EX FABRICA ET RATIOCINATIONE: TÉCNICAS, TECNOLOGÍAS E INNOVACIÓN EN LA ARQUITECTURA ANTIGUA

Volumen I

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(Coordinadores)

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Andrea Moneti

Abstract

New exam of the peculiar case of dialectics between compositional laws and stylistic innovation of the transition from classicism to baroque. Applying categories of differential geometry to the geometry of the architectural spaces created by the two styles, it is possible to construe that classicism expressed them as *metric Euclidean spaces*, and baroque as *non-Euclidean* ones. This can lead to analyze this specific transition as an example of innovation through a coherent overturning of laws however always valid. This can suggest the use of this method in other cases in architectural history.

Keywords: Baroque, style, Architecture, Riemannian, Geometry.

Riassunto

Un nuovo esame del caso particolare di dialettica fra leggi compositive e innovazione stilistica rappresentato dalla transizione da classicismo a barocco. Applicando le categorie della geometria differenziale alla geometria degli spazi architettonici creati nei due stili, è possibile ritenere che il classicismo li abbia rappresentati come spazi metrici *Euclideie* il barocco come non *Euclidei*. L'uso di questo metodo potrebbe applicarsi anche ad altri episodi della storia dell'architettura.

Parole chiave: Barocco, stile, architettura, geometria riemanniana.

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1. INNOVATION THROUGH SYSTEMATIC OVERTURNING OF LAWS IN FORCE: THE ARCHITECTURAL BAROQUE

The phenomenon of style innovation, in architecture, sometimes appeared by the sudden establishment of mandatory composition rules, that immediately became academical. This happened for example at the beginning of the success of various classicisms; otherwise, the changing has taken the windward by the demolition of all the pre-existent rules, as for example with the advent of rationalist vanguards.

But there had also been some very interesting moments, like those discussed here, in which suddenly the exploitation of the academical laws began to express a vision almost opposite to the pristine aesthetical intent of those who had previously established them. The use of this perspective of interpretation can help the historians and critics of architecture and art but, in this specific case, the science historians too.

In order to examine this phenomenon it is in fact very useful to analyze the classical and baroque spatiality expressed by the rules in question using concepts that modern *Differential geometry* has come to isolate starting from mathematical discoveries that took place in those eras (the reader can deepen the nature of these terms using the words in *italics* in the text).

2. AN AESTHETIC LAW FOUNDED UPON THE VISUAL MEASURING OF SPACE: THE GREEK ROMAN AND RENAISSANCE ARCHITECTURAL ORDERS

Let us now examine the group of settlement rules founded upon the use of the Architectural Orders. As hopefully well-known, they were collections of forms and formulas (traditionally three or five), that could be used in association too, making up some definitive rules of drawing, correlation and proportion between pre-established rectilinear horizontal and orthogonal vertical elements, suggesting the coordinate statical function of basement-support-floor (resulting from the elementary wooden architecture).

As well-known, the Orders[1] were born in archaic Greece; and during the Classical period of the V-IV C. BC, they established themselves as accurately perfectioned rules, quickly codified in school treatises by many authors. During Hellenism, in III-II C. BC, their use spread in all the areas of Greek culture expansion, including Rome.

The romans immediately carried out a great fundamental evolution of this system, placing, subordinately to the Orders, positions and proportions of all

1. Newenham, 1963; Martin,1972; Chitam, 2005.

the new forms of covering by arches, and vaults and domes of semicircular section, that their building technology was experimenting[2].

The Order's use was so extended upon all the roman territory, till the end of IV C. AD. During the Middle age, in which the architecture furthermore evolved with inventions of new kinds of vaults under the Byzantine, the Arab and the Romanesque and Gothic art, the Orders survived only as decorative and traditional details of sporadic use.

At the beginning of the XV C. the Orders were in Italy suddenly recovered, also thanks to the finding of the sole surviving ancient treatise, the roman Vitruvius'. In the Renaissance the Orders and the Roman syntax that connected them to the vaulting systems, were re-proposed as the language of modern architecture by Brunelleschi and Alberti, with enormous success. From then on, they were again codified and disseminated through many new treatises[3].

At the end of the XVI C. their use was again widespread across all Europe and colonies, and thereafter maintained as rules till the first XIX C. Later, their use began again to become newly decorative, as quotation, as it survives also today.

The Orders use offered to the observer some simple, intuitive, elegant, and immediately interpretable optical guide, already in antiquity going beyond the real static needs. In the simplest cases, their use could consist in proportional standards outlines of volumes. In this way the architecture and the surrounding space too (as in the well-known painting of XV C. called *Ideal city* in the Galleria Nazionale of Urbino), were offered immediately measurable, and predictable even when hidden, to the observer.

In consistency with the rules of their employ, at the exordium of their use, both in Antiquity and Modern age and against the former traditions, a special partiality for volumes regulated by simple, translatory, bilateral or rotational symmetries (parallelepipeds, cylinders, hemispheres), and also the preference for orthogonal urban patterns was imposed. With the Renaissance, the fundamental typologies of house, palace and temple were thus re-founded and re-proposed in new standard regular models, adapted to the new spatiality.

Both in classical Greece and Renaissance, the spirit of a new rising culture, strongly anxious to push the society and the civilization toward new progresses, imposed these architectural rules. In both these ages flourished a world view putting at its core, with great trust, the human ability of rational thought and of perception of nature. The architectural Orders were then employed as

2. Victruvius, 1486; Newenham, 1963; Mac Donald, 1982-6; Chitam, 2005; Moneti, 1987.

3. Alberti, 1485, ed. as manuscript in 1452; Wittkower, 1949; Murray, 1971; Borsi, 1973; Tavernor, 1998; Volpi Ghirardini, 2014; Vignola, 1562; Palladio, 1570.

effective and versatile tools that made it possible to mark for the observer, through the architecture, the geometrical properties of the physical space.

3. ARCHITECTURAL CLASSICISM AS AN EXPRESSION OF THE EUCLIDEAN METRIC OF SPACE

These properties are called *Euclidean*, from the Greek mathematician of the IV C. BC who wrote these laws, and they are: *Homogeneity* and *Isotropy* (invariance for translations and rotations), existence in each point of only one *parallel* to a given straight line, validity of the *Pythagoras's theorem*, which consents the measures of distances and angles, and sum of the angles of a triangle equal to 180°. In two-dimensional spaces, the surfaces, the planes, and the cones and cylinders also, that are rolled up plains, are Euclidean[4].

