in \Pi$.

Otherwise, if $z \notin A$, z would be composite, z would have failed the alpha test, and no further testing would be required as the first condition would not have been satisfied.

Beta test:

Betas are all products of alphas. In order for z to be a beta, two conditions must be met:

- $z \in A$, as tested above;
- $z \geq |1|25$, because the first beta has a value of $|1|25 = |1|\alpha_1^2 = |1|\beta_1$.

Considering the highest inhibition a single alpha can perform is $|z|\alpha_i^2 = |z|\beta_i$, when seeking to test $z_i \stackrel{?}{=} \beta_h$ where one does not know the values of betas, one must reason in terms of products of alphas, where $\alpha_i \cdot \alpha_j = \beta_h$ and $|1|\alpha_j^2 = |1|\beta_l$ with a $|\alpha_j|1$ "ceiling" calculated as follows:

$$\lfloor |\sqrt{z}|1 \rfloor \cong |\alpha_j|1 \tag{2}$$

All beta inhibitions occurring in a $|\lfloor \sqrt{z} \rfloor|z \cdot |\alpha_1|z$ rectangular plane, we seek to know how many alphas are found between $|\alpha_1|1$ and the $\lfloor |\sqrt{z}|1 \rfloor$ ceiling. Given that alphas occur in an alternate period for which the average period is 3, the sought quantity of alphas is determined as follows:

$$\lfloor |\sqrt{z}|1/3 \rfloor = \gamma \tag{3}$$

However, gamma will not indicate the closest $|\alpha_i|1$ to the ceiling since $\alpha_\gamma \cdot 3 \neq \lfloor |\sqrt{z}|1 \rfloor$. In order to find the $|\alpha_i|1$ value of the closest α found near the $|z|1$ position of gamma, one needs to observe if $\lfloor |\sqrt{z}|1 \rfloor$ is even or odd.

If even, $\lfloor |\sqrt{z}|1 \rfloor$ may be located in between α' and α'' of a given pair of alphas, or immediately before α', or immediately after α''.

If $\lfloor |\sqrt{z}|1 \rfloor$ is odd, it may be located equidistantly between two pairs of alphas (and is a multiple of 3), or can be one of the two alphas in a pair. The closest alpha can be then found by adding 1 to an even $\lfloor |\sqrt{z}|1 \rfloor$ and testing for an alpha as per above, or if $\lfloor |\sqrt{z}|1 \rfloor$ is odd, by simply dividing by 3.

Testing for the position of $\lfloor |\sqrt{z}|1 \rfloor$ within a pair of alphas including the found alpha is done by adding 2 to $\lfloor |\sqrt{z}|1 \rfloor$, and running the alpha test. We name the closest alpha to $|\lfloor \sqrt{z} \rfloor|1$ "α_γ", so that $\{\lfloor |\sqrt{z}|1 \rfloor - 3\} \leq \alpha_\gamma$.

Gamma also indicates the maximum number of iterations the beta test hereunder can be run with the $|1|\alpha_i$ values ranging from the closest alpha to $\lfloor |\sqrt{z}|1 \rfloor$ down to $|\alpha_1|\alpha_i$. Furthermore, the pertinent betas liable to inhibit z are limited to a very few, and the number of these few is shown by gamma, hence the importance of finding the closest alpha to $\lfloor|\sqrt{z}|1\rfloor$. The pertinent $|\alpha_i|\alpha_j$ beta interactions being limited to gamma, are found within the $|\lfloor\sqrt{z}\rfloor|1 \cdot |\alpha_1|z$ plane, along a hemiparabolic concave path of the closest beta to z, starting at $|\lfloor\sqrt{z}\rfloor|1$, and ending in $|\alpha_1|z$. One could eventually extend the path to include $|3|z$ and $|2|z$ in the proto autocatalysis zone, if the alpha test had not been performed beforehand.

Since $|\alpha_i|\alpha_k = \beta_h$, that we know the value of $|\alpha_\gamma|1$, and that the inhibition of a beta occurs when $\beta_h = z$, then the beta test is:

$$\{z/\alpha_\gamma\} \stackrel{?}{=} \alpha_i \tag{4}$$

If $\{z/\alpha_\gamma\} \notin Q$, then $\{z/\alpha_\gamma\} = \alpha_i \Rightarrow \ |\alpha_\gamma|\alpha_i = z\,;\beta_h = z\,; z$ is composite because β_h inhibits z.