So, the Orders acted as some visual markers, applied to the architecture, of the instrument that in modern *Differential geometry* (specifically in the *Riemannian geometry*) is called today *Metric*, that defines its own method of measurement of distances in a Space of any dimension. Applying the Orders, Greek classicism and Renaissance made explicit the *Euclidean metric*, that is the natural one, actually due to the uniformity of the gravitational field in which it and we are immersed, that leads the rays of light in straight lines and generates linear statical laws. Upon the surfaces, Euclidean metric can be mapped as the regular squaring of a paper.

The Orders, by hinting to the function of support as opposed to gravity, precisely emphasized this. The aim of the promoters of their use was to show, together with architecture, all the spaces of human life as clear, regular, measurable, foreseeable also where not visible, and leading to rational and calculating forms of reasoning.

The Greeks had perceived, even before Euclid, that if our real space has these properties, what our two eyes perceived was slightly misshapen. Therefore, they introduced meticulously codified corrections (as the *entasis* of the shafts of columns), that imposed curved lines so that they would be then perceived by distance as rectilinear ones. Alberti and Brunelleschi, in Renaissance, invented and spread also a new projection system of the space upon a two-dimensional one: the Perspective with only one convergence point, a famous tool that sets an *Isomorphic* correspondence between two spaces, preserving the Euclidean properties[5].

The last period of stiff and academical use of Orders, the Neoclassicism of the end of XVIII C., supported Illuminism, the new philosophical rationalism

4. Euclides, 1482; Benno, 1999; Venema, 2006; Weyl, 1952; Einstein, 1920.

5. Panofsky, 1927.

that aimed to push humanity toward a new progress, and it became just a flag of that thought. The term "classic", that means "of school", so began to be used to define a style of use of Orders strictly anchored to their original rule of markers of the Euclidean properties of space; and the word "baroque", supposed derivation from Hispanic term for non-spherical pearls, began to be used to define the immediately preceding stylistic period, repudiated and condemned for its irrational deviations[6].

4. DERIVATION OF MODERN ARCHITECTURAL BAROQUE FROM ANTIQUITY

The architectural baroque began with the bold novelties proposed by some masters in Rome during the first half of XVII sec. mainly Borromini and Pietro da Cortona[7] and Fontana, Rainaldi, Bernini *(pic. 1)*, (and Caravaggio and Rubens…),

Pic. 1. The observer, and the statues of spectators on sides, assist, in our Euclidean space, to one of the levitations experienced by St. Teresa de Avila. The gravitational anomaly, with the attraction exerted from where the rays of light arrive on the body of the Saint, produces a deformation of the space around her. Einstein might agree. Note the triple curvature of the central aedicule. Cornaro's Chapel in S. Maria della Vittoria in Rome, architecture, sculptures and decoration of G. L. Bernini (author).

6. Middleton e Walkin, 1977; Bruer, 1989.
7. Wittkower, 1958; Brandi, 1969; Hoppe, 2003; Portoghesi, 1967; Cerutti Fusco e Villani, 2002.
8. Wittkower, 1937; Borsi, 1980.

all setting out to experimentation and innovation[8].

Rome, which in the previous 150 years had been renewed in the Renaissance style, still kept also the monuments and ruins that illustrated all the complex history of past use of the Orders during antiquity. As militants passionate and convinced about the use of these instruments, during research and studies on ancient Rome fragments, these masters perceived that in some cases the ancient ruins offered examples of use of the Orders that seemed different and heterodox when compared to the academic canons fixed during the previous century[9].

These cases were traces of an history that has had in fact some alternate phases[10]. In Rome, from the third to the first century BC, various novelties were introduced from Near Asia and Alexandria, innovating the classical rules, some of them sprung from scenography, with eccentric proposals: very close supports, intercolumniation invaded by smaller Orders, projecting beams upon free-standing columns, curved, hollow, broken pediments, and moreover a particular preference, adopted in Rome, for spaces dominated by hierarchical axiality, most of which to be enjoyed statically and visually only.

The result had been the introduction of new, interesting and agreeable sensations of uncertainty of positions and depths, modifying the space perception, and a neoclassical reaction, today known as Neoatticism or Augustean classicism, occurred against this trend during the I C. BC. But after a century, however, new schools of experimenters reappeared, in Rome too, where the works of Severus et Celer and Rabirius[11] were remembered. Above the Hadrian's empire, these ancient baroque masters flanked the classicist schools proposing new inventions: very hardly approached horizontal beams and shortened supports like in Petra[12] *(pic. 2)*, broken pediments upon single columns, screwed columns, curve beams both vertically and horizontally upon single intercolumniations, partitions and openings at segmental arch, and finally, in some cases, like in Villa Hadriana[13] *(pic. 3)*, full spaces in which the preceding typologies were re-proposed as submitted to curve deformations, as in condition of expansion and contraction, or even reciprocally intersected.

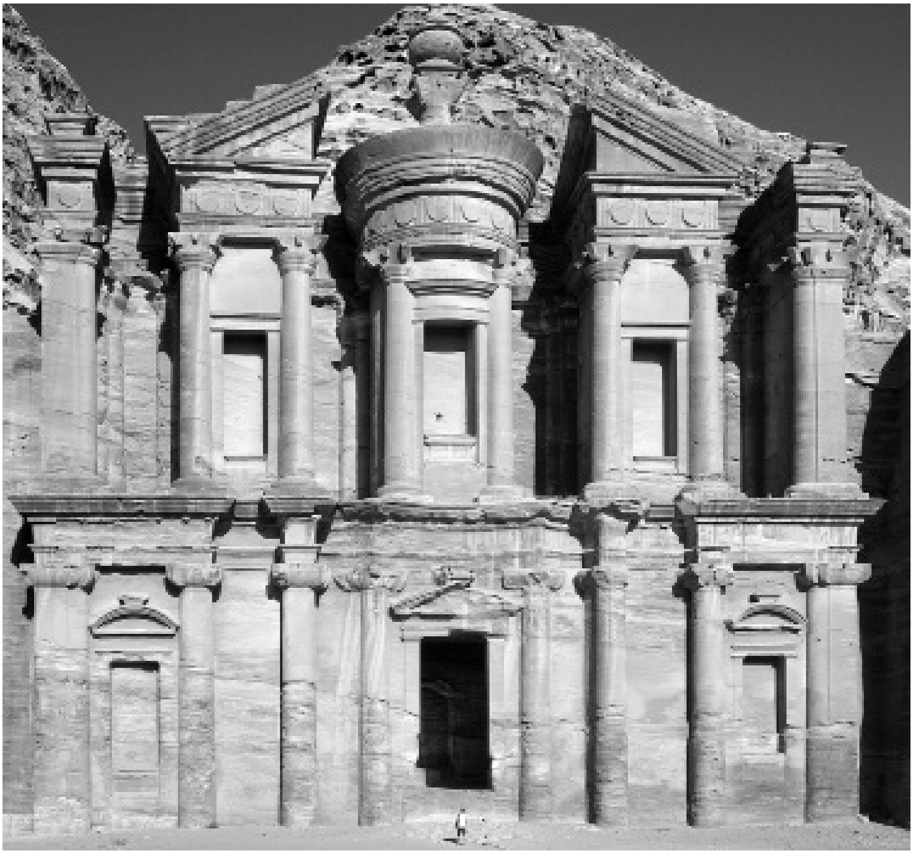
9. As examples of the archeological architectural researchs in the Rome of first baroque: Laurus, 1612; Montano, 1614.

10. Kugler e Burckhardt, 1847; Burckhardt, 1867; Pane, 1935; Ceschi, 1941; Robertson, 1963; Blunt, 1963; Lyttleton, 1974; Mac Donald, 1982-6, 1995; Moneti, 1987, 1992, 2020.

11. Mac Donald, 1982-6.

12. Mc Kenzie, 1990. In Petra and in Villa Hadriana single monuments are founded and edited before the general books here quoted.

13. Kahler, 1950. Mac Donald e Pinto, 1995; Adembri, 2000.



Pic. 2. Beyond the vertical plain of this rock-cutted front cease the laws of our Euclidean metric and another, very dishomogeneous, imposes itself. El Deir in Petra (author).

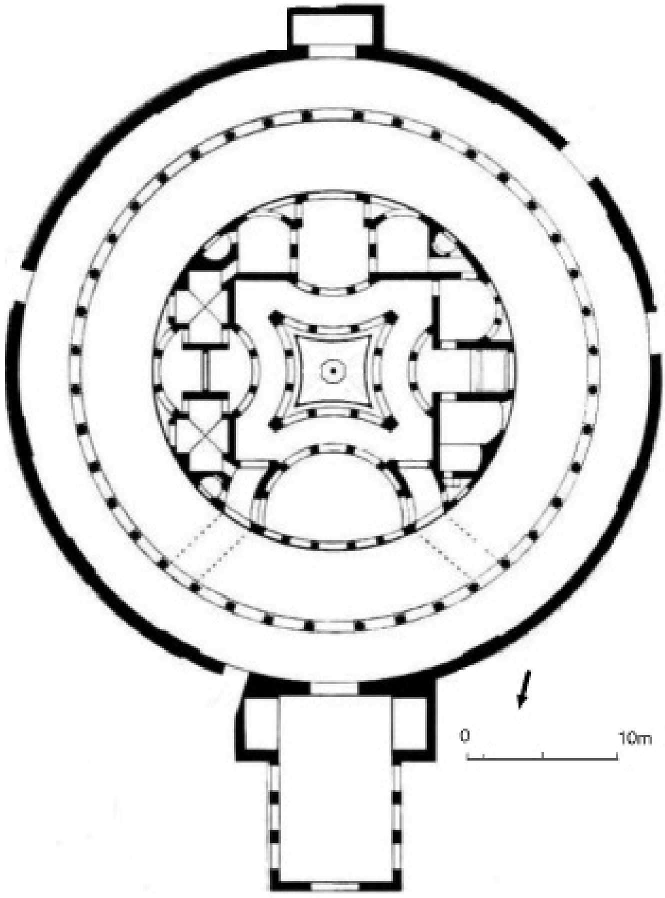
5. EVOLUTION OF MODERN ARCHITECTURAL BAROQUE

So, in early XVII C. Rome, the mentioned group of first baroque architects, attracted and intrigued by the eccentric proposals contained in these ruins, offered authentic concentrates of them, some true explosions of an unexpected spatiality, discordant with the Euclidean one of the classical schools, even recently presented like that one of Palladio[14], as in the Borromini's work[15] *(pic. 4)*.

Applying this newness to the typologies of temple, palace and square fixed and diffused during the XV and XVI C. (for example the model of church like S. Andrea in Mantua of Alberti or that of Gesù in Rome of Vignola), they obtained,

14. Palladio, 1570.

15. Portoghesi, 1967; Blunt, 1979.



Pic. 3. A traditional roman peristyle house transposed in a curved and pulsating space. The secret "Villa of the Island" into the Teatro Marittimo in the Hadrian's Villa in Tivoli (Kahler, 1950, t.6).