If $\{z/\alpha_\gamma\} \in Q$, then $\{z/\alpha_\gamma\} \neq \alpha_i \Rightarrow \ \beta_h \neq z\,; z$ is not inhibited by any $|\alpha_\gamma|\alpha_i = \beta_h$ and may be prime.

In order to test the inhibition of all of the betas along their inhibition path, we simply place z as the numerator, and $|\alpha_j|$ as the denominator, where j will range from gamma to 1, in decrements of 1.

For example, let $z = 2003$:

- $\lfloor|\sqrt{z}|1\rfloor = 44 \cong |\alpha_j|1$
- $\lfloor|\sqrt{z}|1\rfloor + 1 = 45$ and $45/3 = 15 \ \notin Q \Rightarrow 45 \neq \alpha$
- $\lfloor|\sqrt{z}|1\rfloor - 1 = 43$ and $43/3 = 14\,.\overline{33} \in Q \Rightarrow 43 = \alpha$
- $\lfloor|\sqrt{z}|1/3\rfloor = \gamma = 14 \Rightarrow \alpha_\gamma = \alpha_{14} = 43 = \alpha''$

There will be 13 decrements to calculate using (4): $\{z/\alpha_\gamma\} = 2003/43 = 46.58139 \in Q \neq \ \alpha_i$

- $\alpha_{\gamma-1} = \alpha_{13} = 41 = \alpha'$; $\{z/\alpha_{\gamma-1}\} = 2003/41 = 48.85365 \in Q \neq \ \alpha_i$
- $\alpha_{\gamma-2} = \alpha_{12} = 37 = \alpha''$; $\{z/\alpha_{\gamma-2}\} = 2003/37 = 54.13513 \in Q \neq \ \alpha_i$
- $\alpha_{\gamma-3} = \alpha_{11} = 35 = \alpha'$; $\{z/\alpha_{\gamma-3}\} = 2003/35 = 57.22857 \in Q \neq \ \alpha_i$

And so on, until either $\{z/\alpha_{\gamma-n}\} \notin Q$, indicating z is composite, or $\alpha_{\gamma-n} = \alpha_{\gamma-13} = \alpha_1$ is reached with all $\{z/\alpha_{\gamma-n}\} \in Q$, indicating z is prime.

In the present example, all $\{z/\alpha_\gamma\,, z/\alpha_{\gamma-n}\} \in Q$ indicating 2003 is not inhibited by any beta, and therefore, z is prime. In the case of the next alpha, 2009, the results of $\{z/\alpha_{\gamma-n}\}$ show that both $\{\alpha_{13}\,(41)\,, \alpha_2(7)\} \notin Q$, meaning 2009 has two betas $\{|41|49\,, |7|287\}$ inhibiting it, and therefore, it is a composite integer.

As mentioned above, all alphas with an odd index are the first of a pair, and those with an even index are the second of a pair. Consequently, going from one alpha to another amounts to merely adding or subtracting 2 or 4 from the pertinent alpha, depending of which alpha (α', α'') one is starting from, and in which direction one wishes to move.

When none of the $|\alpha_i|\alpha_j$ interactions produce a positive integer, z is said to have failed the beta test. Consequently z cannot be a composite integer and therefore must be a prime one.

Closest neighboring prime

The next question we solve is identifying the neighboring primes to any given positive integer ≥ 25.

In order to achieve this, we must first identify the neighboring alphas, and then check if they are beta-inhibited. The closest non-beta inhibited alpha is the closest prime. We have shown how to do this in the above section.

Number of primes between 1 and z_i

This is obtained simply by first counting how many alphas are found between 1 and z_i; second, by counting how many pertinent betas are found in the same area; third, by subtracting the number of pertinent betas from the total of alphas, since $Z^+ \backslash \{|2|z_h, |3|z_h, \beta\} \equiv \Pi \,; z_h, \beta < z_i$. The pertinent betas are those whose $|1|\beta$ value is inferior or equal to $|1|z_i$. We refer to the number of primes as epsilon (ε) where $\Delta_{\beta_h}^{\alpha_i} = \varepsilon$.