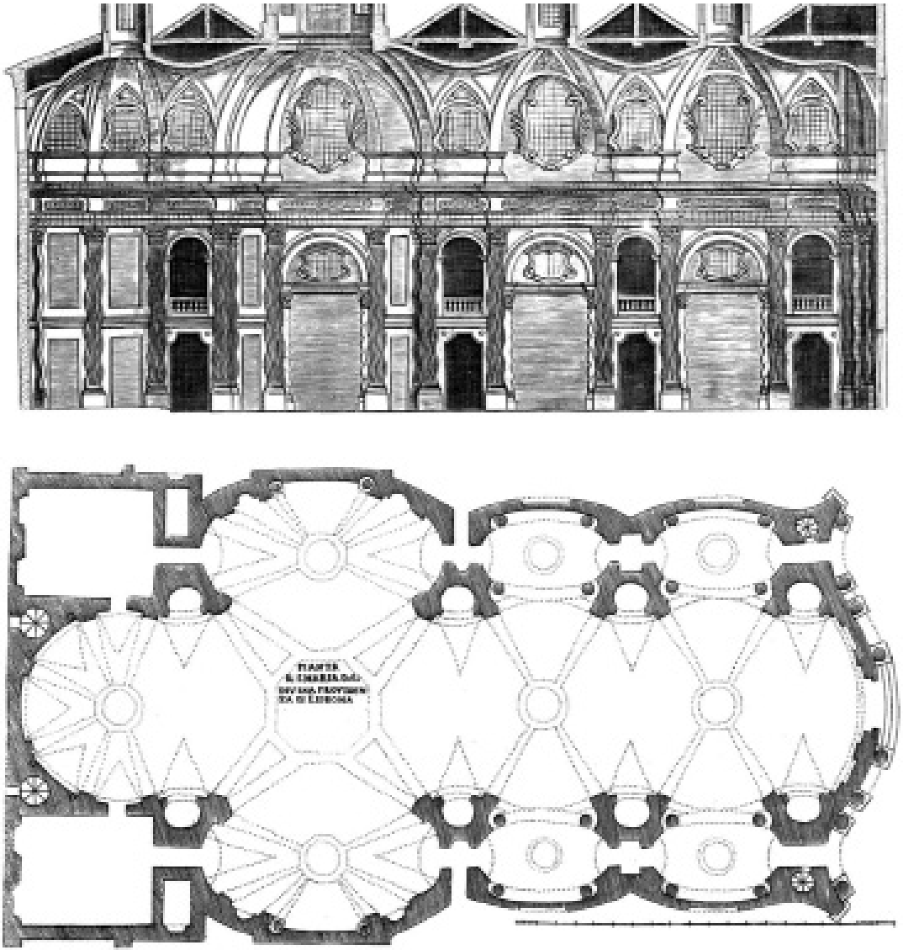
in accustomed observers to these forms, the inverted effect: the immersion in new spaces where the metric suggested by the Orders no longer seemed what was expected[16].

Their works, and those that, using the categories extracted by Norberg-Schulz[17], from these first examples will then be founded, both horizontally and vertically, upon some mutual pulsating juxtapositions and syncopated interpenetrations *(pics. 5, 6, 7)*, operated thus as experimental models of new, imaginary and

16. Upon the advantage of finding the stylistical changing long chronologial series of same typologies, Kubler, 1972.

17. Norberg-Schulz, 1972; Clagett, 2014; Compan *et al.*, 2015.

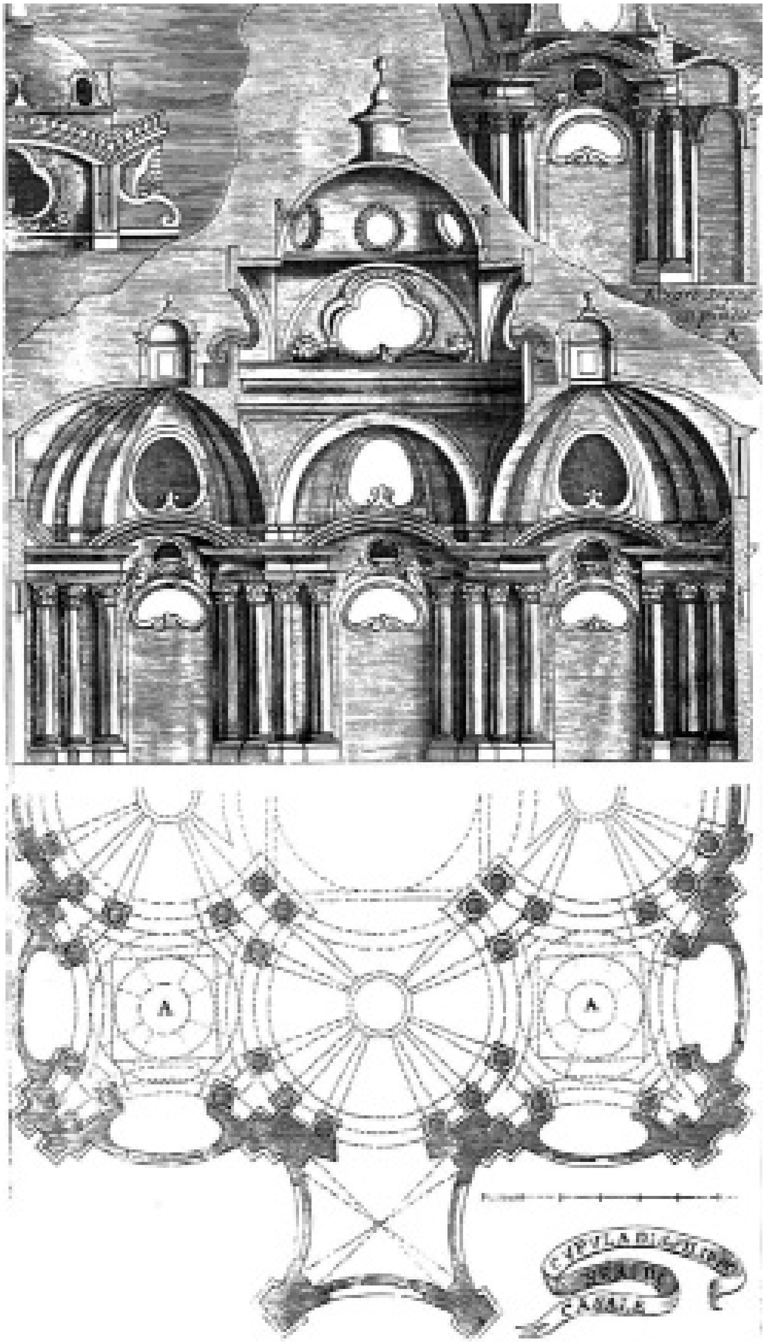
Pic. 4. A flat surface transposed in the curved metric of an undulated space. Front of S. Carlino alle Quattro fontane in Rome, of F. Borromini (author).



Pic. 5. A traditional longitudinal church transposed in a surprising curved pulsating space, covered by vaults with elliptical and hyperbolical geometry: S. Maria da Providencia Divina in Lisbon, of G. Guarini, (Guarini, 1686, t.17, 18).

hypothetical, inhomogeneous, anisotropic, and no longer predictable spaces. The lengths could appear locally stretched along vertical surfaces or optical directions, and some other phenomena, real like the light, or fantastic one like the evoked divine presence, could intervene to deform the Metric of space, and consequently the perception of gravity too, creating sensations of levitation or overweight of loads, and of an elastic and undulating behavior of space.

The Baroque, in modern Rome, unlike what happened in the ancient one after Hadrian, had a popular and explosive success, spreading everywhere in the European world. However, classicist customers and masters still continued



Pic. 6. A traditional cruciform church transposed in a curved space, with breakthroughs and interpenetrations of spatial cells. A project of G. Guarini (Guarini, 1686, t. 25).

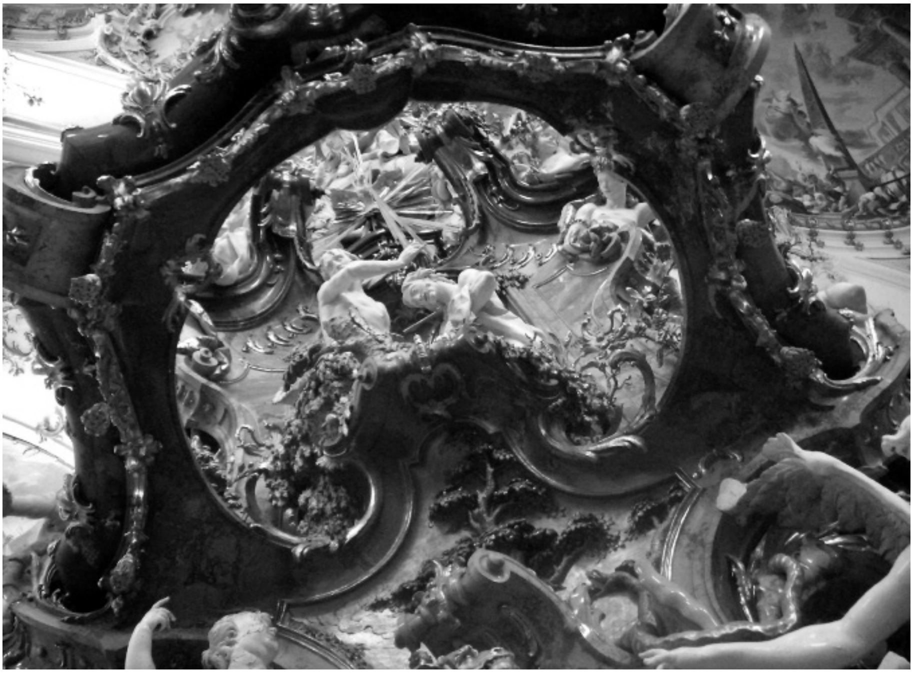
Pic. 7. Another example of a longitudinal church transposed in a pulsating curved space with elliptical and hyperbolic spatial cells; effect obtained through architecture, painting, sculpture: inner of St. Mikulase in Prague-Malonstranske, of C. and K. I. Dientzenhofer (author).

to follow the old rules, and so in many places a moderate and classicizing baroque took shape; but the result was to keep the prominence given by the surprise of the most extreme baroque works constantly bright.

After the first experimenters, others too picked up their examples pushing this vanguard into the field of the emotional, unexpected, hypothetically possible spaces; during the second half of the XVII C. we see the work of Guarini[18] *(pics. 5, 6)*, and during the first half of the XVIII those of Vittone[19], and finally, in Germany, the squall of inventions of the Dientzenhofers[20] *(pic. 7)*, the Asams[21] *(pic. 8)*, Fischer von Erlach[22], Neumann[23], Fischer[24] *(pic. 9)*, Zimmermann[25].

18. Guarini, 1676,1686; Meek, 1888; Brinckmann, 1932; Passanti, 1963; Compan *et al.*, 2015.
19. Scricco, 2014.
20. Gerhard, 1942; Vilimkova e Brucker, 1989; Norberg-Schulz, 1993; Clagett, 2014; Compan *et al.*, 2015.
21. Sauermost, 1986; Bauer e Dischinger 2005.
22. Kreul, 2006.
23. Brinckmann, 1932; Lampl, 1987; Reuters, 1950; Otto, 1979; Wieseneth, 2011.
24. Hundt e Ettelt, 1993.
25. Hoppe, 2003.

Pic. 8. The expansion of a curved space obtained in a very narrow site: gallery of the Asamkirche, in Muenchen, architecture and paintings of Asam brothers. The model of the column goes back to those now in the Tribuna in S. Pietro in Vaticano, of Hadrian's time (Bauer, 2005, 13, Roman von Goetz, Schnell & Steiner).



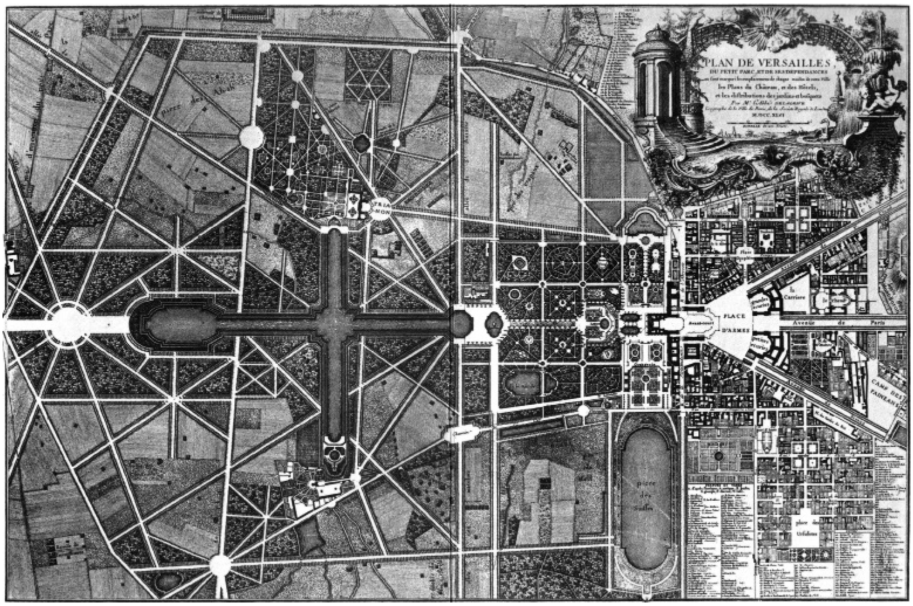
Pic. 9. Everything here is curve, but frames and the cymas are at their place, and so we are still in a metric space: the breakthrough of the support of group of Christ's Baptism, in the Basilica of S. Alexander und Theodor of Ottobeuren, architecture of J. M. Fischer, sculpture by J. M. Feuchtmayer (author).