A straight-forward calculation is possible when $z_i < 175$, because before this integer, only two alphas can produce a beta. Considering $175 = 5 \cdot 5 \cdot 7 = 25 \cdot 7 = 5 \cdot 35$, two different alpha factors produce a same beta. We call these "multi-beta values" because a same beta value will have more than two alpha factors, and therefore $\beta_i = |\alpha_h|\alpha_i = |\alpha_i|\alpha_j$. Consequently, past 175, a quantitative approach will not produce the exact numbers of primes. We are currently working on a more elegant "qualitative" approach through value, other than systematically testing $\beta_{|\alpha_h|\alpha_i} - \beta_{|\alpha_i|\alpha_j} \stackrel{?}{=} 0$ and related subroutines to sum the number of $|\alpha_i|\alpha_j$with a same $\beta_i$value needed to add back into our above epsilon in order to obtain an exact count.

Number of primes between z_i and z_j

This is computed by first calculating the number of number of primes from 1 to z_i obtaining ε_i. We do the same for z_j, generating ε_j, where $\varepsilon_i < \varepsilon_j$. Then $\Delta_{\varepsilon_j}^{\varepsilon_i} = \varepsilon_k$, renders the quantity of primes between two given positive integers.

FURTHER RESEARCH AND CONCLUSION

One can consider the autosoliton phenomenon as a form of sieving, where the autosoliton has a period of $|\pi|1$; the stable region ranges from $|\pi|1$ to $|\pi|\pi - 1$ and includes the inhibitions of smaller primes going from $|\pi|2$ to $|\pi|\pi - 1$; the hysteretic region begins at $|\pi|\pi$. This is seen respectively at $|1|z$ as $|1|\pi$; $|1|\pi$ to $|1|\pi^2 - 1$; and finally, $|1|\pi^2$. Our autosoliton approach may give further insight to the algorithms of Luo (1989) as well as Quesada *et al* (1992), and should allow for their increased speed and efficiency.

Some interesting phenomena appear in the occurrence of betas ($\beta \geq 175$) of more than 3 multiplicands such as $\{\alpha_i^n \cdot \alpha_j\}$ or $\{(\alpha_h \cdot \alpha_i) \cdot \alpha_j\}$, which seem to follow a pattern. For example, the first thirty of this subset of composites are:

Table 1: First thirty multi-beta values.

175	=	$5 \cdot 5 \cdot 7$	595	=	$5 \cdot 7 \cdot 17$	845	=	$5 \cdot 13^2$
245	=	$5 \cdot 5 \cdot 7$	605	=	$5 \cdot 11^2$	847	=	$7 \cdot 11^2$
275	=	$5 \cdot 5 \cdot 11$	625	=	5^4	925	=	$5^2 \cdot 37$
325	=	$5 \cdot 5 \cdot 13$	637	=	$7^2 \cdot 13$	931	=	$7^2 \cdot 19$
385	=	$5 \cdot 7 \cdot 11$	665	=	$5 \cdot 7 \cdot 19$	935	=	$5 \cdot 11 \cdot 17$
425	=	$5 \cdot 5 \cdot 17$	715	=	$5 \cdot 11 \cdot 13$	1001	=	$7 \cdot 11 \cdot 13$
455	=	$5 \cdot 7 \cdot 13$	725	=	$5^2 \cdot 29$	1015	=	$5 \cdot 7 \cdot 29$
475	=	$5 \cdot 5 \cdot 19$	775	=	$5^2 \cdot 31$	1025	=	$5^2 \cdot 37$
539	=	$7 \cdot 7 \cdot 11$	805	=	$5 \cdot 7 \cdot 23$	1045	=	$5 \cdot 11 \cdot 19$
575	=	$5^2 \cdot 23$	833	=	$7^2 \cdot 17$	1085	=	$5 \cdot 11 \cdot 31$

Concentrating on the occurrence of primes, the now explained phenomena can be simply considered to be the interscalar occurrence of unique $|z_j|1$ shown at the smallest interval scale of $|1|z_j$, where no other smaller $|z_i|1$ may occur along the path from $|z_j|1$ to $|1|z_j$. Given the uniqueness of $|1|\pi$, the authors are led to consider $1/|1|\pi \stackrel{?}{=} \{\quad\}$, and by extension, John Conway's surreal numbers (2001).

Our approach is novel in the sense that the authors come from disciplines other than mathematics (population genetics and management), and that we adopted a "grounded" methodology, starting from the partition of positive integers as suggested by Stein (1976, p. 21).

For quite obvious reasons algorithms are not provided, although they can be easily derived from our proof.

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