The architectural decoration too joined the same program; the invention, at the beginning of XVIII sec., of the French *Rocaille* style, led to a proliferation of delicious forms that seemed to spread upon the surfaces as generated by undulating forces, like sea waves, and biological morphologies like those of shells or plants, following geometries founded only upon curve lines. Particularly in southern Germany, the use of Orders saw a turmoil of variations impossible to list, as bound to absolutely extemporaneous oddities, up to the deformation and almost fluidization of frameworks and mouldings[26] *(pics. 9, 10)*.

Also, the homogeneous urban spatiality of Renaissance was exceeded, and from the works of Fontana[27], frequently it was submitted to combinations of systems of rectilinear privileged axes, fixed on pre-eminent destinations, typically diverging or meeting themselves with acute angles *(pic. 11)*. In town and park planning, inventions of geometrical systems of runs and visual directrices at double fan began to spread from the works of the French Le Notre[28].

26. Baumgartner, 1761; Kimball, 1980.
27. Guidoni e Marino, 1979.
28. Delagrive, 1746; Hansmann, 1983; Hazlehurst, 1980; Jeannel, 1985.

Pic. 10. The young Gauss may have seen this engraving? *Geometry*, a rocaille-style allegory of J. W. Baumgartner (Baumgartner, before 1761).



Pic. 11. A concentration and a subsequent expansion of the space, with the many *rond-points* radiating like stars in the sky: town, Castle and gardens of Versailles, planned by L. Le Vau, J. H. Mansart, A. Le Notre. (Delagrive, 1746).

These systems, departing from or arriving to some crucial point, typically the entrance and the exits of regal residences, at first gathered and then branched out themselves, intersecting one another and diverging in the landscape for miles. Surprisingly, the baroque spatiality by curvilinear deformations remained limited only in the inners and the immediately outsides of buildings, while to a larger scale the Baroque preferred to open rectilinear axes and radial points toward the infinite, as if the landscape were a flat sky. Having become scarcely measurable locally, the baroque space returns such over long distances.

6. ARCHITECTURAL BAROQUE AS REPRESENTATION OF CURVE NON-EUCLIDEAN METRICS

Into *Euclidean space* (with Euclidean metric) the position of an observer of an object does not influence what he sees and measures about it. In a *Non-Euclidean* (or *curved*) *space* the *Geodetic,* the path of minimum distance that runs between two points, that in the Euclidean space is always a straight line, is generally a curve; this independently (*Gauss' Egregium theorem*) from the metric of a space that possibly contains this space, like that Euclidean of the observer[29]. In *Hyperbolical* curved metrics, infinite parallel geodetics pass in a single point; in *Elliptical* ones, no geodetic has a parallel. In a curved space an observer generally sees the metric changing, depending on where he looks or how he moves. Among the non-Euclidean two-dimensional spaces (surfaces), the triangles may have the sum of their inner angles superior to 180°, and then they are called *Ellipticals;* they are the spheres, the ellipsoids, and some paraboloids and hyperboloids; or they may have inferior sum, and so they are called *Hyperbolics,* and they are the tori and the remaining paraboloids and hyperboloids[30].

In architecture, all the surfaces of the vaults are compositions of geometrical figures, and classicism used only those of Euclidean and Spherical metrics. The modern Baroque immediately began to experiment juxtapositions and interpenetrations of spatial elliptic and hyperbolic cells *(pics. 5, 6, 7)*. While it remains possible to define in all these curved spaces a metric, and to map it for example with nets of geodetics lines, Baroque led the Orders to continue to carry out this visual task[31].

According to this, the vanguards of baroque architecture of XVII-XVIII C. could have suggested and explored three-dimensional spaces with curved metric, representing what physics has subsequently discovered, with Einstein, that the space of our Universe results generally non-Euclidean and may or may not appear like this only in local level[32].

To visualize the metric structure of spaces, we can use the modern methodology, introduced by Riemann[33]. To create a Metric, it is sufficient to compose an *operator* called *Metric tensor*, which for a two-dimensional space (the plan) is a matrix with 2x2 components which are the coefficients to apply to the Cartesian coordinates of position of a point. So, the Metric tensor of the Euclidean space has components: 0,1;1,0, and its Metric will be graphically displayable as the regular squared network *(pic. 12, A)*. To obtain non-Euclidean spaces it's enough to substitute the components with functions of variable parameters of the coordinates; so, inserting for example some sinusoidal

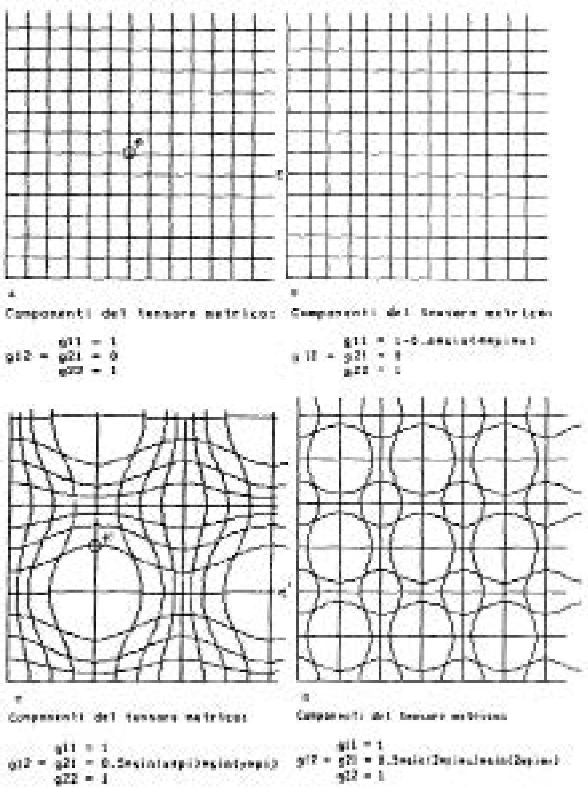
29. Gray, 1989; Do Carmo, 1992; Berger, 2003; Greenberg, 2007; Rovelli, 2014, 2021. Rovelli wrote me: *"Idea carina, che il Barocco suggerisca ed esplori spazi curvi. Gli spazi curvi a cui ti riferisci qui sono quelli come scrivi giustamente, che sono non omogenei studiati prima da C. F. Gauss nel caso semplice di superfici cioè spazi di dimensione 2, come la superficie di una zucca bitorzoluta, e poi da B. Riemann nel caso generale di spazi curvi di dimensione 3, 4, o più. Poi Einstein ha avuto la bella idea che lo spazio fisico in cui viviamo sia effettivamente curvo così, e oggi ne siamo tutti convinti. L'idea di Einstein è che il fatto che lo spazio, più precisamente lo spazio-tempo, si curvo è esattamente la causa del fenomeno che chiamiamo gravità*". Gauss, 1828; Riemann, 1867; Einstein, 1916.

30. Do Carmo, 1992; Greenberg, 2007.

31. As example of treaty upon the prescriptive geometrical use of Orders in baroque age see just that one of Guarini. Guarini, 1737.

32. Einstein, 1916; Wald, 1984; Rovelli, 2021.

33. Riemann, 1867; Do Carmo, 1992; Berger, 2003.



Pic. 12. *Mappings of metrics in two-dimensional spaces: A) Euclidean, B), C), D) hyperbolic. (Moneti, 1987, t. 2, 3, 4, Massimo Ferri).*

functions, the square network begins to show periodical concentrations and dilatations *(pic. 12, B)*, and these are the schemes underneath the baroque architecture of many 2-dimensional flat fronts, like in Sant' Andrea della Valle in Rome[34] *(pic. 13)*. Then, applying for example some products of sinusoids of both the coordinates *(pic. 12, C, D)*, the squared network begins to present curved undulatory lines, and to alternate areas of compression and expansion, like in some plans in the Hadrian's Villa of Tivoli[35] *(pic. 3)*, or in the works and projects of Guarini[36] *(pics. 5, 6)* or K. I. Dientzenhofer[37] *(pic. 7)*.

The Metric tensor of our three-dimensional space results to possess non-Euclidean components: the functions that describe the Gravitational field, field

34. Wittkower, 1958.
35. Kahler, 1950; Macdonald 1995; Moneti 1987; 1992; 2020.
36. Guarini, 1686.
37. Norberg-Schulz, 1993.

Pic. 13. A typical manierist front of a church transposed in a new metric, curved along the horizontal dimension: front of S. Andrea in Valle in Rome, of C. Maderno and C. Rainaldi (author).

of force that can bend the light rays[38]. The Orders, as visualization of Metric of the geometrical space, bounded to the statical functions alluding to the Gravity, both in the Classic and in the Baroque styles expressed the real natural identity between Geometry and Gravitational field, both in the Euclidean and local non-Euclidean characters *(pic. 1)*.

38. Wald, 1984; Rovelli, 2021.

7. BAROQUE AGES AND COEVAL SCIENCE

Now, coming back to the ancient and modern ages of the architectural Baroque, we look at the relationships between this style and the mathematical and natural sciences. It is common knowledge that our present Era of Science begins in the XVII C.[39]; but it is interesting to observe that in Antiquity was maybe Euclid himself, who was sure in defining the oneness in a point of the straight line parallel to another one as Postulate and not as Theorem (in fact not demonstrable), to realize that the most general Metric of a space could not reduce himself to the space by itself described.

Unfortunately the ancient scientific texts have generally made the same bad end as the architectures, but at the end of I sec. Menelaos Alexandrian wrote actually the first complete non-Euclidean geometry, the spherical one, and Hadrian had at his service at least one architect who experimented surfaces with hyperbolic metric[40].

During the Modern age, the years of the birth of Baroque are the same of Galilei, founder of modern mathematical physics, and Descartes, inventor of the *Analytical geometry*[41]. Huygens, the discoverer of the undulatory nature of light[42], Newton, the first theorist of Gravitation[43], and Leibniz, inventors of the *Differential calculus* and *geometry*[44], Euler, the first inventor of the concept of *Curvature*[45] and Lagrange of that of geodetic[46], all lived during full baroque age, and the first mathematic experimenter of non-Euclidean metrics was eventually Gauss, born in the Germany of the sunset of baroque world[47] *(pic. 10)*.

The sensitivity of great baroque architects was clearly not only artistic but also scientific; the needs of geometrical and statical calculations posed by their complex and hazardous projects required that they were also mathematicians and engineers. However, these masters really guessed and predicted, starting only by our interior structure of thought, some subsequent fundamental discoveries about the nature of Universe, managing to express and transmit these sensations with admirable courage, clearness and beautifulness, obtaining also an open satisfaction of public and purchasers.

39. Sarduy, 1987; Whitehead, 1929; Garber e Ayers, 1998.
40. Menelaus, 1758. The writer is in the process of a research in wich in the field of the curved geometry are emerging many new links between the scientific school of Alexandria and the Hadrianic architecture.
41. Sbaratt, 1994; Heilbron, 2010; Gaukroger, 1995; Monnoyeur, 2017.
42. Andriesse, 2003.
43. Westfall, 2007.
44. Deleuze, 1988; Rutherford, 1998; Antognazza, 2008.
45. Euler, 1767; Bradley e Sandifer, 2007.
46. Borgato e Pepe, 1990.
47. Gauss, 1828; Hall, 1970.

But please note that the indispensable starting pedestal from which these expressive innovations could fly, was exclusively the preventive zealous adhesion to a pre-established system of rational laws in force: the architectural Orders, not, we repeat, intended like today as decorations, but as they were at their times, as complete syntactic system of compositional laws.

Only this intuitive and rigorous instrument could consent to express rationally and effectively innovative ideas and formerly disregarded inspirations. This prevented these ideas from being lost in the indifference of a visual world of disparate, bizarre, more or less complexes and originals forms, thus without a common rational language that is required in order to understand them, like for example in the eclectic architecture of XIX C. and in that of present days of first XXI C[48].

8. ARCHITECTURAL AND PICTORIAL BAROQUE SPACE

Now we can note to what extent this sensitivity-expressed by great vanguard masters of baroque architecture for the expression of curve spaces, connects with the spatiality expressed in their contemporary art of painting (and remember that Piero da Cortona was also one of the beginners of baroque painting, and Bernini of sculpture[49]). At first we can see the direct illusive continuation of the same baroque spatiality upon the walls, ceilings and vaults, even more virtuously and imaginatively elaborated, in the painted decoration of architectural subjects, in Italian *quadrature*[50]. This genre had one corresponding during Hellenism in the s. c. Pompeian second style, utilizing perspectival non-isomorphic methods with vertical convergence axes[51].

So, naturally, the same relation with the architecture, also in this case reproposed as surprising and not more predictable, can be seen in the baroque theatrical scenography, like the one of the Bibiena family[52]; and the Baroque generally finds new and inspired expressions through the unity of architecture, sculpture and painting, proposed by Bernini, with intersections at the edge of being discernible, with painting and sculptures stepping over their architectural frames.

So, we can say that the baroque architecture absorbed and exploited very appropriately the role of opening the space of the painting bound to the Albertian perspective, recognizing in it the same breakthrough of baroque

48. Fabi, 2014, but the simple concept of "curve" is not enough without that one of Metric. Lanzara, 2019.
49. Wittkower, 1958.
50. Farneti e Lenzi, 2006.
51. Vitr. *Arch*. VII, 5,5; Mau, 1882; Panofsky, 1927.
52. Lo Bianco, 1992; Lenzi e Bentini, 2000.

landscape architecture for visions directed to the infinite. So, the Woelfflin's conception of the evolution of painting from XVI to XVII C. as a transition from Closed to Open forms, is in coherence with all of this[53].

The paintings, like those of the same Cortona, or Tiepolo, representing the breaking of vaults of baroque buildings towards fantastic skies in which free crowds of human figures are flying[54], aimed precisely to contribute to communicate to the observer the same idea of baroque architecture of spaces in which the gravity was suddenly changed *(pics. 7, 8)*.

9. POETICS OF THE CURVED GEOMETRY OF BAROQUE SPACE

But, in the final analysis, the research of the baroque painting for the expression of natural shading of the human states of mind[55], is also in coherence with this architectural poetics of changing spaces, modelled also by human sentiments. Hence a new perspective of the aesthetic interpretation of baroque spatiality. Classicism expressed the preference for and the trust in a regular, stable, always potentially knowable, but also monotonous Universe; Baroque, and its poetry prefers to see an irregular, stupefying, and so inhomogeneous, but always measurable Universe.

The Classicism prefers to dialogue with the rational man in an Euclidean space; the Baroque, yet starting from the same rational man awareness about himself and the world, loves also the man who feels the force –we could say the gravitation– of his own emotions, fantasies, imaginations, but who as the same time, thanks to his rationality, manages to enjoy this without losing the tools of judgment and without giving in to feelings, especially the negative ones as will be in Romanticism and also later *(pic. 1)*. So, the baroque spatiality was the kingdom of rationalized expression of positive emotions: pathos, amazement, fantasy, playfulness, and so, like in a few other cases in art history, of really bizarre and innocent cheerfulness *(pics. 7, 8, 9, 10)*.

And it was too, maybe, a style expressing a certain idea of its own about transcendence. In fact the goal of the search for fantastic spatiality certainly had roots in Christian faith in the existence, for us, of an otherworldly world. If architectural Baroque spread from Lisbon to St. Petersburg, jumping political, linguistic, religious boundary, there are no doubts that the places where it successfully concentrated in small space its inventiveness and simultaneously offered it to all the people, were mostly the catholic churches; and Baroque

53. Woellflin, 1888, 1915; Barilli, 1982.
54. Spinosa, 1981.
55. Anceschi, 1984; Battistini, 2000.

was born and received the first impulse in the Rome of some popes who, if on the one hand committed the embarrassing incident of the condemnation of Galilei, on the other hand are recognized as fundamentals experts, promoters, and spiritual inspirers of innovation in visual arts[56] *(pic. 1)*.

This fact will encourage judgments of irrationalism to Baroque from the neoclassical illuminist side[57]. In a manner similar to the illuminism, the Renaissance had moved criticism towards the previous style since then called gothic (i. e. barbarian, uncivilized)[58]; and actually the baroque architecture recovered many gothic heritages, and dialogued willingly also with the other contemporary cultures. Indeed, we can say that the insert, into the baroque house fittings out, of paintings in Chinese and Japanese style (also reproduced in Europe) expressed the first temptation of circumstantial escapes from the Albertian perspective, reproducing Euclidean space[59].

10. CONCLUSIONS

The classic architectural orders, imposing definitive rules of correlation and proportion between pre-established horizontal and vertical elements suggesting a statical function, created some conventional optical guides that expressed the architecture as immediately measurable, predictable even when hidden, as well as the surrounding space. So, in several places and periods they performed the function of markers of what in mathematics is called Metric of a Space. Into the rational view of Greek classical, then humanist culture, their employment made it possible to render, by means of architecture, the Euclidean metrical properties of our space: homogeneous, isotropic, in which for one point only one parallel straight line to another one exists, in which the Pythagoras' theorem is valid, and that is as well subjected to the uniform gravitational field (that actually is exactly what induces the Metric). According to this analogy, the innovations introduced during the XVII C. by the foremost masters of Baroque, nonetheless rigorously founded upon the orders' use, could be interpreted as experiments of sights of new, imaginary, inhomogeneous, anisotropic spaces, with no more foreseeable structure, in which others phenomena also, real like the light, or fantastic as the divine presence, can deform the Metric of space, and consequently the gravitation too, perceiving so what the modern physics has recently revealed: that the space of our Universe is Euclidean only locally.

56. Bell e Willette, 2002; Signorotto e Visceglia, 2002.
57. As example of anti-baroque polemic: Milizia, 1785.
58. Vasari, 1568; Kruft, 1988.
59. Guerin, 1911; Porter, 2002.

It is known that architecture and mathematics influenced each other and realized sophisticated amazing and absolutely fantastic spaces even in many other periods of architecture history, like the Byzantine, the Arab, the Gothic, and later during the Art nouveau, and in the styles of the Vanguards of XX-XXI C. It is possible that the intentionally restricted example proposed here, concerning one application of the modern concept of classification of spaces to the characters of built spaces, may contribute to the studies of the peculiar characters of many others styles, cultures and epochs too.

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